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Thermal Regime of Ice Covered Swedish Lakes

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Lars Bengtsson
Dept. Water Res. Eng., Lund University, Sweden

Thorbjörn Svensson
Dept. Environmental Sciences, Univ. of Karlstad, Sweden

Temperature conditions and heat fluxes in ice covered lakes are discussed analyzing measurements in eight Swedish lakes. Heat fluxes from sediments and heat fluxes from water to ice are determined from temperature profiles. The contribution of solar radiation is estimated from heat-budget calculations. It is found that the heat content of most of the lakes changes very little when they are ice covered, but that the lake-water temperature slightly increases. All heat fluxes are small. The heat flux from the sediments is the highest flux in early winter, but is later in the winter balanced by the heat loss from the water to the underside of the ice. Solar radiation is an important heat source in late winter, when the snow cover is thin.

Introduction

Although lakes in northern Europe, northern US and Canada are ice covered for several months of the year, little interest has been devoted to ice covered lakes. This is probably because there is not much biological activity and hardly any water movements in an ice covered lake. The ice cover protects the water from direct wind mixing and insulates against intense heat loss from the water to the atmosphere. However, some movements take place. The wind sets up the ice cover and induces seiches, which influence mixing conditions. Through-flow induces currents and affects stratification within the water body. The bottom water gains heat from the bottom sediments. Therefore, most lakes become warmer from the time of freeze-over to the time of break-up. The heat flow from the sediments induces very slow currents moving along the bottom towards the deeper part of a lake. Some solar radiation may penetrate the ice and some heat is lost from the water to the ice. In this paper the thermal regime of ice covered lakes is treated. Measurements from Swedish lakes are analyzed. Heat flows from the sediments to the lake water, heat losses through the ice and by the outflow and solar radiation through the ice are discussed.

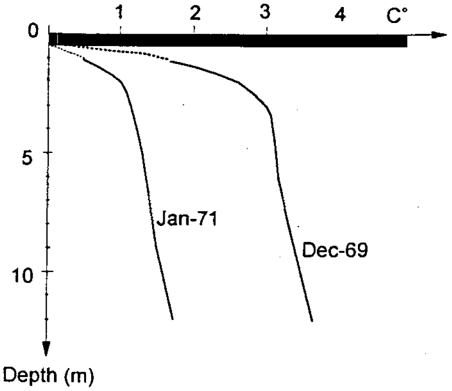

Fig. 1. Temperature profiles in Lake Velen two weeks after freeze-up.

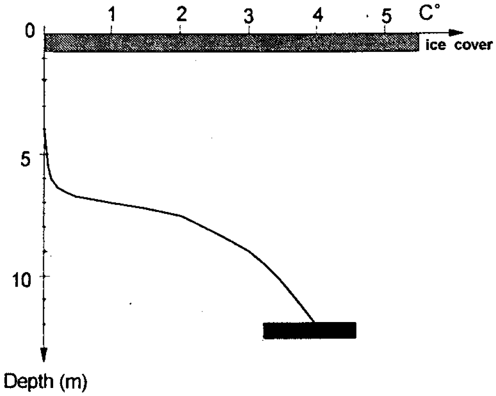

Fig. 2. Temperature profile in Lake Prästholm, April 1978.

Water Temperature in Ice Covered Lakes

Heat loss from lake water through an ice cover to the atmosphere results mainly in ice thickening and not in water cooling. The lake water beneath the ice cover is stably stratified, with temperatures increasing towards (or because of increased salinity) above 4°C near the bottom. If soon after the autumn turn-over the weather is very cold and there are no winds, ice is formed when the main part of the water body is still near 4°C, as is shown in Fig. 1 for Lake Velen two weeks after freeze-up in December 1969. From then on the ice cover acts as an isolation cover and the lake water remains warm. On the other hand, if it is windy during autumn and early winter with air temperatures fluctuating around 0°C, the entire water mass is cooled down much before the lake freezes over, which is also illustrated for Lake Velen in Fig 1. Lakes in northern Sweden, where freeze-up takes place early in the winter, are often warmer in the winter than lakes in southern Sweden, which may only be ice covered for a few weeks.

If there is considerable river inflow to a narrow lake during winter, the light river water of 0°C temperature will flow just beneath the ice cover. A pronounced thermocline may develop separating cold through-flow river water and warm bottom water as is shown for Lake Prästholm in Fig. 2.

As was pointed out by Hutchinson (1957), once a lake, which does not have any warm inflow, is frozen, winter heating takes place. Numerous investigators have recorded this phenomenon, *e.g.* Likens and Ragotzkie (1965) from Wisconsin, Bilello (1968) from Michigan. In some relatively shallow lakes bottom water temperature above 4°C has been reported, *e.g.* Hutchinson (1941), Thanderz in Falkenmark (1973). It seems that diffusion of solutes out of muddy sediments takes place, rising the density so that bottom water can exist at above 4°C without causing instability. Lower bottom temperature than 4°C is frequently reported.

Heat Balance of Ice Covered Lakes

When a lake is ice covered, the thermal exchange between water and atmosphere results mainly in the buildup or thawing of the ice cover, but causes little changes in water temperature. The small heat exchange with the bottom, heat from groundwater inflow, solar radiation penetrating the ice, and, even if almost molecular, the heat transfer from water to ice becomes decisive factors in determining the water temperature of ice-covered water bodies. The heat balance for the water of an ice covered lake is per unit area

$$\frac{dHC}{dt} = \int H_b \, dA_b - H_i A_s + \rho c (Q_i T_i - Q_o T_o) + R_i A_s \tag{1}$$

where

HC – heat content of lake water beneath the ice cover,
t – time, $H_b \equiv$ heat flux from bottom sediments to lake waters,
A_b – bottom area,
H_i – heat flux from lake water to ice cover,
A_s – surface area,
ρ – density of water,
c – specific heat of water,
Q_i – inflow,
Q_o – outflow,
T – temperature with index i for inflow and o for outflow,
R_i – solar radiation that penetrates through the ice cover.

For Lake Velen, Sweden, from which measured data are discussed later, the increase of the heat content over the ice covered period is usually somewhat less than 300 cal/cm^2 (12×10^6 joule/m^2) with reference to the surface area, HC/A_s, which almost coincides with the number (290 cal/cm^2) given by Bilello (1968) for Seneca Lake, Michigan. However, the rate at which the heat content increases depends on the climate and on the morphometric properties of a lake.

During the summer the lake water gains heat. Some of the heat is transferred to the bottom sediments. When in late autumn and early winter the lake water becomes colder than the surface sediments, heat stored in the sediments is continuously lost back to the lake water. As the sediment temperature decreases and the temperature of the bottom water increases in the course of the winter, the rate of the heat flow from the sediments to the lake water decreases. The heat budget of the sediments of Lake Velen, *i.e.* the heat that is yearly stored and released from the sediments, corresponds to about 2,500 cal/cm^2 (100×10^6 joule/m^2).

The heat flux from the water to the ice has often been considered as negligible. However, a pronounced temperature gradient usually develops close to the underside of the ice. Even if the heat transfer is purely molecular the temperature gradient for Lake Velen shown in Fig. 1 indicates a heat flux of about 1 W/m^2. As the surface water becomes warmer during the winter, the heat flux to the ice is increased. As will be shown in proceeding sections, higher values have been estimated for other Swedish lakes.

The inflow-outflow usually has a cooling effect on a lake. The inflowing water is very close to 0°C, but depending on the outlet conditions the water leaving the lake may have at least a temperature of some tenth of a degree. If for example a lake has a surface area of 3 km^2, the through-flow is 0.3 m^3/s (hydraulic load 10^{-7} m/s) with inflow temperature 0°C and outflow temperature 0.5°C, the net heat loss due to the through-flow is 0.2 W/m^2, which is one order of magnitude lower than the heat flux from the sediments and the heat flux to the underside of the ice referenced above. If the lake is dominated by through-flow so that the inflow-outflow in the above example is 30 m^3/s (hydraulic load 10^{-5} m/s), the lake water of the top metres cools down to very close to 0°C, and thus the heat loss is reduced.

Solar radiation can contribute a substantial amount of heat to the water beneath the ice cover when the ice is black ice, free from snow. If the ice is snow covered, much of the incoming solar radiation is reflected at the snow surface and the reminder part is almost completely absorbed in the snow. Hathersley-Smith *et. al.* (1970) discuss examples of direct warming by incoming radiation occurring in lakes of the high Arctic. Pivovarov (1972) gives extinction coefficients of about 1 m^{-1} for black ice and 7 m^{-1} for white ice. Intense incoming solar radiation during spring may be 200 W/m^2. If 10% is reflected at the ice surface, the radiative flux to the lake water through 50 cm white ice free from snow is 5 W/m^2. The flux penetrating 50 cm black ice is 120 W/m^2. Thus, when black ice is free from snow, the heat gained from the atmosphere exceeds by far the heat released from internal sources. The atmospheric heat gain is partly compensated for by increased heat loss from water to ice.

If there is alluvial material on the bottom of a lake, groundwater may contribute to temperature rise in an ice covered lake. However, even if the groundwater inflow to a lake of 3 km^2 surface area is 0.03 m^3/s (hydraulic ground water load 10^{-8} m/s) and the groundwater is 5°C, the heat flow per unit surface area is only 0.2 W/m^2. Since, in Sweden this is a large groundwater flow for a lake of 3 km^2, it seems that the internal source of heat from bottom sediments is more important than heat contribution from groundwater. The lakes which are discussed in more details in this paper have muddy sediments on the bottom. For these lakes the groundwater inflow has to be very minor. Water balance calculations for these lakes do not indicate any groundwater intrusion.

Investigated Lakes

Lake Velen in mid Sweden was intensively studied in the late 60's and early 70's as reported by Falkenmark (1973 *ed.*). The study included continuous water temperature measurements in several verticals and measurements of sediment temperature profiles. The lake has an area of 2.8 km^2 with a length of about 6 km and a maximum width of 1.1 km. The maximum depth is 18 m and the mean depth is 6.5 m. The main lake basin is separated from the northern basin, 9 m deep, by a sill at 6 m depth. The annual average discharge from the lake is 0.3 m^3/s. In most winters it is as low as 0.1 m^3/s. Winter data are available from 1969 to 1972.

Lake Prästholmselet and *Lake Degerselet* are two lakes in the Råne River basin in the very north of Sweden, with large through-flow 6-12 m^3/s in relation to the volumes of the water bodies. Lake Prästholmselet is 3.0 km^2 with a mean depth of 9 m. Lake Degerselet is larger 10.9 km^2 with a mean depth of 6 m. Measurements were made in 1976-78.

Near Göteborg, the ice covered lakes *Tulebosjön, Landvettersjön* and *Bredvattnet* were studied 1983-86. The results are reported in Swedish, Svensson (1987). Lake Tulebo is small, 0.3 km^2 and almost cone shaped with a maximum depth of 8 m. The mean winter discharge is about 0.2 m^3/s. Lake Landvetter is 2.6 km^2. It has two subbasins. The maximum depth of the northern part is 21 m, of the middle-southern part 12 m with a separating sill at 8.5 m. The mean discharge during the investigation period was 2.2 m^3/s. The lake is regulated. Lake Bredvattnet is small, 0.3 km^2 with maximum depth 8.5 m and mean depth 3.4 m. Its upstream catchment is only 0.6 km^2, which means that the river through-flow is very small.

Lake Boren is a rather large 25 km^2 and shallow lake, maximum depth 12 m, with large through-flow, 40 m^3/s, situated in the Motala River basin just east of Lake Vättern. Winter measurements were made 1984-86. In *Lake Erken*, just north of Stockholm, measurements were carried out 1984-85. The area of the lake is 23.7 km^2, the maximum depth is 20 m, the mean depth 9 m.

Sediment Heat Flux

The lake sediment gains heat in the summer and releases heat to the lake water in the winter. Thus, the sediment temperature varies periodically over the year with a high amplitude near the lake bottom and a smaller amplitude deeper into the sediments.

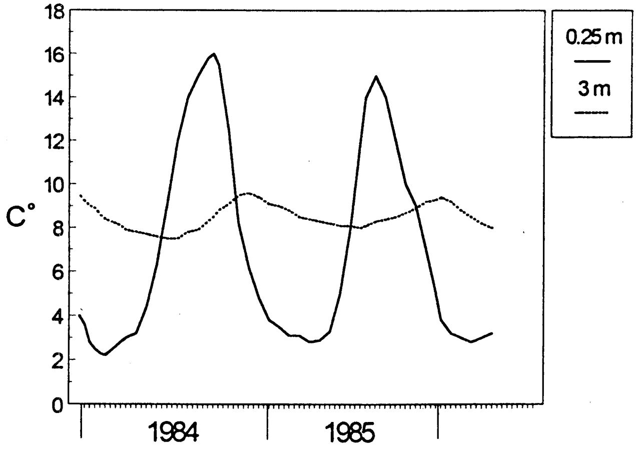

Fig. 3.
Sediment temperature 0.25 and 3 m below the bottom at 7.2 m depth in Lake Tulebo.

The sediment temperature variation at 0.25 m and 2.80 m below the lake bottom where the water depth is 7.20 m in Lake Tulebo is shown in Fig. 3. The bottom water temperature varies between 1-3°C in the winter to 10-12°C in the summer. The sediment temperature at 0.25 m below the bottom varies from 3-4°C to 10-11°C giving an amplitude of 3.5°C. The temperature at 2.80 m below the bottom is almost constant 7°C, with an annual amplitude of 0.5°C. On shallow water, 3.5 m, the temperature amplitude is much higher. The sediment temperature 0.25 m below the bottom varies approximately between 3°C and 17°C (amplitude 7°C), and at 2.80 m below the bottom between 7°C and 11°C (amplitude 2°C).

The sediment of Lake Tulebo is detrius gyttja with layers of sand at the shallow bottom and with clay at the deep bottom. The organic content is about 20% except for the surficial sediment layer at 3.5 m depth where it is 7%. From samples, the heat conduction coefficient was determined to 0.6 W/m, °C for the sediment (down to 50 cm) from the deep bottoms, and for the shallow bottoms to 0.8 for surficial (5-15 cm) sediment and 0.6 W/m, °C for deeper sediment (15-35 cm under the lake bottom). The heat flow to the water is determined from the temperature gradient below the water-sediment interface. The variation of the heat flux over the ice covered season is given for Lake Tulebo in Table 1.

Table 1 – Heat flux from sediments to water in Lake Tulebo (W/m^2)

		Nov	Dec	Jan	Feb	Mar	April
3.5 m	1983/84	4.0	4.0	4.0	3.0	2.4	0
7.2 m		2.0	2.5	2.4	1.5	1.0	0
3.5 m	1984/85			4.1	2.3	1.6	
7.2 m				1.9	1.0	1.6	
3.5 m	1985/86		3.5	3.5	2.5	1.8	1.8
7.2 m			2.0	2.5	1.5	0.8	1.0

Table 2 – Heat flux from sediments to water in Lake Landvetter (W/m^2)

		Jan	Feb	Mar	April
3 m	1983/84	6.5	2.6	2.3	1.5
8 m		3.6	2.8	2.2	0.5
16 m		3.3	2.1	1.4	0.5
3 m	1984/85	9.0	7.0	5.7	4.4
8 m		3.6	2.4	2.4	1.7
16 m		1.9	1.3	1.4	1.0
3 m	1985/86	9.6	7.2	4.8	3.2
8 m		3.2	2.0	1.8	1.5
16 m		2.2	1.9	1.7	0.6

The same kind of measurements as the ones in Lake Tulebo was made also in the larger and deeper Lake Landvetter. The temperature of the bottom water is 2-3°C in the winter and 5-6°C in the summer, but increases to about 10°C during a short convection period in the autumn. At shallow bottoms and at mid-depth the temperature range is from about 3-4°C to 14-16°C, and in the deepest part of the lake 3 to 8°C. The heat conduction coefficient was determined on soil samples to be 0.7 W/m, °C from the bottom surface down to 0.6 m below the bottom at all depth, although slightly higher at mid-depth. The sediment heat flux as calculated from measured temperature profiles in the surficial part of the sediments is presented in Table 2.

Sediment temperature profiles were measured also in the small Lake Bredvattnet. The water below the ice cover tends to be 4°C from about 2 m depth downwards already some weeks after freeze-up. Therefore, the heat flux from the sediments to the lake water is about the same at different depth. It was determined to be 2.5 W/m^2 in very early winter and about 1.5 in mid-winter.

In Lake Boren sediment temperatures were measured only at 5.2 m water depth, where the heat flux was determined to be 2.0-2.6 W/m^2 in February-March of 1985 as well as for the same period in 1986. Sediment temperature profiles were measured in Lake Erken during a student course in mid-winter 1985 and in the period March-April 1984. The heat flux was as for most of the lakes about 2 W/m^2.

The temperature in the sediment of Lake Velen was measured down to 4.5 m below the bottom at 4 stationary positions in the lake and at 6 other positions using a mobile equipment. The equipment and the profiles are presented in Falkenmark (1973). At depth exceeding 10 m, the sediment temperature deeper than 2 m below the lake bottom is 6-7°C throughout the year. At shallower depth, 2-6 m, the temperature 2 m below the bottom varies between 5 and 10°C. The surficial (0.25 m) annual sediment temperature range is 4-7°C in the deepest part, 4-9°C at mid-depth and 3-17°C at 3 m depth.

The surficial sediment of the bottom of Lake Velen consists of detrius gyttja of 90% water content by weight, corresponding to a porosity of 90%. This mud layer is

Table 3 – Heat flux from sediments to water in Lake Velen (W/m^2)

		Jan	Feb	Mar	April
3 m	1970	2.5	2.0	1.7	1.6
3 m	1971	3.2	2.0	1.7	0.5
11 m			1.3	0.9	
17 m			0.8	0.8	
3 m	1972	3.3	2.9	2.0	

0.5 m in the shallow parts underlain by till and 3 m in the deepest part underlain by silty clay. Since the water content is high, the conduction coefficient is assumed to be that of water. The calculated heat fluxes are shown in Table 3.

In the northern riverine lakes, Lake Prästholmselet and Lake Degerselet, sediment temperature profiles were measured in 1977-78. The gradients and the heat fluxes were lower than in the other lakes, the heat flux on deep as well as on shallow water being about 1 W/m^2.

Summarizing the measurements from the studied lakes it can be concluded that as an overall mean, the heat flux from sediments in shallow water is 4 W/m^2 in early winter and 2 in late winter, and on bottoms deeper than 8 m 2.5 W/m^2 in early and 1.5 in late winter. The sediment heat flux depends on the sediment temperature variation over the year, which is clearly related to the lake water temperature variation at different depth.

Heat Flux to the Underside of the Ice

Especially in late winter the water temperature gradient near the ice is quite sharp. Since heat transfer must take place at least at a molecular rate, a minimum value of the heat flux from the water to the underside of the ice can be found from temperature measurements near the underside of the ice. Here some results from such measurements are reported. The temperature was measured with a thermistor probe mounted on a stand and lowered through a hole in the ice and pushed laterally below the solid ice. The thermistor probe could be positioned vertically within 0.1 mm and be as close to the ice as 1 mm.

Temperature profiles from 1985 near the ice in the middle of Lake Tulebo are shown in Fig. 4, Svensson (1987). The first profile from early February gives a molecular heat flux of 2.7 W/m^2, and the other two heat fluxes of about 5-9 W/m^2. The mean value for the whole lake excluding measurements near the outlet was 3 W/m^2 in early February and about 8 W/m^2 during March. Measurements in the middle of the lake the following winter showed heat flux ranging from 3 to 10 W/m^2 in the period late November to late February, which is clearly higher than the heat flux from the sediments.

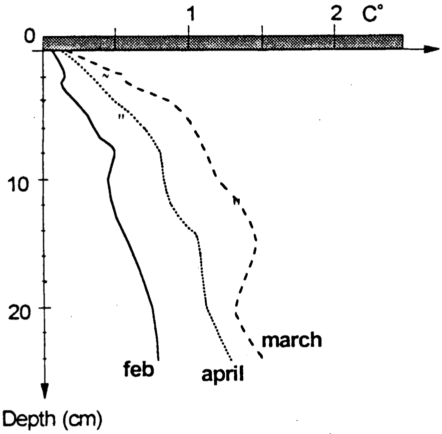

Fig. 4. Temperature near the ice cover in Lake Tulebo, Feb.-April 1985.

Detailed temperature profiles near the ice in Lake Landvetter were measured twice about 1 March 1984. The temperature increased linearly to about 1°C at about 7 cm below the ice and was then rather constant down to several metres. The molecular heat flux was about 5 W/m^2 at the first occasion and varied over the lake being in the range 6-10 W/m^2 during the second measurement occasion. Measurements at three occasion the following winter from 12 February to 1 April, showed a mean heat flux of about 6 W/m^2.

From the temperature profile near the ice in Lake Bredvattnet, the molecular heat flux to the ice was determined to be 2.1 W/m^2 on 27 February 1985 and increased to 4.8 W/m^2 on 1 April. The temperature profiles were linear from the ice and about 1 m down. The following winter the heat flux was 1.5-2 W/m^2.

Also in Lake Erken some temperature measurements were carried out close to the underside of the ice using a mobile thermistor probe, which was pushed laterally under the ice after having been submerged through a drilled hole. Very small temperature gradients were measured in January and February 1985. In early March the temperature 0.15 m under the ice in the central part of the lake was 0.28°C, which means a gradient of 1.9°C/m and a molecular heat flux of 1.0 W/m^2. In the previous year, April 1984, the temperature profile was linear down 0.5 m below the ice, where the temperature was 1.6°C, which means that the molecular heat flux should have been 1.7 W/m^2.

In Lake Velen, the water temperature was registered continuously in four verticals for several years, but no thermistor was placed closer to the water surface than one metre. Knowing the ice thickness and assuming a linear temperature profile between the underside of the ice and 1 m level, it is still possible to estimate a minimum heat flux. This minimum heat flux was about 1.8 W/m^2 throughout the winter.

In the following winter, the temperature at 1 m in the middle of the lake was 0.9°C from freeze-up in early January until about 1 April. The ice thickness was 0.25 m already a few days after freeze-over and increased only slightly through January and February. In mid March the ice was 0.38 m thick. There was almost no snow on the ice. The minimum heat flux, as defined above, can be calculated to be 0.6 W/m^2 in early winter and 0.8 in March. Also in the winter of 1972, the 1 m level temperature was about 1°C and the ice thickness about 0.3 m, although covered with about 0.1 m snow. Again, the temperature profile indicates a heat flux of somewhat less than 1 W/m^2.

In Lake Boren the water temperature was only measured from 1 m below the water surface and downwards, and the temperature at 1 m was only 0-0.1°C. The molecular heat transfer from the water to the ice was thus very small.

In the two riverine lakes in the Råne River basin, the river water is found close to the ice, which means that the water in the lakes is very close to 0°C at one to several metres below the ice. In Lake Prästholmselet the temperature was exactly 0°C down to 5-6 m. In Lake Degerselet the temperature increased downwards already from 1 to 3 m depending on position in the lake. A temperature profile from Lake Prästholm was shown in Fig 2. Although there are currents of about 1 mm/s in these two lakes, for the major part of the winter no heat is transferred from the water to the ice, since the water temperature near the ice is at the freezing point.

In April the temperature above the thermocline in the two lakes is slightly above freezing. A temperature of 0.15°C was observed in 1977 and of 0.05°C in 1978 in a 5 m thick riverine surface layer in Lake Prästholmselet. The surface layer temperature of Lake Degerselet was in April 1977 as high as 0.3°C. From 3 m depth the temperature increased downwards.

In a turbulent boundary layer the sensible heat flux is given implicitly by the relation

$$Nu \equiv C\, Re^{m}\, Pr^{m} \tag{2}$$

where

Nu – Nussell number,
Re – Reynold's number and
Pr – Prandtl number, which is 13.6 for fresh water of 0°C.

The coefficient C is 0.023 for smooth turbulent flow, but according to Haynes and Ashton (1979) 0.034 for rough turbulent flow under ice with uneven surface. The exponents are $m = 0.8$ and $n = 0.4$. The heat flux to the ice, H_i, is

$$H_i = C_i\, v^{0.8}\ (\mathrm{m/s})\, D^{-0.2}\ (\mathrm{m})\, T(^\circ\mathrm{C}) \tag{3}$$

where v – flow velocity, D – vertical extension of flow, which for the discussed situations is from the underside of the ice down to the thermocline, T – bulk flowing water temperature, and where in SI-units $C_i = 2240$ for rough flow and 1622 for smooth flow.

The distance from the ice to the thermocline in Lake Prästholmselet was 4 m, the mean velocity 0.002 m/s and the temperature of the cold water layer 0.15 and 0.05°C

Table 4 – Mean heat fluxes from water to ice in Swedish lakes (W/m^2)

Lake	Jan-Feb	Mar-April	area km^2	depth m	Q m^3/s
Tulebo	6-10	10	0.3	3	-
Landvetter	6-10	6	2.6	7	2
Bredvattnet	2	2-5	0.3	4	
Boren	0	1	25.0	5	40
Erken	0	1-2	24.0	9	
Velen	1-2	1-2	2.8	7	
Degerselet	0	0-2	10.9	6	8
Prästholm	0	0-1	3.0	9	8

for the two investigated years. Assuming smooth flow, the heat flux is calculated to 1.2 W/m^2 and 0.4, respectively. The velocity in Lake Degerselet was estimated as the river flow divided by the cross sectional area above the thermocline to be about 0.001 m/s, which indicates a heat flux from the water to the ice of 2.0 W/m^2.

Often in bulk aero-dynamic formulations, the sensible heat flux in a turbulent layer is given as proportional to the temperature and the stream velocity, H_i = coef φ c_p T v ($\varphi \equiv$ density of water, $c_p \equiv$ specific heat capacity). Hamblin and Carmack (1990) used this approach in riverine ice covered lakes with a proportionality coefficient of 0.8×10^{-3} and 1 m as the reference level of the water temperature. When this simplified formula is applied to the data from the two Swedish lakes, the computed heat fluxes are about 60% of the fluxes found using the Nusselt-Reynolds relation.

Now summarizing the results obtained on heat flux from water to ice, it is seen that in late winter the heat flux to the ice is in all the studied lakes as large as the sediment heat flux, and in two lakes significantly larger being 6-10 W/m^2. The result are summarized in Table 4. In the riverine lakes the cold river water runs close to the ice and the heat lost to the underside of the ice is except in late April minor.

Heat Transfer with Through Flow

A shallow lake with large river through-flow relative to its aerial extension tends to be very cold in the winter. Lake Boren and, of course, the two northern riverine rivers are examples of this. The heat loss through the outlet and the heat introduced with the river water at the inlet are calculated straightforward provided the river water temperatures are known.

The throughflow in the two riverine lakes was 6-10 m^3/s corresponding to a hydraulic load, Q/A, of about 1×10^{-6} m/s for Lake Degerselet and 3×10^{-6} for Lake Prästholmselet. It was only in late April that the temperature conditions in Lake Prästholm were such that the throughflow contributed to a heat loss, 2.0 W/m^2, comparable to the heat flux from the sediments. Thus, although a rather big cold river runs through a small lake, the heat lost by the throughflow does not even in late winter dominate the heat balance.

In the shallower but larger Lake Degerselet, the water temperature at the outlet was about 0.1°C until early April, which corresponds to a heat loss per unit area of 0.3 W/m^2. In April both the temperature of the in- and out-flowing water increased to 0.2-0.4 °C, which means that the net heat loss with the through-flow remained small.

Also in Lake Boren the river flow is large, 40 m^3/s, which relative the area of the lake is 1.6×10^{-6} m/s and comparable to the two riverine northern lakes. However, the temperature in Lake Boren is rather evenly distributed in the vertical , seldom exceeding 1°C even at the bottom. The net heat loss with the throughflow was 1.2 W/m^2 in early winter. In late winter, the in- and outflowing water had the same temperature, 0.3-0.4°C.

The river flow in Lake Landvetter also corresponds to a specific discharge, hydraulic load, close to 10^{-6} m/s. In 1984 after a solid cover had formed in January, the inflow temperature varied between 0.0 and 0.7°C. The temperature at the outlet increased from about 0.7°C in January to 1.7°C prior to that the ice cover thickness was reduced. In January and February, when the outflow was about 6 m^3/s, the heat loss due to the river flow was 4-5 W/m^2 and in late winter, with outflow less than 2 m^3/s, 2 W/m^2.

In 1985 the inflow water temperature was 0.0°C from the freeze-up at about new year until 1 April. The outflow temperature was again 0.7-1.0°C in January and February, but increased throughout March to 1.7°C. By the time of break-up in late April, the in- and outflow water temperatures were almost the same, 2.8°C. The heat lost until 1 April varied in the range 1-2 W/m^2, being lower than the previous year because of lower through-flow, only about 2 m^3/s. In the following year, the inflow temperature was again very close to 0.0°C. The heat loss was about 4 W/m^2 and during a week of high flows, 13 m^3/s, in March, as high as 9 W/m^2.

Also in Lake Tulebo the hydraulic load is about 10^{-6} m/s although the winter discharge is as low as 0.3 m^3/s. The in- and outflow varies much in such a small lake, which influences the thermal conditions in the lake. While the winter inflow temperature always is very close to 0°C, the outflow may be 2°C or higher but was in 1985 and in 1986 always 0.6-0.9°C. During the three winters 1984-1986, the heat lost by the river flow varied from 0 to 3 W/m^2, but seldom exceeded 1 W/m^2. In early 1985 it was 2.3 W/m^2, in late February 0.3 W/m^2 and in March very close to zero. In early 1986 the heat loss was 0.5 W/m^2 but was as high as 3 W/m^2 in March.

In the other three lakes, Lake Bredvattnet, Lake Velen and Lake Erken the river through-flow is very small. The lake itself constitutes 40% of the river basin for Lake Bredvattnet, 6% for Lake Velen and 17% for Lake Erken. No river flow observations are available for Lake Bredvattnet during the study period, but since the upstream catchment is not much larger than the lake itself, the influence of river flow on the thermal conditions in the lake must be minor.

Table 5 – Mean heat loss with river in Swedish lakes (W/m^2)

Lake	Jan-Feb	March	Q/area m/s
Tulebo	1-2	0-3	10^{-6}
Landvetter	2-4	1-2	10^{-6}
Bredvattnet	-	-	10^{-8}
Boren	1	0	2×10^{-6}
Erken	0-0.1	0-0.1	4×10^{-8}
Velen	0-0.4	0-0.4	10^{-7}
Degerselet	0.3	0.5	10^{-6}
Prästholm	0	0	3×10^{-6}

In Lake Erken, the water temperature at the outlet was 0.2-0.3°C in February and March of 1985. The outflow from the lake was high during this period, 0.6-1.1 m^3/s. The temperature of the minor inflow was 0.0°C. The heat lost by the river outflow corresponded to 0.02-0.06 W/m^2, which, although the river flow was higher than normal, is low compared to the heat flux from the sediments. In the preceding year the outflow was only measured at a few occasion in March and April. The water was warmer than in 1985, 0.7-1.2°C, but the outflow was less. The heat loss did not exceed 0.06 W/m^2, which was the maximum heat loss by the outflow in 1985.

The water temperature at the outlet and the inlet of Lake Velen was measured only about once a month. In every year the January outflow temperature was 0.9-1.1°C, and the February-March temperature 1.1-1.4°C. The inflow is coming from a small upstream lake. Most of the time the inflowing water is about 1°C and close to the temperature of the outflow of Lake Velen, but it may be near 0°C. The winter discharge is about 0.3 m^3/s corresponding to a hydraulic load of 10^{-7} m/s, which is only 1/10 of that for most of the other lakes discussed here. Through the winter until the beginning of snowmelt, the heat lost by the river through-flow corresponds to 0-0.4 W/m^2.

The discussion on heat flow with river flow is synthesized in Table 5. When the hydraulic load is low, less than 10^{-7} m^3/s, the heat loss with the throughflow is negligible compared to the heat flux through the ice and from the bottom sediments. When the hydraulic load is of the order 10^{-6} m/s, the through-flow may, but not necessarily, significantly influence the water balance.

Solar Radiation through the Ice

Most of the solar radiation is reflected from a snow cover, and the remainder is almost completely absorbed in the upper part of a snow pack. Thus, very little solar radiation should reach the ice beneath a snow cover. Also the transmissivity of white ice is low, so only if the ice is black ice and free from snow a significant fraction of the incoming solar radiation should reach the lake water. However, since the intensity of the solar radiation reaching the snow surface, or the ice if the ice is free from snow, is ten and in the late phase of the ice covered period almost hundred times the heat flux from the sediments, it is only required that a very small fraction of the incoming solar radiation reaches the water below a snow/ice cover for the temperature of the water to be strongly influenced by the short wave radiation. The extinction coefficient for coarse snow is about 30 m^{-1} for wave length 0.35-0.7 μm and 200 for 0.7-1.2 μm, for example Bergdahl (1977). However for wet snow the transmissivity is higher. Grenfell and Maykut (1977) reported an overall extinction coefficients of 10 m^{-1} for melting snow. For snow, ice laboratory measurements have shown the extinction coefficients for the above given two spectrum ranges to be 30 and 50 m^{-1}, Bergdahl (1977). From field measurements Adams (1978) found the transmissivity to be higher and reported an emissivity of 4 m^{-1}. Solar radiation penetrates black ice to a large extent; the extinction coefficient is about 0.2 for the shortest wave length and 2 m^{-1} for the range 0.7-1.2 μm. The spectral characteristics of ice changes with time as the ice is changing character. If there are bubbles or impurities in the ice, the extinction coefficients are somewhat higher. About 25% of the solar radiation is of the lowest wave length, 50% in the mid-range and the other 25% longer than 1.2 μm. For long wave lengths, exceeding 1.2 μm, all the radiation is absorbed in the snow and in the ice. However, the spectral composition is changing with depth when the radiation penetrates into the snow and the ice. Therefore, overall extinction coefficients are usually used when penetration of solar radiation is estimated.

The solar radiation was measured below the ice in Lake Tulebo at 12 occasions in the springs of 1984, 85 and 86 using a submerged pyranometer. In 1984 and 1986 the ice was free from snow and in 1985 the snow depth on the ice was only 4-5 cm. The composition of the ice cover with respect to ice type was not determined. Since, all the three years, snow fell during and the two weeks just after freeze over, some of the ice should have been white ice. The extinction coefficient was determined to be in the range 3.5-7 m^{-1} when the ice thickness was 20-25 cm. In 1986 when the ice was 30-50 cm thick, the coefficient was 3-4 m^{-1}. In 1985 with a thin 4-5 cm snow cover on 40 cm of ice, the fraction of solar radiation was four times measured to be 18% when the incoming solar radiation was 35-120 W/m^2. Assuming the albedo of the thin snow pack to be 0.2, and the extinction coefficient of the ice, as for the other two years, to be 3 m^{-1}, the extinction coefficient for the snow must have been about 6 m^{-1}.

From the temperature measurements in the studied lakes, the solar energy penetrating through the ice into the lake water can be determined from heat-balance calculations. Such calculations are only performed from January through March to avoid periods just after freeze-up in early winter and periods of snow melt and very intense solar radiation in spring. These calculations are summarized in Table 6.

The heat balance and the variation of its different terms over the ice covered period are shown for Lake Tulebo in Fig, 5. It is seen that the heat fluxes are small and

Table 6 – Heat balance calculations – heat fluxes (W/m^2), ice/snow thickness (cm)

Lake	year	month	ice	snow	$\partial HC/\partial t$	H_{sed}	H_Q	H_{ice}	R_S
Tulebo	1984	Feb-Mar	20	0	8	2	-	-6	12
	1985	Feb	40	5	1	2	-	-10	9
		Mar-Apr	50	0	3	2	-	-12	13
	1986	Feb-Mar	40	0	0	2	-1	-11	10
Landvetter	1984	Feb-Mar	20	0	8	3	-2	-7	14
	1985	Feb	40	5	0	4	-1	-7	4
		Mar	50	0	3	3	-3	-7	10
	1986	Feb	20	0	3	4	-3	R_i-H_i=	2
		Mar	40	0	2	4	-3	R_i-H_i=	1
		Mar-Apr	50	0	8	3	-11	R_i-H_i=	16
Bredvattnet	1985	Feb-Mar	40	10	1	2	-	-2	1
	1986	Feb-Mar	30	3	1	1	-	-1	1
Velen	1970	Jan	50	20	0	1	-	-1	0
		Feb-Mar	60	20	0	1	-	-2	1
	1971	Jan	30	5	1	2	-	-1	0
		Feb-Mar	40	0	2	1	-	-1	1
Boren	1985	Feb-Mar	40	20	0	2	-1	-1	0
Erken	1985	Feb-Mar	40	10	-2	2	0	R_i-H_i=	-4
Degerselet	1978	Feb-Mar	80	30	1	1	0	0	0

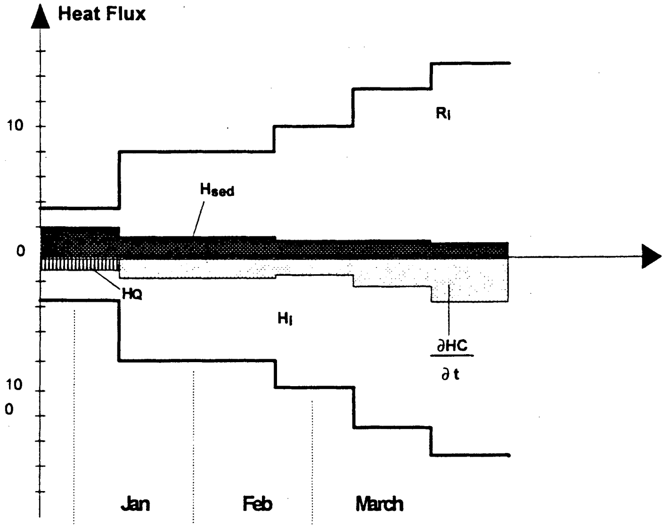

Fig. 5. Heat balance for Lake Tulebo 1984. H_Q = net heat transfer by through-flow, H_{sed} ≡ heat flux from sediments to water, R_i = solar radiation penetrating the ice, H_i ≡ heat flux between water and ice ∂HC/∂t= change of lake water heat content.

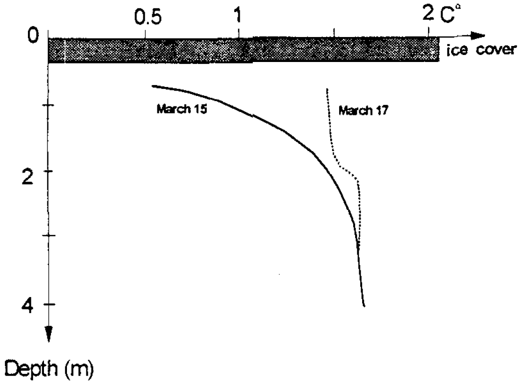

Fig. 6.
Development of temperature profile under the ice of Lake Velen during March 1972.

that , if there is none or only little snow on the ice, the solar radiation and also the heat flux to the underside of the ice are larger than the heat flux from the sediments. It also appears that there is some balance between the solar radiation and the heat lost from the water to the ice.

If it is assumed that the heat flow within the water is due to molecular processes, the temperature profile caused by the penetrating radiation can be determined analytically. A solution for very deep water was given by Matthews and Heaney (1987). Solutions for fixed surface and bottom temperatures in water of restricted depth can be derived. Such analytical temperature profiles show that a rather sharp temperature gradient develops, and that the temperature reaches a peak value not very far below the ice. However, since the density of water is temperature dependent, convection occurs and a region of constant temperature develops. An example from Lake Velen, with temperature data given in a Swedish IHD-report (1973), is shown in Fig. 6. Due to penetrative convection a mixed layer develops. The increase of the heat content corresponds to a net heat flux of 7 W/m^2. The heat loss to the underside of the ice as derived from the temperature profile in Fig 6, assuming molecular heat transfer only, was 1.3 W/m^2 on 15 March and at least 2.7 on 19 March. The heat flux from the sediments was 1.9 W/m^2. Assuming the conditions to be similar all over the lake, the solar radiation reaching the lake water must have been at least 7 W/m^2. Although the heat loss to the underside of the ice increases when heat accumulates in the lake water, the solar radiation is considerably higher than the heat flux to the underside of the ice. For the heat flux to be as high as 7 W/m^2 when only molecular transport conditions prevail, it is required that the water temperature is 4°C as close to the ice as 30 cm.

When comparing the heat balance terms for the studied lakes, it is seen that different heat fluxes dominate in the different lakes, and that there are variations from year to year. In some prolonged periods the heat content of Lake Tulebo and Lake Landvetter increases rather much, 8 W/m^2 for extended periods, while the heat content of all the other lakes is rather constant and not even increasing at all in the biggest lakes, although the heat flux from the sediments is about the same, 2 W/m^2, in all the lakes. The heat content increase in the first two lakes is due to solar radiation penetrating the ice. There was no snow on the ice on these lakes during the periods referred to. At the same time, the conditions at Lake Bredvattnet were similar, but still the contribution of solar radiation was minor. This should be attributed to the fact that the shores of Lake Bredvattnet are steep and the small lake is surrounded by high trees, so that only little radiation reaches the lake when the sun angle is low. The reason why the solar radiation has rather little influence on the heat balance in Lake Velen as compared by for example Lake Landvetter is also partly because the lake is situated in a coniferous forest, but since both lakes are rather large, 2-3 km^2, only minor fractions of the lake area are shaded so the main reason for the small solar radiation effect should be the thin snow cover on the ice of Lake Velen. However, the heat flux from the water to the underside of the ice may have been underestimated in Lake Velen, since the water temperature was not measured closer to the ice than 0.5-0.7 m. This in turn would lead to an underestimation of the radiation through the ice.

Conclusions

From a study of a number of rather small Swedish lakes it is found that the heat fluxes in ice covered lakes are small, of the order some W/m^2, being dominated by heat flux from sediments in early winter and by solar radiation and heat flux from the water to the ice in late winter. In early winter the heat flux from the sediments is higher than the heat loss rate to the ice, 2.5 compared to 1 W/m^2 for all the lakes, although the fluxes are lower for riverine lakes. In late winter the flux from the sediments is reduced to about 1.5 W/m^2 and the heat loss rate to the ice increases. The amount of solar radiation that penetrates the snow-ice cover depends on the snow and ice conditions. The lake water gains heat during the ice covered period, but the heat loss to the ice increases when the water becomes warmer.

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Address:
Lars Bengtsson,
Dept. Water Resources Engineering,
Lund University,
Box 118,
S-221 00 Lund,
Sweden.

Torbjörn Svensson,
Dept. Environmental Sciences,
University of Karlstad,
Sweden.