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Lateral Mixing in Channels due to Secondary Currents

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Different mechanisms generating secondary currents in channels are presented. It is shown by an order-of-magnitude analysis that secondary currents of magnitude 1% of the downstream velocity are as important for lateral mixing as turbulent diffusion. Results from numerical simulations that support the analysis are presented.

Introduction

In the description of spreading of effluents in channels there are usually two or three stages progressing downstream from the source. In the last stage the pollutant is considered as totally mixed in the cross-section and the problem is treated as one-dimensional. Before this last stage, cross-sectional mixing has to take place.

Most often for channel flows in the environment, the channel geometry is such that the lateral dimension is much greater than the vertical dimension. This means that for practical purposes the pollutant can be considered as vertically well mixed close to the source and the interest is then focused on the lateral spreading. For turbulent conditions, lateral mixing is often considered as mainly due to turbulent diffusion.

Although, by virtue of geometrical constrictions, the flow in a channel is essentially unidirectional, there often exist mean velocity components in the cross-plane. There are a couple of possible mechanisms that may produce these cross-plane (or secondary) currents. It is the purpose of this paper to show that even very small average motions in the cross-plane are as important as turbulent diffusion for lateral mixing.

Mechanisms Generating Secondary Currents

For turbulent flows, cross-plane gradients of the normal Reynolds stresses may generate secondary currents when the cross-section is non-circular or when the roughness of the wall varies along the perimeter. Another type of secondary current which is commonly encountered in open channel flow occurs in bends. Bend generated secondary currents are a result of the interaction between centrifugal forces and the cross-stream pressure gradient force.

Cross-stream wind shear stress will generate a third type of secondary current. The surface velocity induced by wind is often approximated as a few per cent of the wind velocity. Although this is likely to be an over-estimate for channels because of the sheltering effects of banks and vegetation, it is probable that this type of secondary current is strong enough to be important for lateral mixing.

A fourth kind of secondary current is generated in a channel rotating around an axis which is not parallel to the downstream flow direction. It is produced by the interaction of the Coriolis force and the cross-stream pressure gradient force. Open channels and streams are examples of rotating channels, since they are fixed to a local rotating system, the earth. It was shown by Larsson (1986a) that secondary currents of the order of 1% of the downstream velocity are generated by the earth's rotation in deep, slowly flowing channels. Coriolis generated secondary currents will be used in this paper to illustrate the importance of secondary currents on lateral mixing.

Order-of-Magnitude Analysis

The influence of secondary currents on lateral mixing will now be analyzed and the analysis will be supported by numerical simulations in the next section. The problem which will be used as an illustration, is to find the longitudinal distance needed for complete lateral mixing of a pollutant which is released from a fixed, vertical line source of constant strength located on the channel centre line. When the lateral mixing is strong this distance is short.

The downstream distance needed for lateral mixing by turbulent diffusion is scaled as

$$L^D = \frac{wB^2}{D_T} = \frac{wB^2}{\beta\Gamma wH} = \frac{B^2}{\beta\Gamma H} \qquad (1)$$

in which w – downstream velocity; B – channel width; D_T – turbulent diffusivity; ß – $D_T / (w^*H)$; Γ – w^*/w; w^* – friction velocity; H – channel depth.

What value the constant of proportionality should have depends on the somewhat arbitrary choice of definition of "well mixed". Assuming uniform downstream velocity and constant diffusivity it can be shown analytically, see e.g. Fischer *et al.* (1979), that if the pollutant is defined as well mixed when the concentration is within 10% of the final value everywhere in the cross-section, then the constant is approximately 0.075.

If there are secondary currents in the channel these will affect the situation. The downstream length scale connected with the lateral spreading due to secondary currents is simply

$$L^S = w\left(\frac{B}{u}\right) \equiv \frac{wB}{\alpha w} \equiv \frac{B}{\alpha} \qquad (2)$$

in which u – lateral secondary velocity and α – u/w.

The ratio between the two length scales is

$$\frac{L^D}{L^S} \equiv \frac{B}{H}\,\frac{\alpha}{\beta\Gamma} \qquad (3)$$

Typical values for the parameters are $\alpha = 0.01$, $\beta = 0.1$ and $\Gamma = 0.05$. With these values the last parenthesis in Eq. (3) is of order unity and the ratio $L^D/L^S = B/H$. Since the aspect ratio, B/H, normally is greater than one, the ratio L^D/L^S is also greater than one (for the assumed parameter values). This means that lateral mixing due to secondary currents is in an order-of-magnitude sense as efficient as mixing due to turbulent diffusion.

It can be seen from Eqs. (1) and (2) that the relative importance of the secondary currents increases with increasing width. The reason for this is that while the distance for lateral mixing in the case of secondary currents is proportional to the width, B, the corresponding length is proportional to B^2 in the diffusion case.

With a similar argument it can be shown that the relative importance of the secondary currents increases when the pollutant source is located off the centre line. If the source is right at the channel side, the situation is analogous to a centre line source in a channel of double width, provided that the channel sides are considered as reflecting walls with respect to the dispersing matter. Because of the proportionality to B^2 and B respectively, the distance required for cross-sectional mixing increases when the source is moved from the centre line to the channel side by a factor 4 for the diffusion case but only with a factor 2 for the secondary current case.

Numerical Simulations

The concentration equation, is solved numerically in this study using the computer code PHOENICS, see Spalding (1981). The program solves, not only the concentration equation, but also the equations of motion together with the continuity

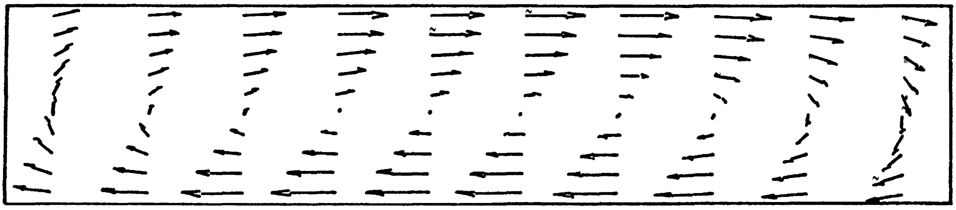

Fig. 1. Secondary flow pattern in the cross-section of a rotating channel, Rossby Number = 137, aspect ratio $(B/H) = 5$.

equation and the transport equations for turbulent kinetic energy, k, and the rate of dissipation of k, ε. The latter two variables are used to compute the eddy viscosity which is assumed to be equal to the eddy diffusivity.

The numerical model was used by Larsson (1986b) to simulate the flow in a rotating channel and it was shown capable of computing Coriolis induced secondary velocities qualitatively well and with a fairly good accuracy.

The geometry used in the present simulations is a rectangular cross-section, with B/H (width/depth) = 25/5 m. The average downstream velocity is 0.1 m/s. The boundary condition for the concentration is that there is no normal flux at any boundary. For the velocities, wall functions, are used to relate the wall stresses to the near-wall velocities. For k and ε local equilibrium is assumed to prevail close to the walls. For all velocities and for k, the free surface is considered as a symmetry boundary while for ε a fictive wall is assumed just above the surface. The latter boundary condition ensures that the eddy viscosity profile goes to zero close to the surface. For further information on the boundary conditions used, see Larsson (1986b). For original source of information concerning boundary conditions, see Launder and Spalding (1974).

In the simulations the flow was first allowed to develop a certain distance downstream in order to obtain uniform flow conditions. Thereafter all variables were "frozen" except the concentration which was introduced evenly along the vertical centre-line. The evolution of the cross-stream concentration distribution was then studied. Particular interest was focused on the depth averaged concentration.

Similar simulations were made both with and without the channel being rotated around a vertical axis. Such rotation generates secondary currents as has been described earlier in the paper. The secondary flow pattern is shown in Fig. 1. The particular rotation speed chosen was $\Omega = 7.29 \times 10^{-5}$ rad/s, equal to the rotation of the earth. This gives a Rossby number ($Ro = w/(2\,\Omega H)$) of 137 and maximum horizontal secondary velocities of about 3% of the downstream velocity.

The downstream development of the depth averaged concentration is presented in Fig. 2 for the non-rotating case. The concentration at the side wall and in the middle is shown. Although neither the eddy diffusivity nor the downstream velocity was constant over the cross-section, the results can be seen to conform with the

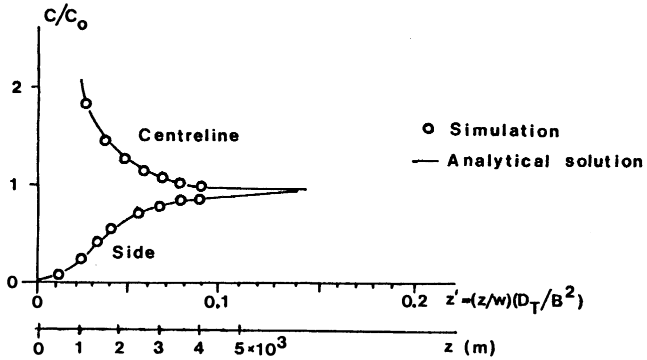

Fig. 2. Development of concentration downstream of a constant source on the channel centre line. No secondary current.

analytical solution which as mentioned above is based on the assumption of constant diffusivity and uniform velocity.

The results from the simulation with the secondary currents present reveal a much faster lateral spreading. The depth averaged concentrations at the channel sides and on the centre line are presented in Fig. 3.

A bulk measure of the efficiency of the lateral spreading is the distance from the source to the point in the channel where all depth averaged concentrations are within 10% of the final value. This was for the non-rotating case 3,600 m and only 600 m for the rotating case, which means that the lateral mixing is much more efficient in the latter case. Since the eddy diffusivity was lower in the rotating case, the increased mixing is evidently due to transport by the secondary currents.

From inspection of a plot of the lateral distribution of the simulated depth-averaged concentration for the rotating case it might be thought that the mixing process is of a diffusive type. But at closer examination this conclusion does not hold. In Fig. 4 the downstream development of the depth averaged concentration

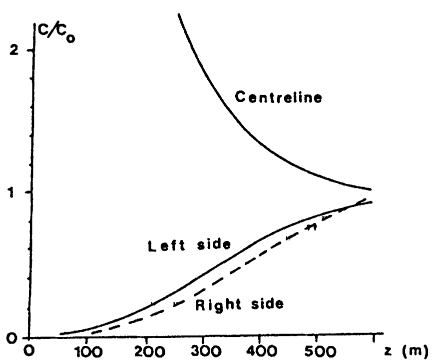

Fig. 3. Development of concentration downstream of a constant source on the channel centre line. Secondary currents present, $Ro = 137$.

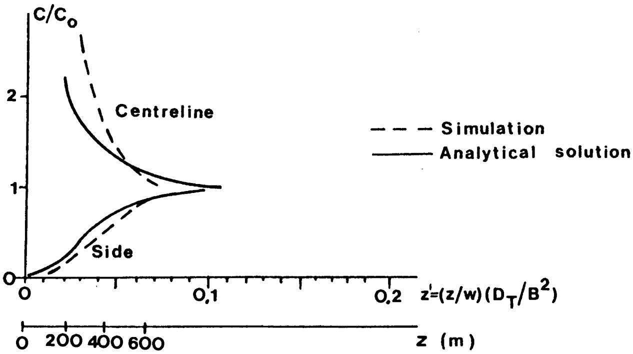

Fig. 4. Development of concentration downstream of a constant source. Comparison between simulation (secondary currents present) and analytical solution (no secondary currents).

at the side and at the centre line is plotted together with the analytical solution for the constant-diffusivity version of the problem. The behaviour of the downstream development of the concentration shows that the lateral mixing is not of a diffusive type in this case.

Further simulations were made for identical conditions except that the rotation rate was varied in order to accomplish different Rossby numbers and thereby also varying magnitude of the secondary currents. The results from the simulations are summarized in Fig. 5 which shows the distance for cross-sectional mixing as a function of Rossby number. The distance is non-dimensionalized with the corresponding distance at no rotation. It can be seen that the distance needed for lateral mixing is halved for Rossby numbers of approximately 300, corresponding to secondary velocities of about 1 % of the downstream velocity. It should be noted that these results are for channels of aspect ratio equal to 5. The corresponding Rossby numbers would be somewhat higher for wider channels and lower for more narrow channels.

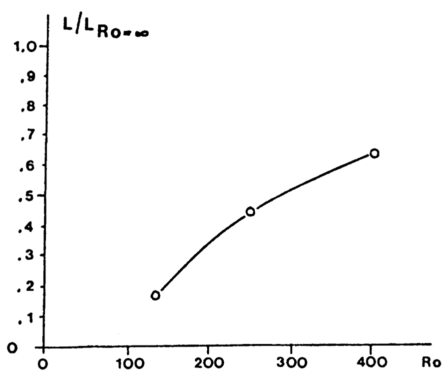

Fig. 5. Distance needed for lateral mixing in a rectangular channel as a function of Rossby number. Aspect ratio, $(B/H) = 5$.

Conclusions

The analysis and the numerical simulations which have been presented above show that secondary currents of the order of 1 % of the downstream velocity are equally important for lateral mixing in channels as turbulent diffusion. This result holds irrespective of what the particular cause of the secondary currents is. For Coriolis induced secondary currents the distance needed for complete lateral mixing is only half of that at no rotation when the Rossby number is about 300, the exact figure somewhat depending on the aspect ratio of the channel.

When the secondary currents are important for the mixing it is not correct to use a diffusion type of mathematical description of the processes. This does not give the correct behaviour of the solution. Especially it was shown above that the distance for cross-sectional mixing increases linearly with the width when the secondary currents are important while it is proportional to the square of the width when the mixing is solely due to diffusion. This means that the relative importance of the secondary currents increases with the width of the channel as well as with the distance from the source to the channel centre line.

The relatively strong influence of even very small secondary currents is a likely reason why so diverging experimental results on the lateral dispersion coefficient can be found in the literature.

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