

# Modelling the impact of urbanization on flood frequency relationships in the UK

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## ABSTRACT

This paper investigates the effect of urbanization on the three key statistics used to establish flood frequency curves when combining the index flood method with the method of L-moments for estimating distribution parameters, i.e. the median annual maximum peak flow (the index flood), and the high-order L-moment ratios L-CV and L-SKEW. An existing procedure employing catchment descriptors was used to estimate the three statistics at ungauged sites in the UK. As-rural estimates of the three statistics were obtained in 200 urban catchments and compared to the corresponding values obtained from observed data. The (log) differences of these estimates were related to catchment descriptors relevant to the urbanization process using linear regression. The results show that urbanization leads to a reduction in L-CV but an increase in L-SKEW. A jack-knife leave-one-out experiment showed that the adjustment factors developed were generally better at predicting the effect of urbanization on the flood frequency curve than the existing adjustment factor currently used in the UK.

**Key words** | FEH, flood frequency estimation, index flood, L-moments, urbanization

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## INTRODUCTION

The UK standard method for establishing flood frequency relationships (or curves) is based on statistical analysis of annual maximum series (AMS) of instantaneous peak flow, and was first described in the Flood Studies Report (FSR) (NERC 1975) and later updated in the Flood Estimation Handbook (FEH) (Institute of Hydrology 1999). It allows estimation of T-year peak flow values at any gauged or ungauged catchment larger than 0.5 km<sup>2</sup>. Recently, the FEH method has again been updated by the Environment Agency (2008) as documented by Kjeldsen & Jones (2009a,b). The method is based on regional frequency analysis using L-moment ratios, and is an adaptation of the index flood method as presented by Stedinger *et al.* (1993), Hosking & Wallis (1997) and the Institute of Hydrology (1999). The objectives of this study are to investigate the effect of urbanization on flood frequency relationships, and to use this information to develop a new set of procedures for adjusting the FEH flood frequency

curve for the effect of urbanization when applied in an ungauged catchment.

Urbanization is a radical form of land-use change. The construction of impervious surfaces (roads, pavements and roofs) inhibits the natural infiltration capacity, while the increased conveyance capacity reduces the catchment response times. It is well established in the literature that the effect of urbanization can be detected in the magnitude of individual annual maximum series of peak flow (Packman 1980; Sheng & Wilson 2009), and thereby lead to changes in the flood frequency characteristics. It is also generally considered that the effect of urbanization is to increase the low return period floods more than the high return period floods. These effects have been accepted qualitatively for several decades (Hall 1973), but the ability to predict the effect in an ungauged catchment is still limited. Summarizing data from published literature, Hollis (1975) found that (compared to the pre-urban flood

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response) a 35% impervious area would lead to an increase of the mean annual flood of about 275%, whereas the 100-year flood would increase by about 80%. Comparable effects were reported by Beighley & Moglen (2003).

Analyzing annual maximum series from 115 urban catchments in the UK, Robson & Reed (1999) developed a model to predict the ratio of the median annual maximum peak flow as estimated from a catchment in an urban or rural state. They found that this ratio could vary between no effect (1) and up to a factor of about 20, depending on the degree of urbanization and the underlying soil type. However, the factor 20 was largely a result of extrapolation from observed data, and the effect from observed data was confined to an increase of 100% and less. A more dramatic effect of urbanization is expected when an urban area is built on a permeable, non-responsive, soil type than when built on less permeable soils, e.g. clay. The FEH (Robson & Reed 1999) also suggested that the effect would gradually diminish as the return period increases and, at very high return period, no effect could be detected. This latter assumption was not verified by evidence derived from observed data.

A substantive issue when attempting to study the effect of urbanization on flood characteristics is the need to consider the temporal development of urbanization. A number of studies have suggested an approach based on naturalization of flood series from urbanized catchments, either through statistical methods (McCuen 1989; Moglen & Shivers 2006) or through more detailed hydrological modelling using rainfall-runoff models (Beighley & Moglen 2003). Both methods require substantial knowledge of the temporal development of urbanization. However, no such systematic data on temporal urban development are readily available in the UK which rules out such detailed adjustments of individual catchments. This study has taken a different approach, where a value of the urban extent is sought which is representative of the period spanned by the observed record. In practice, the extent of urban development in each catchment is back-dated from a level recorded in a national survey around the year 2000 to a level corresponding to the mid-record. For example, for a record spanning the period 1980–2000, the urban extent is back-dated to a level representing the mid-level at year 1990. The actual back-dating itself is based on a national model of urban development.

In this study, a set of urban adjustment factors for the median and high-order L-moment ratios (L-CV and L-SKEW) of the annual maximum series were developed by comparing estimates in urban catchments obtained directly from observed data with the best estimates of the as-rural values of the flow statistics were obtained using FEH procedures as if the catchment was rural and ungauged. The difference between the observed flow characteristics and the as-rural estimates can then be related to the data on urban extent available at each site. This methodology has some similarities to the adjustment procedures presented by Sauer *et al.* (1983) and Moglen & Shivers (2006) and allows the resulting models to be used in conjunction with existing UK models for prediction of flow statistics in rural catchments.

The following sections provide information on the FEH procedures used to obtain as-rural estimates, details on the urban adjustment procedures and the data used in this study and the procedure used for back-dating the urban extent for each catchment. Finally, a set of urban adjustment procedures are derived and their predictive ability assessed using alternative existing procedures. The results suggest that the procedures developed in this study are better at predicting the effect of urbanization on the flood frequency curve than the existing methods.

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## IMPACT OF URBAN EXTENT ON L-MOMENT RATIOS

### As-rural estimates in urban catchments in the UK

The as-rural estimates are obtained using the latest development of the FEH index flood methodology as presented by the Environment Agency (2008). A key element of the FEH is the use of the index flood method, where the flood frequency curve is defined as a product of a site specific index flood  $\xi$  (defined as the median annual maximum flow in the FEH) and a dimensionless growth curve which describes the relationship between the dimensionless flood and the exceedance probability (often expressed as the return period  $T$ ), denoted  $z_T$ .

The FEH recommends the three-parameter Generalized Logistic (GLO) distribution for flood frequency estimation in the UK. Using the GLO distribution, the flood frequency curve (or the quantile function or inverse cumulative

distribution function) for estimating the  $T$ -year peak flow,  $Q_T$ , is given as

$$Q_T = \xi + \frac{\alpha}{\kappa} (1 - (T - 1)^{-\kappa}) = \xi \left[ 1 + \frac{\beta}{\kappa} (1 - (T - 1)^{-\kappa}) \right] = \xi z_T \quad (1)$$

where  $\xi$ ,  $\alpha$ ,  $\beta$  and  $\kappa$  are model parameters and the growth curve is defined as the term within the square brackets. Note that according to the definition in Equation (1), for a return period of two years the growth curve takes a value of 1; the 2-year peak flow value is therefore equal to the median annual maximum flood, i.e. the flood exceeded on average every other year.

In the context of the index flood method, the disproportional effect of urbanization on low and high return period flood (Hollis 1975) is expected to result in higher values of the index flood but flatter growth curves in urban catchments than in corresponding rural catchments.

The GLO model parameters are estimated using a variant of the method of L-moments (Robson & Reed 1999). The location parameter  $\xi$  is defined as the median annual maximum flood and the two parameters controlling the growth curve ( $\beta = \alpha/\xi$  and  $\kappa$ ) are estimated using L-CV and L-SKEW. Specifically, when an estimate of the  $T$ -year peak flow is required in an ungauged catchment, the as-rural estimates of the median, L-CV and L-SKEW can be obtained through the improved FEH methodology (Environment Agency 2008) summarized below.

The as-rural estimate of the median annual maximum flood ( $\text{m}^3 \text{s}^{-1}$ ) is estimated from a set of catchment descriptors as

$$\xi = 8.3062 \text{ AREA}^{0.8510} 0.1536^{(1000/\text{SAAR})} \times \text{FARL}^{3.4451} 0.0460^{\text{BFIHOST}^2} \quad (2)$$

where  $\xi$  is the median (denoted QMED by Robson & Reed 1999), AREA is the catchment area ( $\text{km}^2$ ), SAAR is the standard average annual rainfall measured during the reference period 1961–1990 (mm) and FARL is an index of flood attenuation due to upstream reservoirs and lakes and can take values between 0 (strong attenuation) and 1 (no attenuation). Values of FARL are based on lakes, reservoirs, ponds and other static water bodies as digitized from a 1:50,000 scale map and made available on a

50 m  $\times$  50 m grid, thus excluding water bodies less than 50 m across. Finally, BFIHOST is an index of baseflow as defined by HOST soil classes (Boorman *et al.* 1995) and can take values between 0 (very impermeable soils) and 1 (very permeable soils). More details on each catchment descriptor are provided by Bayliss (1999).

As-rural estimates of L-CV and L-SKEW are obtained using a regional statistical method known as pooled analysis where estimates at the ungauged site are obtained as weighted averages of L-moment ratios from a collection of other sites considered to be hydrologically similar to the site of interest. Hydrological similarity is defined in terms of catchment descriptors, including AREA, SAAR and FARL (all defined above), and FPEXT which is an indicator of the extent of floodplains in the catchment. A summary of the pooling-group method is provided in the appendix and a comprehensive description is provided by Kjeldsen & Jones (2009b).

## Developing models for urban adjustments

In a study of urbanized catchments in the US, Sauer *et al.* (1983) considered the difference between estimates of flood statistics in urban catchments obtained directly from flow data with the corresponding as-rural estimates of the same statistics, and related this difference to a set of catchment descriptors. Sauer *et al.* (1983) based their method on models linking the  $T$ -year peak flow directly to catchment descriptors rather than statistical moments (as in this study). Here the effect of urbanization on the median, L-CV and L-SKEW (the three summary statistics used for estimating the GLO model parameters) is investigated by comparing (i) estimates of these statistics obtained directly from observed data in the urban catchments,  $y^{(A-S)}$  with (ii) the corresponding as-rural estimates as obtained from the FEH method outlined above, denoted  $y^{(A-R)}$ . The difference between the two log-transformed statistics (here represented as a vector containing data from all sites used in the model fitting) are related to a set of catchment descriptors through an ordinary least-squares regression model:

$$\ln \left[ y_{\text{obs}}^{(A-S)} \right] - \ln \left[ y_{\text{cds}}^{(A-R)} \right] = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \quad (3)$$

where  $\mathbf{X}$  is a matrix of catchment descriptors,  $\boldsymbol{\theta}$  is a vector of regression model parameters and  $\boldsymbol{\varepsilon}$  is a vector of random and

independent regression errors. The subscripts obs and cds have been added to emphasize that the estimates are obtained from observed data (obs) and from catchment descriptors (cds), respectively.

As the sample estimate of L-SKEW can take negative values, a constant of 1 was added to all estimates of L-SKEW to allow log-transformation. The catchment descriptors included as explanatory variables in the  $\mathbf{X}$  matrix in Equation (3) should ideally be describing aspects of urbanization in each of the considered catchments.

The FEH (Robson & Reed 1999) provided a calibrated version of Equation (3) for adjusting the median for the impact of urbanization where the urban adjustment factor (UAF) is applied to the as-rural estimate of the median to obtain the corresponding median for the urban catchment. No similar model was developed for the L-moment ratios or the growth curve. Instead, as part of the FEH, Robson & Reed (1999) presented a non-parametric adjustment factor, assuming that for a very large flood (arbitrarily defined as having a return period of 1,000 years) the degree of urbanization would have no influence on the growth curve. The adjustment factor was defined as

$$z_T^{(U)} = \text{UAF}^{-(\ln T - \ln 2)/(\ln 1000 - \ln 2)} z_T^{(A-R)} \quad 2 \leq T \leq 1000 \quad (4)$$

where  $z_T^{(A-R)}$  is the as-rural estimate of the growth curve for the  $T$ -year return period as defined in Equation (1) and  $z_T^{(U)}$  is the resulting estimate of the growth curve in the urban catchment. Note that the superscript ( $U$ ) represents a predicted value of the growth curve rather than an observed value, indicated with the superscript ( $A - S$ ) in Equation (3).

When applying an automated version of the FEH procedure to the entire UK, Morris (2003) found the growth curve adjustment to be inconsistent on a small number of catchments that were both heavily urbanized and permeable at the same time.  $T$ -incoherence could occur on these catchments, defined as cases where  $z_{T=1000}^{(U)} < z_{T=2}^{(U)}$ . Morris (2003) suggested that the adjustment to the rural growth factor should be defined as

$$z_T^{(u)} = 1 + \frac{\left(z_T^{(A-R)} - 1\right) \left(\frac{z_{1000}^{(A-R)}}{\text{UAF}} - 1\right)}{\left(z_{1000}^{(A-R)} - 1\right)} \quad 2 \leq T \leq 1000 \quad (5)$$

rather than through Equation (4) to avoid this  $T$ -incoherence.

## DATA

### Annual maximum series of peak flow

The hydrological dataset used in this study consists of annual maximum series instantaneous peak flow data from 602 rural catchments used to develop the improved FEH methods for producing as-rural estimates, and a corresponding dataset of 206 annual maximum series from urbanized catchments not included in the development of the improved FEH tools. A summary of the two datasets is shown in Table 1.

### Catchment descriptors

Digital catchment descriptors are available for all catchments in the UK larger than 0.5 km<sup>2</sup> (CEH 2007). The number of different catchment descriptors that could potentially be included to explain the difference between the at-site and as-rural estimates is large, but only a subset of variables previously found to have links to the effect of urbanization has been included in this analysis.

A key catchment descriptor is the proportion of the spatial extent of urbanization, available in all UK catchments larger than 0.5 km<sup>2</sup> and derived from digital land-cover data (Bayliss *et al.* 2006). This index is referred to as URBEXT<sub>2000</sub>, where the subscript 2000 indicates that the land-cover data represent the catchment state as observed between 1998 and 2000. The underlying land-cover map uses two classifications of urbanization—urban and suburban—made available on a national 50 m grid. The urban class contains large areas of concrete and tarmac typically found in city centres and major industrial and commercial sites. The suburban class describes grid squares where a

**Table 1** | Summary of AMS of instantaneous peak flow from the rural and urban dataset

	Rural	Urban
Number of gauges	602	206
Shortest record length (years)	4	3
Longest record length (years)	117	120
Average record length (years)	32.7	35.9
Number of annual maximum events	19,679	7,401

mixture of built-up area and permanent vegetation is found, such as city suburbs and small towns and villages. The  $URBEXT_{2000}$  index is a composite index of urban and suburban extent. It is defined as the fraction of the urban class plus half the fraction of the suburban class, assuming that half of a grid square defined as suburban is covered by vegetation (Bayliss *et al.* 2006).

Packman (1980) argued that the effect of urbanization on the flood frequency relationship should be related to separate changes in runoff volume (or percentage runoff) and catchment lag-time. It is generally accepted that the catchment lag-time is related to the proportion of urbanization in a catchment (NERC 1975; Packman 1980; Sheng & Wilson 2009). Based on work by Packman (1980), an updated version of an index quantifying the effect of urbanization on percentage runoff—the percentage runoff urban adjustment factor (PRUAF)—was defined by Kjeldsen (2009) as

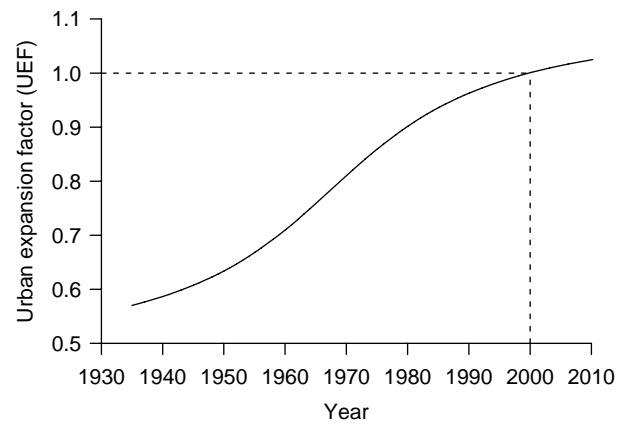
$$PRUAF = 1 + 0.47 URBEXT_{2000} \left( \frac{BFIHOST}{1 - BFIHOST} \right) \quad (6)$$

where both BFIHOST and  $URBEXT_{2000}$  are as defined above.

Other possible catchment descriptors related to the urban development describe the relative location and the urban areas (URBLOC) and the concentration of the urban areas (URBCONC). More details on both descriptors are provided by Bayliss *et al.* (2006), but they were not found to improve the description of the median or the L-moment ratios in this study.

### Adjusting observed records for urbanization

The lack of systematic and comparable data describing the temporal development of urban extent covering the period of most gauged records rules out a detailed adjustment of each individual data series. Instead, the values of the descriptor  $URBEXT_{2000}$  were backdated for all catchments to coincide with the midpoint of the observed record of each individual data series, using a general UK urban expansion factor (UEF). The underlying model describing UEF was developed by Bayliss *et al.* (2006) by combining different official datasets on the total area of land in UK



**Figure 1** | Urban expansion factor (UEF) defined in Equation (7). Note that the model is defined to return a value of one for the year 2000.

under development. The UEF is defined to have a value of 1 at the year 2000 and is given as

$$UEF(\text{year}) = 0.7851 + 0.2124 \arctan\left(\frac{\text{year} - 1967.5}{20.32}\right) \quad (7)$$

where the evaluation of the arctan function is based on radians. The UEF was developed to cover the period 1935–2000, thus the constant 1967.5 in Equation (7) represents the mid-point of this period. The UEF model is illustrated in Figure 1.

## RESULTS

The effect of urbanization was investigated separately for the median, the L-CV and the L-SKEW using ordinary linear regression models. Before the regression models were evoked, an exploratory analysis was conducted for each of the two L-moment ratios. It was determined if an urbanization effect could be expected, and the differences between the at-site and as-rural estimates were compared to the corresponding estimates obtained from the 602 rural catchments. The latter comparison of residuals was undertaken to ensure that the FEH methods can provide reasonable as-rural estimates of the L-moment ratios in the urban catchments. Of course, this assumption can only be tested indirectly as no as-rural estimates can be obtained from data in the urban catchments.

## The median

The regression model for predicting the median in rural catchments from catchment descriptors shown in Equation (2) was developed by Kjeldsen & Jones (2009a) as a log-linear regression model. This investigation will therefore be based on the residuals obtained as the difference between the log-transformed at-site and the FEH as-rural estimates of the median in the urban catchments. Note that 6 of the 206 urban catchments were excluded from this analysis. These catchments were all located in an area northwest of London and the as-rural estimates (Equation (2)) of the median were significantly larger than the observed at-site values. The reasons for these discrepancies are not fully understood but are likely to be related to the complex hydrology of the area dominated by chalk.

A first assessment of the effect of urbanization on the median is shown in Figure 2 where histograms of (log) residuals obtained from the 602 rural catchments from Kjeldsen & Jones (2009a) are compared to the corresponding residuals obtained from the 200 urban catchments. To further assess the impact of urbanization, two subsets of the urban dataset were used according to whether  $URBEXT_{2000}$  is smaller (155) or larger (45) than 0.150.

The resemblance of the two sets of residuals (urban and rural) in Figure 2 indicates that the effect of urbanization on the median can be expected to be limited. However, while

still scattered around 0, the urban residuals have a slight tendency for more positive values than the rural residuals. This tendency is more pronounced for the more urbanized catchments, which indicates that the urban residuals contain some structural information describing the variation in flood statistics between catchments not found in the rural dataset. It should be noted that even for the very urbanized catchments, the at-site median can still be smaller than the predicted as-rural value. This shows that the effect of urbanization is not necessarily unidirectional, and that anecdotal evidence of a reduction of peak flow values as a result of attenuation from hydraulic infrastructure appears evident in the data analyzed here.

The final form of the regression model linking the effect of urbanization to a set of catchment descriptors was the result of an iterative process where not every step is reported here. Throughout the process, the existing FEH model was used as a benchmark against which to measure other potential models. Note that the variable selection is constrained by the need for the urban adjustment factor to produce a value of 1 for  $URBEXT_{2000}$  equal to 0, i.e. no adjustment for a completely rural catchment. The exploratory analysis found only a connection between the effect of urbanization and two variables:  $(1 + URBEXT_{2000})$  and  $PRUAF$ . Other transformations of  $URBEXT_{2000}$  were attempted, such as  $(1 + URBEXT_{2000}^2)$ , but were not found to improve the description of the data. A summary

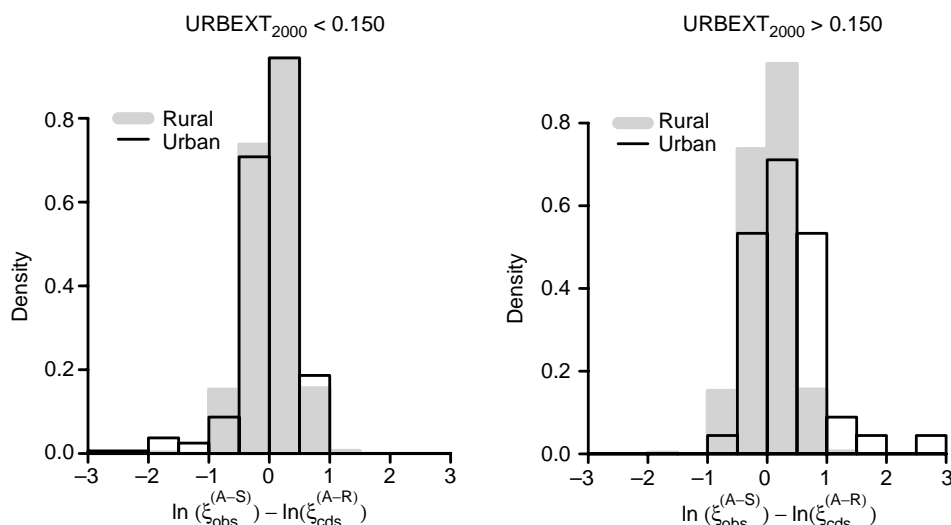


Figure 2 | Histogram representing the residuals from estimates of the median for (1) 602 rural catchments and (2) 200 urban catchments.

**Table 2** | Six different regression models linking the (log) difference between at-site and as-rural estimates of the median annual maximum to catchment descriptors

Model no.	Variables	Parameter	Std. dev.	t-value	p-value	r <sup>2</sup>	s
1	ln [1 + URBEXT <sub>2000</sub> ]	1.67	0.21	7.99	1.09 × 10 <sup>-13</sup> (***)	0.24	0.382
2	ln [PRUAF]	2.51	0.24	10.42	<2 × 10 <sup>-16</sup> (***)	0.35	0.353
3 (FEH)	ln [1 + URBEXT <sub>2000</sub> ] ln [PRUAF]	1.07 1 (fixed)	0.20	5.43	1.65 × 10 <sup>-7</sup> (***)	0.35	0.352
4	ln [1 + URBEXT <sub>2000</sub> ] ln [PRUAF]	0.37 2.16	0.29 0.39	1.29 5.98	0.197 1.02 × 10 <sup>-8</sup> (***)	0.36	0.352
5	ln [1 + URBEXT <sub>2000</sub> ] × ln [PRUAF]	9.89	0.91	10.90	<2 × 10 <sup>-16</sup> (***)	0.37	0.347
6	ln [1 + URBEXT <sub>2000</sub> ] ln [PRUAF] ln [1 + URBEXT <sub>2000</sub> ] × ln [PRUAF]	0.32 0.57 6.80	0.29 0.67 2.45	1.11 0.84 2.78	0.269 0.404 0.006	0.38	0.347

Significance levels:  $p < 0.01$  (\*\*\*),  $0.01$  (\*\*),  $0.05$  (\*). No asterisk indicates a significance level larger than  $0.05$  (not significantly different from zero).

of the regression statistics for the considered models is shown in Table 2.

The last of the models in Table 2 (model 6) is the most comprehensive model and includes both explanatory variables plus a term representing the interaction between the two variables. Despite having a smaller residual standard error than any of the other models, the  $p$ -values for the coefficients on  $(1 + \text{URBEXT}_{2000})$  and the interaction terms are relatively large, suggesting that these explanatory variables are not contributing significantly to the description of the data. Considering both model simplicity and descriptive ability, the results in Table 2 indicate that either Model 2 or Model 4 is the preferred model.

Model 5 could provide a reasonable compromise between model complexity and performance. However, this particular model structure will result in very high values of urban adjustment when applied to catchments with high values of BFIHOST (permeable) as well as a high degree of urbanization. Note here that the dataset contains few catchments which combine high BFIHOST values with high values of URBEXT<sub>2000</sub>; extrapolation is therefore likely to be necessary for practical use. For extrapolation to such catchments, the estimates from Model 5 will be an order of magnitude larger than the corresponding estimates from the existing FEH model.

It was finally decided to adopt Model 4 as it provides a reasonable model and is consistent with the existing FEH model:

$$\xi_{\text{cds}}^{(u)} = \xi_{\text{cds}}^{(A-R)} (1 + \text{URBEXT}_{2000})^{0.37} \text{PRUAF}^{2.16} \quad (8)$$

The results in Table 2 suggest that the term  $(1 + \text{URBEXT}_{2000})$  in Model 4 add little to the ability of the model to describe the data. An alternative choice of model could therefore have been Model 2, describing the effect of urbanization using PRUAF only, i.e.

$$\xi_{\text{cds}}^{(u)} = \xi_{\text{cds}}^{(A-R)} \text{PRUAF}^{2.51} \quad (9)$$

This model was not chosen since Model 4 demonstrates a closer resemblance to the existing FEH model.

### The L-moment ratios

A generalized method for adjusting growth curves for the effect of urbanization was presented by Packman (1980) who stressed that extrapolation beyond return periods of 50 years should be considered 'largely intuitive'. The adjustment method later published by the FEH went one step further, hypothesizing that for very extreme floods of return period 1,000 years, the effect of urbanization on the peak flow magnitude is negligible. The growth factor of the urban catchment is therefore equal to what it would have been if the catchment was not impacted by urbanization. In this study, the effect of urbanization on growth curves will be investigated primarily by examining the effect on each of the L-moment ratios (L-CV and L-SKEW, which control the growth curve according to Equation (1)), rather than the growth curve itself.

### Investigating applicability of generalized rural models in urban catchments

Using the recently developed improved FEH pooling-group method (Kjeldsen & Jones 2009b), pooled L-moment ratios (as-rural estimates) can be derived for each of the urban catchments. By considering the urban catchment to be ungauged, the pooled estimates represent the best available estimate of what the L-moment would be at the site if it was not influenced by urbanization, i.e. as-rural. It should be noted that the pooled estimates of L-moment ratios are estimated as if the site of interest is ungauged. These estimates are therefore associated with a higher uncertainty than the corresponding at-site estimates obtained directly from the data at each site (Kjeldsen & Jones 2006).

No compelling evidence was found that the L-moment ratios from the six catchments initially excluded from the analysis of the median were outliers, and thus they were retained in this analysis. Using only catchments with a record length in excess of 20 years (177 catchments), a first

tentative assessment of the impact of urbanization on the L-moment ratios is shown in Figure 3. The difference in L-moment ratios between the at-site estimate and the as-rural estimate obtained from the pooling-group method is plotted for two subsets of the urban data defined according to the level of urbanization. The first subset consists of 150 catchments which, according to the classification scheme by Bayliss *et al.* (2006), are categorized as being slightly to moderately urbanized ( $0.030 < \text{URBEXT}_{2000} \leq 0.150$ ). The second subset includes 27 catchments categorized as being heavily to very heavily urbanized ( $0.150 < \text{URBEXT}_{2000} \leq 0.600$ ).

A comparison of the histograms in Figure 3 indicates that the effect of urbanization manifests itself in lower values of L-CV and higher values of L-SKEW than would be expected for rural catchments. The figures also suggest that this effect is more pronounced for higher values of  $\text{URBEXT}_{2000}$  than at lower values. The effect of urbanization is generally considered to be a larger proportional

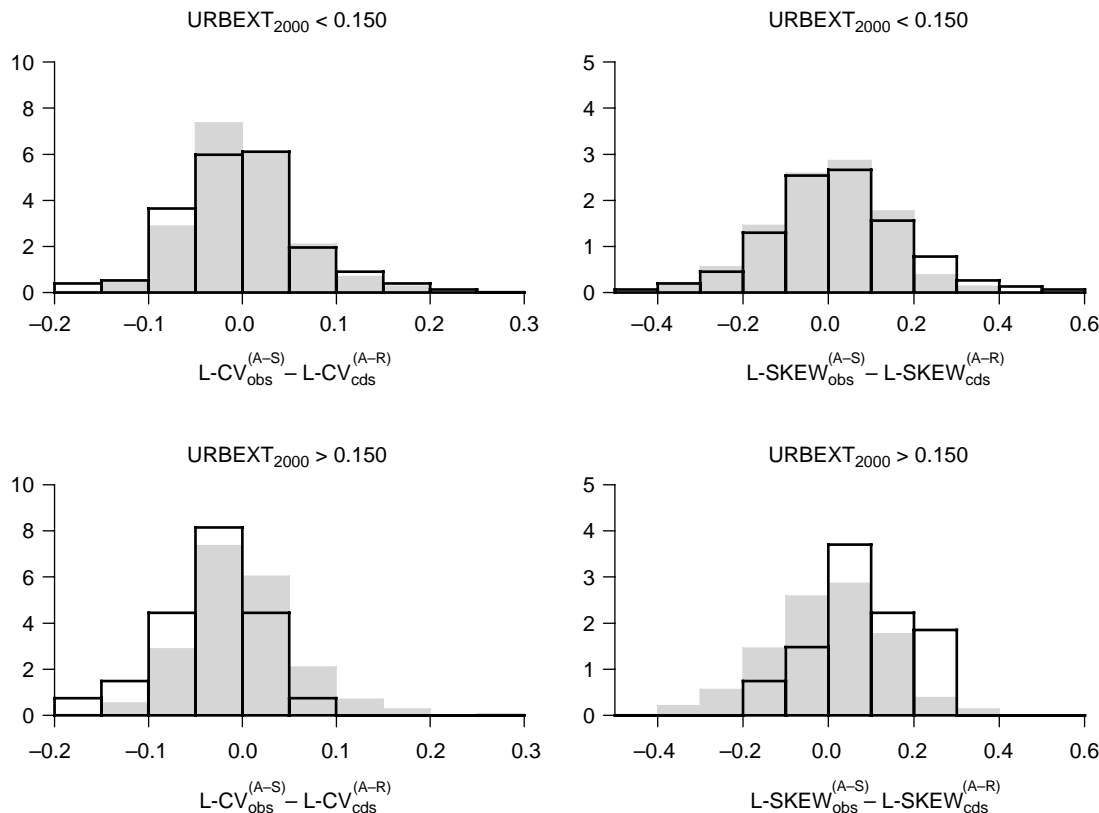


Figure 3 | Histograms representing the residuals from the rural catchments (grey) and the corresponding residuals for the urban catchments (black lines) for L-CV and L-SKEW.



increase in more frequent floods than the more rare floods. Packman (1980) argued that this effect would lead to a reduction in the standard deviation (thus L-CV) but did not extend the argument to include the coefficient of skewness (or L-SKEW). However, it seems reasonable to assume that the effects of the disproportional increase would lead to samples with a greater tendency for positive skewness. Thus, the lowering of L-CV found in this study supports the previous findings that urbanization results in a flatter growth curve (e.g. Packman 1980), whereas the effect of urbanization on L-SKEW has not been reported elsewhere (to the author's knowledge).

A straightforward comparison of the at-site and pooled L-moment ratios is complicated by the fact that the pooling-group method was developed using the rural dataset, but did not include the urban dataset. As a result, the residuals (at-site minus as-rural estimates) from the urban catchments are expected to have a slightly higher degree of variability than the residuals from the rural catchments. Also, the observed difference between the at-site estimate from an urban catchment and the corresponding pooled estimate will be caused by different factors including: (1) the effect of urbanization, (2) bias in the pooling-group method

because a particular urban catchment might not be well represented with regard to its catchment descriptors in the dataset of rural catchments available for pooled analysis, and (3) sampling uncertainties in the estimates due to limited record lengths. An implicit assumption of this analysis is that the last two factors have an insignificant influence compared to the effect of urbanization itself.

Systematic variation in residuals of L-CV and L-SKEW related to catchment descriptors other than urbanization was investigated by plotting the residuals against each of the catchment descriptors used for defining hydrological similarity, as shown in Figures 4 and 5. The polylines in each figure represent the outermost convex hull as defined by the rural dataset.

Little or no systematic variation with any of the four catchment descriptors can be readily identified for either L-CV or L-SKEW from Figures 4 and 5. Also, the spread of the residuals for the urban catchments for the vast majority falls within the region defined by the rural residuals, thereby adding confidence that the pooling-group method can be assumed to provide as-rural estimates of the L-moment ratios in the urban catchments with a degree of uncertainty comparable to that of the rural catchments.

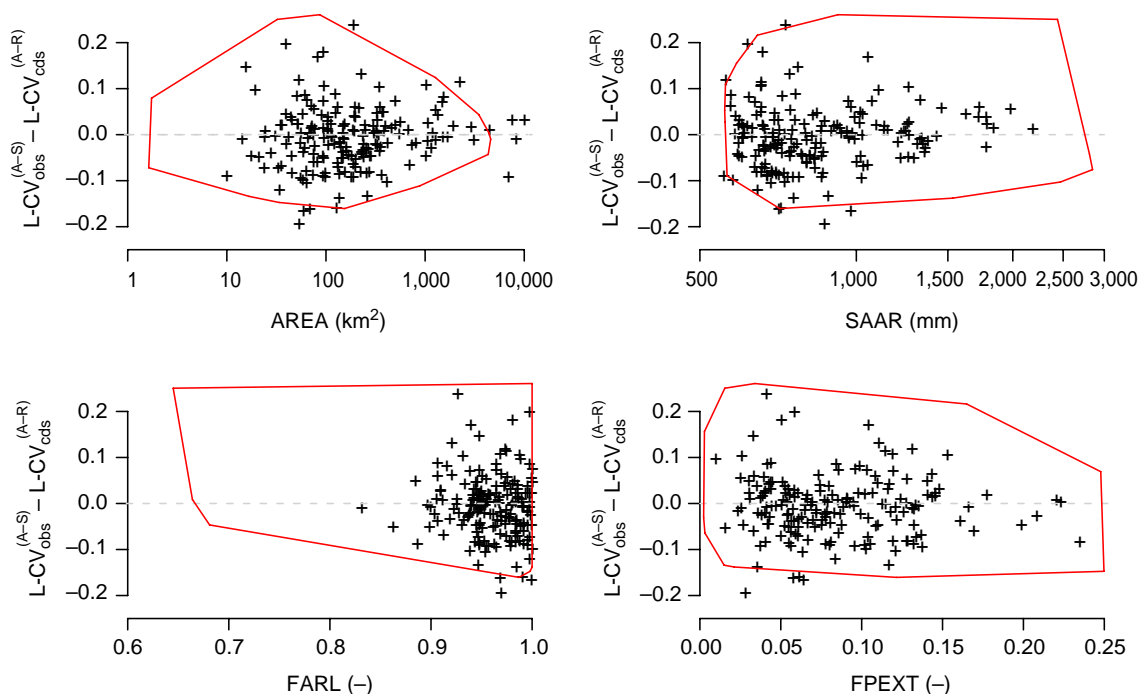


Figure 4 | Comparison of L-CV residuals from the rural (polylines) and urban ('+') datasets.

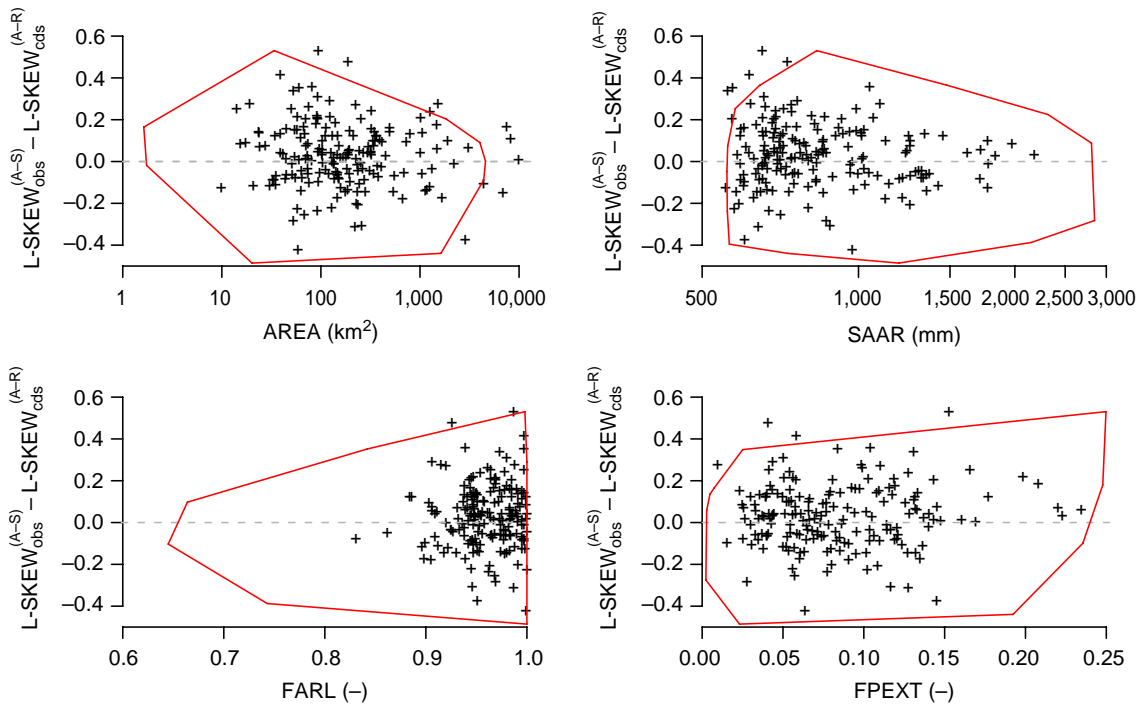


Figure 5 | Comparison of L-SKEW residuals from the rural (polylines) and urban ('+') datasets.

**Model selection**

Initially, an exhaustive search for the best subsets of explanatory variables in Equation (3) for predicting the difference between the urban and rural L-moment ratios was undertaken based on linear regression. Both log-transformed and non-transformed catchment descriptors were considered, but the only significant explanatory variable to be identified for both L-CV and L-SKEW was URBEXT<sub>2000</sub>. Similar to the investigation into the effect of

urbanization on the median, the variable PRUAF was also included but no relationship to the L-moment ratios was identified. The summary statistics of selected regression models for L-CV and L-SKEW are shown in Tables 3 and 4.

From the results in Tables 3 and 4 it can be seen that the relationship between the difference of the (log) at-site (urban) estimates of the L-moment ratios and the corresponding (log) as-rural estimates is generally weak for L-CV and even weaker for L-SKEW. In both cases, the relationship weaker than for the corresponding results obtained for

Table 3 | Models for describing L-CV in urban catchments

Dependent variable	Explanatory variable	Parameter	Std. dev	t-value	p-value	R <sup>2</sup>	s
$\ln [L-CV^{(A-S)}] - \ln [L-CV^{(A-R)}]$	$\ln [1 + URBEXT_{2000}]$	-0.6695	0.1476	-4.57	$1.06 \times 10^{-5}$ (***)	0.10	0.263
$\ln [L-CV^{(A-S)}] - \ln [L-CV^{(A-R)}]$	$\ln [1 + URBEXT_{2000}]$	-0.9177	0.2200	-4.17	$4.74 \times 10^{-5}$ (***)	0.11	0.264
	$\ln [PRUAF]$	0.9675	0.01941	49.84	$< 2 \times 10^{-16}$ (***)		
$\ln [L-CV^{(A-S)}]$	$\ln [1 + URBEXT_{2000}]$	-0.9070	0.2161	-4.20	$4.30 \times 10^{-5}$ (***)	0.97	0.262
	$\ln [L-CV^{(A-R)}]$	0.9713	0.0191	1.50 <sup>†</sup>	0.13 <sup>†</sup>		
$\ln [L-CV^{(A-S)}] - \ln [L-CV^{(A-R)}]$	URBEXT	-0.5893	0.1286	-4.58	$8.64 \times 10^{-6}$ (***)	0.11	0.263
$\ln [L-CV^{(A-S)}]$	URBEXT	-0.7470	0.1795	-4.16	$4.97 \times 10^{-5}$ (***)	0.97	0.262
	$\ln [L-CV^{(A-R)}]$	0.9772	0.0182	1.25 <sup>†</sup>	0.21 <sup>†</sup>		

<sup>†</sup>Test if coefficient significantly different from 1.

Significance levels:  $p < 0.01$  (\*\*\*),  $0.01$  (\*\*),  $0.05$  (\*). No asterisk indicates a significance level larger than 0.05 (not significantly different from zero).

**Table 4** | Models for describing L-SKEW in urban catchments

Dependent variable	Explanatory variable	Parameter	Std. dev	t-value	p-value	R <sup>2</sup>	s
$\ln[\text{L-SKEW}^{(A-S)} + 1] - \ln[\text{L-SKEW}^{(A-R)} + 1]$	$\ln[1 + \text{URBEXT}_{2000}]$	0.1686	0.0704	2.39	0.018 <sup>(*)</sup>	0.03	0.126
$\ln[\text{L-SKEW}^{(A-S)} + 1] - \ln[\text{L-SKEW}^{(A-R)} + 1]$	$\ln[1 + \text{URBEXT}_{2000}]$	0.1014	0.1054	0.96	0.337	0.04	0.126
	$\ln[\text{PRUAF}]$	0.1082	0.1262	0.86	0.393		
$\ln[\text{L-SKEW}^{(A-S)} + 1]$	$\ln[1 + \text{URBEXT}_{2000}]$	0.1754	0.0826	2.12	0.035	0.58	0.13
	$\ln[\text{L-SKEW}^{(A-R)} + 1]$	0.9463	0.0930	0.58 <sup>†</sup>	0.564 <sup>†</sup>		
$\ln[\text{L-SKEW}^{(A-S)} + 1] - \ln[\text{L-SKEW}^{(A-R)} + 1]$	URBEXT	0.1436	0.0615	2.34	0.021	0.03	0.126
$\ln[\text{L-SKEW}^{(A-S)} + 1]$	URBEXT	0.1754	0.0826	2.12	0.035	0.58	0.126
	$\ln[\text{L-SKEW}^{(A-R)} + 1]$	0.9463	0.0930	0.58	0.564 <sup>†</sup>		

<sup>†</sup>Test if coefficient significantly different from 1.

Significance levels:  $p < 0.05$  (\*). No asterisk indicates a significance level larger than 0.05 (not significantly different from zero).

the median (Table 2). For both L-CV and L-SKEW, there is little evidence that using the pooled estimate as a predictor in combination with  $\text{URBEXT}_{2000}$  has any benefits over a model relating the difference directly to  $\text{URBEXT}_{2000}$ .

For L-CV the best model relates the difference directly to  $\text{URBEXT}_{2000}$  without any transformation of  $\text{URBEXT}_{2000}$ . This performs slightly better than the version relating the difference to  $\ln[1 + \text{URBEXT}_{2000}]$ . It is therefore recommended that the urban adjustment procedure for L-CV is given as

$$\text{L-CV}^{(u)} = \text{L-CV}^{(A-R)} \times 0.5547^{\text{URBEXT}_{2000}} \quad (10)$$

For L-SKEW, there is little difference between a model relating the difference to either a log-transformation of  $\text{URBEXT}_{2000}$  or to  $\text{URBEXT}_{2000}$  directly. Thus, to ensure consistency, the urban adjustment factor for L-SKEW is defined as

$$\text{L-SKEW}^{(u)} = \left[ (\text{L-SKEW}^{(A-R)} + 1) \times 1.1545^{\text{URBEXT}_{2000}} \right] - 1 \quad (11)$$

## COMPARISON OF PREDICTIVE CAPABILITY

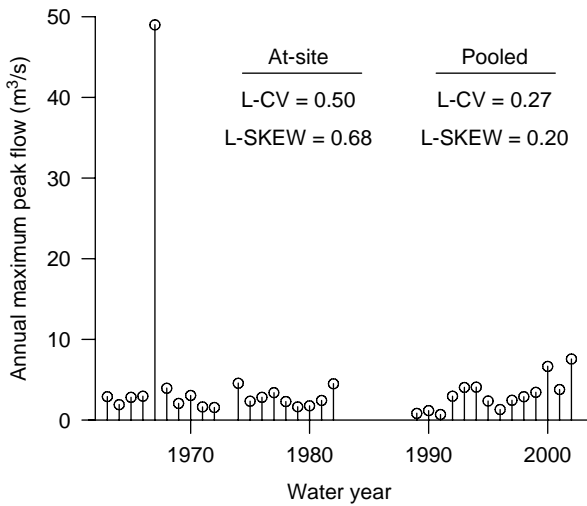
The predictive ability of these adjustment procedures is now compared to the urban adjustment procedures suggested by the FEH (Robson & Reed 1999) and Morris (2003).

A cross-validation experiment based on the leave-one-out technique (Efron & Tibshirani 1993) was carried out to assess and compare the ability of different adjustment

procedures to predict the  $T$ -year growth factor in urbanized catchments. Only the dimensionless growth factors were considered in this experiment, and no competing procedures for adjusting the median were suggested in this study. The leave-one-out procedure was considered necessary in order to compare the L-moment ratio adjustment developed in this study with the calibration-free adjustment procedures suggested by the FEH (Robson & Reed 1999) and Morris (2003). In this study four different procedures were considered:

1. no adjustment (estimate growth curve as if it was a rural catchment);
2. adjust both L-CV and L-SKEW (method developed in this study);
3. the FEH adjustment procedure (Robson & Reed 1999); and
4. the Morris (2003) procedure.

At some gauging stations, the observed annual maximum series include one or two flood events that are very large (8–10 times the median annual maximum runoff) compared to the bulk of the observations in that series. For such catchments, the at-site sample estimates of L-CV and L-SKEW are much higher than the typical average values predicted by the pooling-group method. The annual maximum series of peak flow from catchment 40012 located in southeast England (Figure 6) is an example of such a catchment where the at-site L-CV is 1.89 times the corresponding pooled estimate. If the large event in water year 1967 (18 September 1968) is removed from the series,



**Figure 6** | Annual maximum series for catchment 40012. The extreme event occurring on 16 September 1968 (17 times larger than QMED) is easily identified.

then the at-site estimate of L-CV is reduced to 0.27 and the ratio of the at-site and the pooled estimate is reduced to 1.03.

It would be tempting to remove the catchments from the dataset where the at-site and as-rural estimates are very different. Unfortunately, it is not generally known what causes the difference between the at-site and the as-rural (or pooled) estimate. It could be caused by a number of factors, such as: (1) oddities in the at-site samples (as discussed above), (2) failure of the pooling-group method to accurately represent the at-site L-moment ratios, (3) the residual effect of urbanization, or (4) any combination of the first three reasons. Therefore, any censoring of the dataset will involve some arbitrary decisions.

To reduce (but unfortunately not remove) the influence of these catchments, it was decided to use the absolute difference between at-site and predicted growth factors rather than the squared difference for assessing predictive ability. The cross-validation statistic adopted in this study, based on observations, is defined

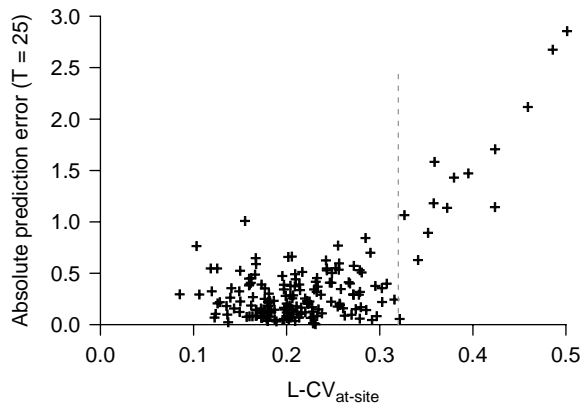
$$\frac{1}{m} \sum_{i=1}^m |z_i - \hat{z}_i^{(-i)}| \tag{12}$$

where  $z_i$  is the observed quantity (here growth factor) at the  $i$ th site and  $\hat{z}_i^{(-i)}$  is the corresponding estimate of the same quantity from a model fitted to the observations with the  $i$ th observation omitted from the dataset. Equation (12) is also known as the cross-validation estimate of prediction error. Table 5 compares the cross-validation statistic in Equation (12) for each of the five methods listed above.

The results in Table 5 suggest that the adjustment procedure developed in this study provides better predictions of the growth curve than both the FEH (Robson & Reed 1999) and Morris (2003) procedures. However, for return periods in excess of 50 years, the unadjusted as-rural growth curve appears to provide an overall better prediction of the urban growth curve. It is worth remembering that the L-moment ratio will have been estimated using annual maximum series with an average record length of 36 years, i.e. the behaviour at long-return periods is mainly a result of extrapolation from the observed data based on the GLO distribution. For comparison, the cross-validation statistic defined as the root sum of squares are also shown in Table 5

**Table 5** | Comparison of cross-validation statistics (absolute difference) for urban growth curve adjustment factors for  $T = 5, 10, 25, 50, 100$  and  $1,000$  year return periods. The numbers in brackets are the root sum of square validation statistics

Method	Return period (years)					
	5	10	25	50	100	1000
1. No adjustment (as-rural)	0.094 (0.126)	0.185 (0.261)	0.358 (0.545)	0.543 (0.887)	0.796 (1.405)	2.550 (5.980)
2. Adjust L-CV and L-SKEW (this study)	0.090 (0.124)	0.182 (0.261)	0.356 (0.548)	0.543 (0.893)	0.799 (1.411)	2.560 (5.984)
3. The FEH adjustment procedure	0.094 (0.129)	0.189 (0.276)	0.368 (0.582)	0.564 (0.945)	0.833 (1.488)	2.647 (6.173)
4. The Morris (2003) procedure	0.094 (0.131)	0.195 (0.286)	0.382 (0.597)	0.581 (0.963)	0.848 (1.504)	2.644 (6.170)



**Figure 7** | As-rural (pooled) estimates of L-CV plotted against at-site estimates of L-CV for 202 urban catchments.

for each method and return period, and the results shown in brackets. The root sum of squares is defined equivalently to Equation (12) as

$$\sqrt{\frac{1}{m} \sum_{i=1}^m (z_i - \hat{z}_i^{(-i)})^2} \quad (13)$$

The growth curve for the catchments with short records and extraordinary large singular events (see for example, Figure 6) are generally much steeper than the pooled growth curve. Any further reduction in growth curve factors, such as those imposed by any of the urban adjustments, is therefore likely to indicate that no adjustment is the preferred option. This effect is further amplified when using the sum of squares rather than the absolute value as the basis for the cross-validation statistics.

In Table 5, the root sum of squares values suggest that no adjustment is the preferred option at a return period of 25 years. The sum of absolute differences, Equation (12), suggests that no adjustment is preferable for the 50 year return period and beyond. To further assess how much the results in Table 5 are affected by the presence of the catchments discussed above, an additional experiment was conducted where these catchments were removed from the dataset. Figure 7 shows the prediction residuals for the 25 year growth factor for each individual catchment plotted against the corresponding at-site estimate of L-CV.

By repeating the cross-validation experiment outlined above, but using only a subset of the data where the at-site sample values of L-CV are less than 0.33 (points to the left of the vertical dashed line in Figure 7), a new set of average prediction errors have been derived and are shown in Table 6.

The results in Table 6 confirm the results reported using the entire dataset that the adjustment to L-CV and L-SKEW developed in this study will generally provide a better prediction of the effect of urbanization on the growth curve than the adjustments suggested by the FEH and by Morris (2003). Again, the use of the sum of squares rather than absolute values reduces the return period for which no adjustment is the preferred option from 1,000 years to 100 years (based on the return periods represented in Table 6) but does not change the overall recommendation that the adjustment procedure developed in this study is preferable to the alternative adjustment procedures. From both Tables 5 and 6 it can be observed that the relative benefit of the

**Table 6** | Comparison of cross-validation statistics (absolute difference) for urban growth curve adjustment factors for  $T = 5, 10, 25, 50, 100$  and 1,000 year return periods derived by not including the 14 catchments with highest at-site L-CV values. The numbers in brackets are the root sum of square validation statistics

Method	Return period (years)					
	5	10	25	50	100	1000
1. No adjustment (as-rural)	0.080 (0.102)	0.151 (0.190)	0.274 (0.342)	0.397 (0.498)	0.555 (0.705)	1.488 (2.044)
2. Adjust L-CV and L-SKEW (this study)	0.076 (0.096)	0.146 (0.183)	0.270 (0.338)	0.396 (0.498)	0.554 (0.710)	1.493 (2.068)
3. The FEH adjustment procedure	0.079 (0.102)	0.151 (0.197)	0.277 (0.370)	0.410 (0.548)	0.581 (0.784)	1.560 (2.266)
4. The Morris (2003) procedure	0.077 (0.096)	0.155 (0.197)	0.289 (0.375)	0.424 (0.556)	0.593 (0.791)	1.555 (2.258)

growth-curve adjustment procedure is reduced as the return period increases. For a return period of 1,000 years, the no adjustment option is the preferred choice. This is consistent with the existing FEH and Morris (2003) methods (Equations (4) and (5)), although both these methods were found to perform poorly at lower return periods and should therefore not be used in general.

## CONCLUSION

Results presented in this paper allow users of the existing FEH procedure for flood frequency estimation in rural catchments to adjust flood frequency curves for the impact of urbanization when estimated in ungauged and urbanized catchments. Following the comparison of several procedures, the recommended adjustment procedure is based on a set of regression Equations (Equation (8), (10) and (11)) linking a set of catchment descriptors to the difference between (log) estimates of the median, L-CV, and L-SKEW obtained from at-site data in urban catchments and the corresponding as-rural estimates obtained from the FEH procedures.

To adjust the growth curve, the approach taken in this study was to directly investigate the impact of urbanization on the relevant L-moment ratios (L-CV and L-SKEW). It was found that increased urbanization has a tendency to reduce L-CV, i.e. cause a flattening of the growth curve when compared to the as-rural estimate. This effect was supported by the findings of other published studies (Hollis 1975). With regard to L-SKEW, the results indicated a slight tendency of increased urbanization to cause an increase in L-SKEW, which will result in more upwardly curved growth curves. This effect was statistically less significant than the effect on L-CV, but has not been reported previously.

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## APPENDIX

The as-rural estimates of the L-moment ratios—L-CV or L-SKEW (both denoted  $t^{(A-R)}$  for convenience in the following)—at an ungauged site are obtained by forming a weighted average of L-moment ratios from a collection of gauged catchments considered hydrologically similar to the site of interest. This collection of sites is also known as a pooling group. The as-rural estimate is defined as:

$$t^{(A-R)} = \sum_{i=1}^M \omega_i t_i$$

where  $M$  is the number of hydrologically similar gauged sites,  $t_i$  is the L-moment ratios at the  $i$ th site and  $\omega_i$  is the weight assigned at the  $i$ th site. Hydrological similarity  $d$  is defined here in terms of catchment descriptors:

$$d_{ij} = \sqrt{3.2 \left( \frac{\ln[\text{AREA}_i] - \ln[\text{AREA}_j]}{1.28} \right)^2 + 0.5 \left( \frac{\ln[\text{SAAR}_i] - \ln[\text{SAAR}_j]}{0.37} \right)^2 + 0.1 \left( \frac{\text{FARL}_i - \text{FARL}_j}{0.05} \right)^2 + 0.2 \left( \frac{\text{FPEXT}_i - \text{FPEXT}_j}{0.04} \right)^2}$$

where AREA is the catchment area ( $\text{km}^2$ ), SAAR is standard annual average rainfall as measured from 1961 to 1990 (mm), FARL is an index of flood attenuation due to upstream reservoirs and lakes and can take values between 0 (strong attenuation) and 1 (no attenuation) and FPEXT is an indicator of the extent of floodplains in the catchment and can take values between 1 (all floodplain) and 0 (no floodplain). The number of sites to be used is determined by

the total number of annual maximum events, which has to exceed 500.

The weights assigned to each gauged site depend on the sampling variability  $c_i$  and distance in catchment descriptor space from the target site  $d_i$  and is defined as

$$\omega_i = \frac{(c_i + b_i)^{-1}}{\sum_{k=1}^M (c_k + b_k)^{-1}}, \quad i = 1, \dots, M$$

where the quantity  $b_i$  is defined separately for L-CV and L-SKEW as

$$\text{L-CV} : b_i = 0.0047\sqrt{d_i} + 0.0023/2$$

$$\text{L-SKEW} : b_i = 0.0219(1 - \exp[-d_i/0.2360])$$

The sampling variance is defined for L-CV and L-SKEW as

$$\text{L-CV} : c_i = 0.02609/(n_i - 1)$$

$$\text{L-SKEW} : c_i = 0.2743/(n_i - 2)$$

where  $n_i$  is the record length at the  $i$ th site.