(e) At the lower sliding speeds the film thickness becomes so small that boundary processes must take over before the collision is complete.

(f) The load capacity generated by the colliding asperities is generally very much higher than the equivalent Hertz load capacity, except at the lowest sliding speed.

(g) Tractions of the same order as the load capacity are generated at the higher overlaps; surface shear stresses of the order of 10^6 psi are generated at the center of the contact region.

3 The conditions of pressure, temperature, shear stress, and shear rate predicted to occur are so severe that the assumptions that the lubricant is an inert, Newtonian, continuum fluid within the contact, that the asperities deform elastically, and that the contribution of the surface shear stress to the deformation of the asperities is negligible become questionable.

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DISCUSSION

H. S. Cheng[2] and Kwan Lee[2]

We like to congratulate Dr. Fowles for solving a very difficult problem in elastohydrodynamic lubrication.

Of particular interest is his remarkable success in using the so-called direct-iterative method for extremely heavy loads. For the rolling and sliding EHD problem, it was found by many previous investigators, that the direct-iteration fails to converge when the maximum Hertz pressure approaches 20,000 to 25,000 psi for rollers lubricated with mineral oils, no matter how small the deacceleration factor is between iterations. Indeed, the inherent divergence of the D-I method at high loads for the rolling problem can be demonstrated mathematically by an error analysis following the method outlined by Ostrowski.[3] In the present paper, Dr. Fowles did not indicate any special techniques in preventing divergence other than using a scaling factor between two successive iterations. This led us to believe that the convergence difficulties in using D-I method are much less critical in squeeze-film type problems than in the pure rolling EHD problems.

In Dr. Fowles' earlier isothermal theory [1], he used a pressure-viscosity coefficient much smaller than that employed in the present paper. The direct quantitative comparison between the isothermal and thermal results may have been inhibited because

[2] Associate Professor and Graduate Student, respectively, Mechanical Engineering and Astronautical Sciences, Northwestern University, Evanston, Ill.

[3] Ostrowski, A. M., *Solution of Equations and Systems of Equations*, Academic Press.

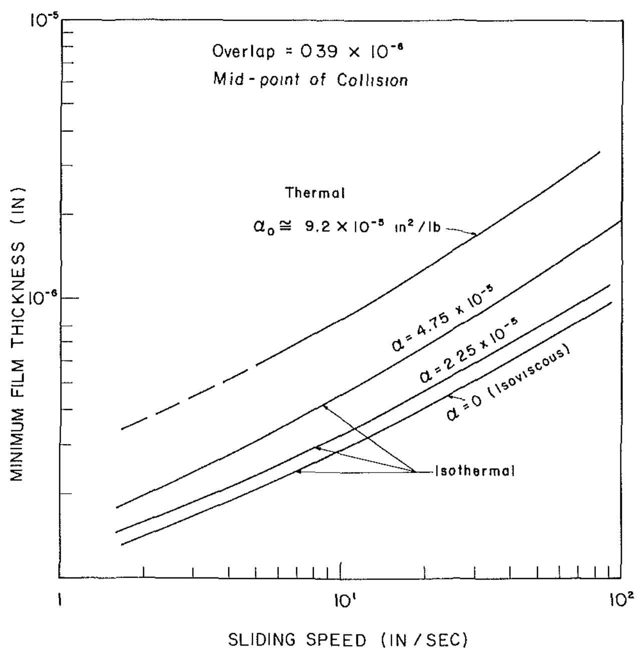

Fig. 16

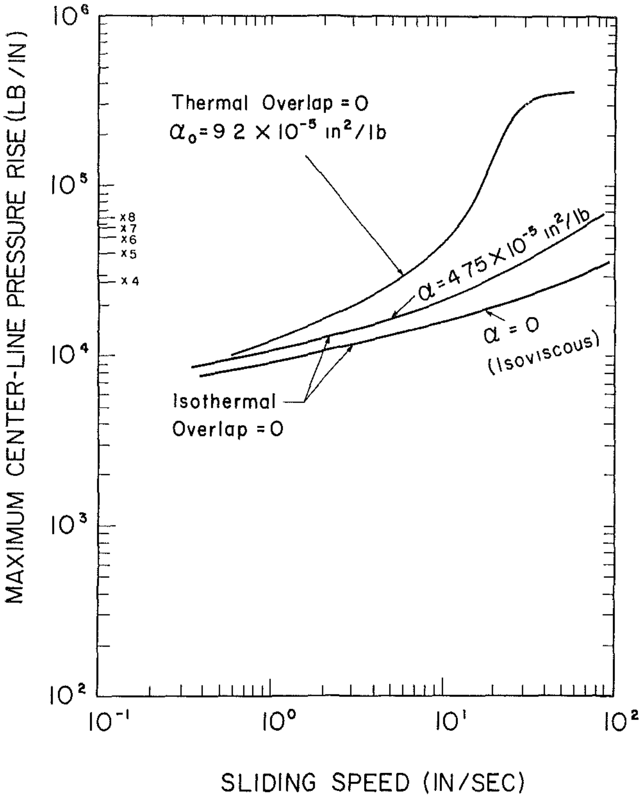

Fig. 17

of this difference in lubricant property. We feel such a comparison is important even if it is based on partial data. For this reason, we have converted some of the nondimensional film thickness and pressure in [1] and plotting them together with the thermal data. Fig. 16 shows the comparison of minimum film thickness at the mid-point of collision (this is not the absolute minimum film thickness during the collision) for the overlap equal to 0.39 × 10^{-6} in. The trend of the thermal curve agrees remarkably well with the isothermal ones. Moreover, these curves also suggest strongly that the actual isothermal curve for α = 9.2 × 10^{-5} in²/lb might be very close to the thermal results for α_0 = 9.2 × 10^{-5} in²/lb (α_0 is the equivalent pressure-viscosity coefficient at ambient temperature).

The comparison of the maximum center-line pressure between the isothermal and thermal data for overlap equal to zero is shown in Fig. 17. These curves substantiate the author's point, that at low sliding the thermal effects on pressure are indeed moderate. Reduction of pressure due to thermal action is only evident at high sliding speeds. Based on these comparisons, one is led to believe that at moderate sliding speeds, the influence of temperature on the film thickness as well as the center-line pressure is rather moderate. Similar findings were also found in [7] and [3] for rolling and sliding EHD contacts.

The traction data in this paper far exceed any measured so far. Based on past experimental traction, there is overwhelming evidence to support the Smith's limiting shear stress concept.[4] Because of the limiting shear behavior of the lubricant, no experimental coefficients of friction have been found to exceed the range of 0.1 to 0.15. Looking at Figs. 14 and 15, the coefficient of friction, for most cases, either approaches or exceeds unity. One wonders whether such high tractive coefficients actually exist in reality.

[4] Smith, F. W., *ASLE Trans.*, Vol. 5, 1962, p. 142.

H. Christensen[5]

There are two sets of problems in the study of lubrication of rough surfaces. The one set deals with the conditions surrounding the elementary event of two-asperity interaction or collision and is the subject of Dr. Fowles' paper. The other set of problems is concerned with the adding-up of all the contributions from the individual interactions taking place at a given instant to determine the effect on the surface as a whole. In doing this the roughness asperities must be described in terms of a range or distribution of heights, curvatures, etc., and one approach to the study of lubrication of rough surfaces is to regard the roughness as a random process characterized by a number of statistical parameters. This approach leads in its turn to a probabilistic theory of lubrication. In such a probabilistic theory of mixed lubrication the elementary event is the interaction or collision of two opposing asperities and it is necessary to possess a model or mathematical description of the conditions surrounding this interaction. One such model that has been used is based upon the simple adhesion theory of friction which operates on two material parameters only: an average "yield pressure" and a "surface shear stress." Dr. Fowles has based his model on elastohydrodynamic theory which seems more suitable in mixed lubrication situations. As it stands the analysis is far too complex to be used directly as a model of roughness interaction in a probabilistic lubrication theory. An important question is therefore whether it is going to be possible to express the results in a simpler form. Such a development has already taken place in ordinary (macro) elastohydrodynamics. From a number of independent numerical solutions a simple power-type law has emerged connecting load capacity with the relevant material and lubricant properties, geometry and kinematic conditions.

Dr. Fowles' analysis shows that the conditions surrounding the two-asperity interaction can be extreme both as regards pressure, friction, and temperature. In an analysis as complex as that presented there is, naturally, a number of questions that can be asked about the assumptions and the author has himself listed a number of such questions. However, it seems to me that rather than trying to refine the analysis further, and thereby compounding the complexity, by incorporating a more complex rheological structure of the lubricant, include shear deformation of the asperities, different boundary conditions in pressure and temperature, etc., a most fruitful line of development would be in the direction of simplification and generalization.

A mathematical model asperity interaction, simple enough to use and at the same time preserving the salient features of elastohydrodynamics would be of great value to the understanding of lubrication of rough surfaces and to the important process of running-in and surface failure of lubricated contacts.

R. S. Fein[6]

Dr. Fowles' paper represents another important step towards understanding how passing asperities can carry load without "touching." It is gratifying that Dr. Fowles' thermal (micro-) elastohydrodynamic theory substantially confirms the film thicknesses predicted by a simpler isothermal analysis.[7] This is illustrated in Fig. 18 which shows the simple analysis results superposed on the author's Fig. 9. The simple analysis should be reasonably valid when the calculated micro-EHD film thickness causes the asperities to deform and the deformation is much less

[5] Head of Section for Machine Dynamics, SINTEF, The Engineering Research Foundation at the Technical University of Norway, Trondheim, Norway. Mem. ASME.

[6] Texaco Inc., Beacon, N. Y.

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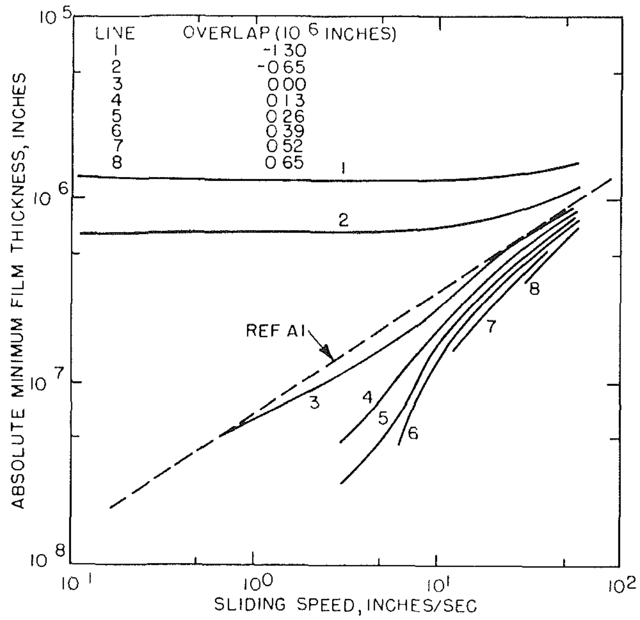

Fig. 18

than the asperity heights. Note near-perfect agreement of the simple theory and the author's 0.00 microinch overlap line. Also, note the agreement within a factor of about two for (1) all positive overlaps when the minimum thickness films are reasonably thick (> $\sim 10^{-7}$ in. or 25A°) and (2) all negative overlaps when the minimum thickness films are at least about equal to the absolute value of the negative overlap.

The rough numerical agreement between the present micro-EHD work and the paper by Fein and Kreuz[7] indicates that the micro-EHD films may not be very sensitive to the detailed model for their formation. In this case, the results suggest that the micro-EHD films are probably extremely important for asperity lubrication. In fact, Dr. Fowles seems to understate this importance by choosing a particularly low viscosity lubricant at atmospheric pressure. As a consequence, his calculated minimum film thicknesses only become significant at the higher sliding velocities. However, if the asperities "collided" within a macroscopic elastohydrodynamic conjunction, the calculated film thicknesses would be large even at extremely low sliding velocities; this would be the consequence of the high viscosity produced by the high ambient pressure within the macroscopic conjunction. Thus it appears that asperity separation by microelastohydrodynamic films is facilitated by macroelastohydrodynamic processes.

The other point where Dr. Fowles seems to understate the importance of micro EHD is in the conclusion that when micro-EHD films are thin "boundary processes must take over before completion of the collision." The question is, what "boundary processes"? To this discussor it appears that a boundary process which provides low wear and friction must in many cases be akin to micro-EHD. It is known that chemical reaction can produce viscous films on bearing surfaces.[8] Analysis indicates that these viscous surface films should act as extensions of the solid when the micro-EHD film of bulk lubricant is sufficiently thick.[8] However, when the bulk lubricant film becomes thin, flow of the viscous boundary films should become significant. By virtue of their high viscosity (or, in general, their resistance to being forced from between approaching asperities while flowing coherently), the boundary films will separate colliding asperities at much lower sliding velocities than the bulk lubricant. In this case, Dr. Fowles' "boundary processes" would be microelastohydrodynamic lubrication by boundary lubricant films.

[8] Fein, R. S., "Chemistry in Concentrated Conjunction Lubrication," Proc. Symposium on *Interdisciplinary Approach to the Lubrication of Concentrated Contacts*, NASA SP-237, to be published.

Author's Closure

The author thanks Drs. Cheng, Lee, Christensen, and Fein for their interesting comments.

The conclusions of Drs. Cheng and Lee that the convergence difficulties in using the direct-iterative (D-I) method seem to be much less critical in squeeze-film type problems than in pure rolling EHD problems is shared by the author. The difficulties do, however, exist and the early attempts at obtaining D-I solutions were quite unsuccessful, divergence occurring quite early in most collision series. It was for this reason that the earlier isothermal theory utilized a modification of Dr. Cheng's Newton-Raphson technique. It was subsequently found that, while the use of suitable scaling factors between iterations delayed the onset of divergence somewhat, the satisfactory choice of methods for estimating the pressure and temperature profiles with which to initiate the iterations at each new time step, and for handling the numerical evaluation of the time derivatives, were of much greater importance. The introduction of the least squares fitting of third-order polynomials for both extrapolation and differentiation as described in the paper resulted in an order-of-magnitude improvement in the stability of the solution scheme.

The comparisons between the isothermal and thermal theories presented by Drs. Cheng and Lee are most interesting. The compatability of the two theories, one based on a modified Newton-Raphson scheme and the other on a direct-iteration scheme, lends confidence to the results of both. In addition, if similar comparisons between the traction and load capacity results exist, a considerable step towards simplifying the results will have been taken. The need for expressing the results in a simpler form is the main point of Dr. Christensen's comments. He points out that such a development has already taken place in macroscopic elastohydrodynamics in the form of an empirical correlation of the results of a number of independent solutions. The main obstacle to a similar procedure in thermal asperity-elastohydrodynamics is the large amount of computer time which would be required to obtain a sufficient number of solutions to develop the correlation. This problem is aggravated by the lack of generality of the thermal solutions. If it can be shown that the influence of temperature on the load capacity and traction is not great except under extreme conditions, then considerable savings in time would result from the ability to use the quicker and more general isothermal theory to develop the required data.

Drs. Cheng and Lee question whether the high tractive coefficients predicted in this work actually exist in reality. This question is really impossible to answer. Certainly they do not exist in macroscopic EHD systems and, while the limiting shear stress concept is an appealing and reasonable explanation, the author does not agree that the existence of such a limiting stress has been established beyond doubt. Further, the prediction of such high coefficients under some conditions at the microscopic level does not necessarily imply similar high coefficients at the macroscopic level. The total load capacity and total traction generated between two sliding rough surfaces, from which the macroscopic traction coefficient is obtained, result from the interaction of a multitude of asperity pairs with various tip radii and overlaps and instantaneously at any point in their collision process. A close inspection of Figs. 2 and 3, and of Figs. 14 and 15, shows that any averaging process over all possible conditions will enhance the load capacity relative to the traction, and therefore result in an overall traction coefficient which is lower than the maximum microscopic values.

That the present absolute minimum film thickness data is in such close agreement with Dr. Fein's simpler analysis is interesting but probably fortuitous. This latter analysis envisions an asperity interaction in the form of a simple hemispherical asperity sliding against an infinitely stiff plane, with the film thickness between the asperity and the plane given by the approximate classical equation for point contact derived by Archard and Cowking.[9] It would probably be more appropriate to use the line contact version of this equation in Fig. 18, thereby increasing the predicted film thicknesses by about 45 percent. This, of course, does not materially change the agreement, but whether any fundamental importance should be attached to it is doubtful. The one analysis yields a time-invariant plateau film thickness in a system governed by entrainment effects (simple sliding) while the other yields an instantaneous minimum film thickness which exists at one point in a system governed by squeeze-film effects only. Dr. Fein's use of this agreement to conclude that micro-EHD films are insensitive to the detailed model of their formation is certainly not justified. Squeeze-film systems are capable of generating pressures, and hence load capacities and tractions if sliding is present, far in excess of Hertz values, to which entrainment systems involving rolling/sliding or simple sliding are closely limited. This is clearly illustrated in Figs. 4 and 14, and is a very important difference since load capacity and traction are the principal results from the point of view of the application of asperity EHD theory to the lubrication of rough surfaces.

[9] Archard, J. F., and Cowking, E. W., "Elastohydrodynamic Lubrication at Point Contacts," *Proceedings of the Institution Mechanical Engineers*, Vol. 180, Part 3B, 1965–1966, pp. 47–56.

The author agrees with Dr. Fein that the use of a relatively low viscosity lubricant and atmospheric ambient pressure tend to minimize the significance of the asperity EHD effects. However, the use of this lubricant was dictated by the availability of expressions describing all its relevant properties as functions of temperature and pressure, and atmospheric pressure seemed a good general condition at which to start this work. Certainly there is a need to extend these solutions to more severe ambient conditions and different lubricants.

The term "boundary process" is intended to include whatever happens after the film thins to a level where the present continuum theory loses physical significance. The possible significance of the formation of viscous boundary films during sliding has already been touched on in the general discussion section of the paper. However, the total lack of knowledge concerning such questions as the thickness, rheology, and rate of formation of these boundary films prevents any progress beyond speculation at this time.