Single and multi-objective optimal design of water distribution systems: application to the case study of the Hanoi system
Lina Perelman, Ariel Krapivka and Avi Ostfeld

ABSTRACT

This manuscript describes the application of two recent methodologies developed by the authors for single and multi-objective optimal design of water distribution systems. The single-objective model is a hybrid algorithm incorporating decomposition, spanning tree search, and evolutionary computation, while the multi-objective algorithm integrates features from multi-objective genetic algorithms with the Cross Entropy combinatorial optimization scheme. The two models are implemented on the Hanoi water distribution system, one of the more explored systems in the research literature, through base runs and sensitivity analysis. The single-objective model produced the best known least cost solution for split pipe design, while the multi-objective model has shown robustness and well explanatory outcomes. Discussion of the accomplished results and suggestions for future research are provided.

Key words | design, management, network, optimization, urban water, water distribution systems

INTRODUCTION

Optimal water distribution system design is probably the most explored problem in water distribution systems optimization.

Numerous models for optimal design of water distribution systems have been published in the research literature during the last four decades. A possible classification for those is: (1) decomposition: these methods are based on decomposing the problem into an “inner” linear programming problem which is solved for a fixed set of flows or heads, while the flows or heads are altered at an “outer” problem using a gradient or a sub-gradient optimization technique (e.g. Alperovits & Shamir 1977; Quindry et al. 1979, 1981; Kessler & Shamir 1989; Fujiwara & Khang 1990, 1991; Sonak & Bhave 1993; Eiger et al. 1994); (2) linking simulation with nonlinear programming: these methods are based on linking a network simulation program with a general nonlinear optimization code (e.g. Ormsbee & Contractor 1981; Lansey & Mays 1989; Taher & Labadie 1996); (3) nonlinear programming: these methods utilize a straightforward nonlinear programming formulation (e.g. Watanatada 1973; Shamir 1974); (4) methods employing evolutionary/meta-heuristic techniques: genetic algorithms (e.g. Simpson et al. 1994; Savic & Walters 1997; Salomons 2001; Vairavamoorthy & Ali 2005; Wu & Walski 2005; Krapivka & Ostfeld 2008), simulated annealing (e.g. Loganathan et al. 1995), the shuffled frog leaping algorithm (e.g. Eusuff & Lansey 2003), ant colony optimization (e.g. Maier et al. 2003; Zecchin et al. 2005); (5) other methods: dynamic programming (e.g. Singh & Mahar 2003), integer programming (e.g. Samani & Mottaghi 2006); (6) and recently multi-objective evolutionary optimization: these methods evaluate the tradeoffs of the least cost design problem with other related design competing objectives (e.g. Prasad & Park 2004; Farmani et al. 2005; Vanvakeridou-Lyroudia et al. 2005; Perelman et al. 2008).

This manuscript describes an implementation and comparison through base runs and sensitivity analysis of two recent published methodologies of the authors for
single (Krapivka & Ostfeld 2008) and multi-objective (Perelman et al. 2008) optimal water distribution systems design on the benchmark water distribution system of Hanoi (Fujiwara & Khang 1990). The Hanoi water distribution system is relatively more complex than the systems explored in Krapivka & Ostfeld (2008) and in Perelman et al. (2008). A short description of the two methodologies follows.

SINGLE-OBJECTIVE OPTIMAL DESIGN

A genetic algorithm (GA) (Holland 1975; Goldberg 1989)—linear programming (LP) scheme was suggested by Krapivka & Ostfeld (2008), following the algorithm of Loganathan et al. (1995) for single-objective optimal design of water distribution systems. The methodology incorporates three main stages: decomposition, spanning tree search, and evolutionary computation. Its description is briefly outlined below.

Step 1: decomposition

In this step the optimal design problem of a water distribution system for one loading gravitational systems is decomposed:

$$\Delta q \in Q \left[ \Theta(\Delta q) := \min_{u} c^{T}u \geq 0 \right]. \quad (1)$$

Subject to:

$$LJ(q_0 + L^T \Delta q)u = 0 \quad (2)$$

$$PIJ(q_0 + L^T \Delta q)u \leq b_1 \quad (3)$$

$$Iu = b_2 \quad (4)$$

Where $\Delta q =$ vector of the circular flows; $Q =$ sub space of the circular flows; $c =$ unit cost vector of the candidate pipe diameters; $q_0 =$ an initial vector of flows satisfying mass continuity at nodes; $u =$ vector of length of the candidate pipe diameters; $L, P, J, I =$ basic loop, path, hydraulic gradient, and identity matrices, respectively; and $b_1, b_2 =$ right-hand side parameter vectors, respectively.

Step 2: optimal spanning tree

At this step all spanning trees of the system are scanned using cyclic interchange (Deo 1989). For each spanning tree the minimum cost is computed using linear programming (LP). The least cost spanning tree is then selected, complemented by its chords with minimum allowable pipe diameters.

Step 3: evolutionary computation

The objective at this stage is to minimize $\Theta \Delta q =$ subject to $\Delta q \in Q$ (i.e. Equation (1)), in conjunction with the least cost spanning tree layout with the chords constrained to their minimum permitted diameters. $\Theta \Delta q =$ is highly non-smooth and non linear thus using an analytical optimization scheme (e.g. steepest descent) will dim to fail (Eiger et al. 1994; Loganathan et al. 1995). $\Theta \Delta q =$ is thus minimized using GA.

The outcome of the single-objective optimal design model is the least cost system design, allowing links to have split pipe diameters (i.e. a link between two nodes can be constructed using two pipes of different diameters). The proposed methodology in its current form is limited to one loading gravitational water distribution systems.

MULTI-OBJECTIVE OPTIMAL DESIGN

Quantitatively, multi-objective optimization can be defined as the problem of finding the vector of decision variables which satisfy a set of constraints and optimizes a vector function whose elements represent the objective functions. Consequently, a multi-objective optimization problem can be formalized (Perelman et al. 2008) as:

Optimize:

$$Y(z) = (y_1(z), y_2(z), \ldots, y_k(z))^T \quad (5)$$

Subject to:

$$a_i(z) > 0 \quad i = 1, 2, \ldots, A \quad (6)$$

$$b_j(z) = 0 \quad j = 1, 2, \ldots, B \quad (7)$$

Where $a_i =$ decision variable; $b_j =$ parameter vectors, respectively.
where: \( Y(\xi) \) = vector function of \( \xi \) objectives, \( z = (z_1, z_2, \ldots, z_\theta)^T \) vector of \( \theta \) decision variables, \( a_i(z) \) = the \( i \)-th inequality constraint, and \( b_j(z) \) = the \( j \)-th equality constraint.

Two objectives are employed in this study: minimum cost versus maximum pressure deficit at a demand node (Farmani et al. 2005). The later serves as a surrogate to system performance, which competes against system cost (i.e. as the maximum pressure deficit at a node reduces cost increases, and vice versa). The model output thus provides a tradeoff between minimum system cost and maximum pressure deficit, allowing a quantitative assessment of cost versus system performance.

In recent years several evolutionary multi-objective methods employing genetic algorithms were developed (e.g. VEGA, Schaffer 1985; MOGA, Fonseca & Fleming 1995; NSGA-II, Deb et al. 2002) and applied for water distribution systems and water resources systems management (e.g. Prasad et al. 2004; Prasad & Park 2004; Reed & Minsker 2004; Farmani et al. 2005; Vamvakeridou-Lyroudia et al. 2005; Perelman et al. 2008).

The proposed multi-objective method integrates features form multi-objective genetic algorithms (Fonseca & Fleming 1995) with the Cross Entropy combinatorial optimization scheme (Rubinstein 1999). The method involves the following steps:

1. **Iteration counter**: set an iteration counter \( t = 0 \).
2. **Initialization**: choose an initial probability vector \( \hat{p}_0 \) with components\( p_{0i} \) \((i = 1, \ldots, m)\) where \( p_{0i} \) is the probability of success of a diameter \( i \) (i.e. selection) at \( t = 0 \), and \( m \) is the total number of available diameters.
3. **Sample solutions**: randomly generate \( N \) sample vectors \( X_i \) \((i = 1, \ldots, N)\), at iteration \( t \) using the probability vector \( \hat{p}_t \) (i.e. generate \( N \) “zero-one” solution vectors each of size \( m \), where a “one” implies a diameter selection and a “zero” otherwise).
4. **Performance assignment**: assign each generated solution vector \( X_i \) a vector of performance objective values \( S(X_i) \).
5. **Rank assignment**: assign each sampled vector \( X_i \) a rank \( R[S(X_i)] \):
   \[
   R[S(X_i)] = 1 + I_i
   \]  
   where \( I_i \) is the number of solutions which dominate solution \( X_i \) in the current iteration.

All non dominated solutions are assigned a rank of one, whereas all the dominated solutions are penalized according to the population density of the non dominating solutions.

6. **Solution sorting**: sort solutions according to their assigned ranks:
   \[
   R_1[S(X_1)] \leq R_2[S(X_2)] \leq \ldots \leq R_N[S(X_N)]
   \]  
   where \( R_1[S(X_1)] \) is the lowest rank value corresponding to design \( X_1 \). The solution ensemble associated with the lowest rank values comprises the best Pareto front of the current iteration.

7. **Updating of the probability vector \( \hat{p}_t \) to \( \hat{p}_{t+1} \)**
   \[
   \hat{p}_{t+1,i} = \frac{\Phi_{t,i}}{\rho N} \quad i = 1, \ldots, m
   \]  
   where: \( \hat{p}_{t+1,i} \) =the \( i \)-th component of the probability vector \( \hat{p}_{t+1} \) at iteration \( t + 1 \); \( \rho \) = the Elite sample percentage (e.g. 1%); and \( \Phi_{t,i} \) the number of times diameter \( i \) is selected at iteration \( t \) within the Elite sample. To prevent from the probability vector to get trapped in a zero-one solution the probability is smoothed using:
   \[
   \hat{p}_{t+1,i} = \alpha \hat{p}_{t+1,i} + (1 - \alpha) \hat{p}_{t,i} \quad \forall i = 1, \ldots, m
   \]  
   where \( \alpha \in (0, 1) \) is a smoothing parameter.

8. **Check convergence criteria**: assemble the Pareto front using the solutions of all previous iterations. Convergence is declared if for a predefined number of subsequent iterations no additional non-dominated solutions are added to the assembled Pareto front. If convergence is attained—STOP and define all current non-dominated solutions as the best approximated Pareto front; otherwise: \( t \leftarrow t + 1 \) and return to step (3) above.

**APPLICATION TO THE CASE STUDY OF THE HANOI WATER DISTRIBUTION SYSTEM**

The case study of the Hanoi network (Fujiwara & Khang 1990, 1991) is one of the more explored benchmark systems in the water distribution systems optimization literature.
Its layout is described in Figure 1. The system is subject to a one demand loading condition, and consists of 34 links and 32 demand nodes supplied by a single reservoir at a constant head of +100 m. The minimum pressure head requirement at all nodes is 30 m. All nodes are at zero elevation. Six candidate pipe diameters 12, 16, 20, 24, 30, 40 (inch) with a Hazen-Williams coefficient of 130 are considered for each of the links. The goals of the single and multi-objective optimization are: minimum cost design and minimizing cost design versus minimizing maximum pressure deficit. The cost $C$ ($) of installing a pipe of diameter $d$ (mm) and length $L$ (m) is:

$$ C = 8.593 \times 10^{-3}d^{1.5}L $$

(12)

**Single-objective optimal design**

Table 1 describes the algorithm application results and comparison to previous studies. The optimal spanning tree chords used were 16, 28, and 31 (see Figure 1), following Sonak & Bhave (1993). It can be seen from Table 1 that the algorithm provided the best feasible solution of 6,055,246 compared to previous published results.

Table 2 is a sensitivity analysis on the Hazen-Williams coefficients used for the headloss computations. As noted by Savic & Walters (1997) different Hazen-Williams formulas were used in different studies as of the employment of different units, which implied differences in the resulted optimal solutions.

To perform a full objective comparison to previous studies the same Hazen-Williams coefficients were selected (Table 2). The proposed single-objective algorithm improved the results obtained by Fujiwara & Khang (1990) (6,027,806 versus 6,320,000) and by Sonak & Bhave (1995) (6,027,806 versus 6,045,500). Comparison to Eiger et al. (1994) could not be accomplished as Eiger et al. did not provide all the adopted Hazen-Williams coefficients. Also it should be noted that the optimal solution value increased for employed Hazen-Williams coefficients which produced higher headlosses [e.g. 6,153,140 using the upper bound Hazen-Williams headlosses coefficients of Quindry et al. (1981), compared to all other generated solutions]. The average number of linear programming evaluations was 2500, and the probability of mutation of the genetic algorithm: 0.025.

**Multi-objective optimal design**

Table 3 and Figures 2 and 3 summarize the multiobjective analysis for the Hanoi system. Table 3 describes statistics of 30 runs for a base run and six sensitivity analyses. Convergence was declared if at five subsequent iterations no additional non-dominated solutions were added to the Pareto front.

The following describes the results presented in Table 3. As noted above, six candidate pipe diameters with a Hazen-Williams coefficient of 130 were considered for each of the links. This consequently defined the number of decision variables as: ndv = 204 [34 (number of links) \times 6 (candidate diameters for each link)] and the design space as $6^{34} = 2.87 \times 10^{26}$.

The sample size at the base run was $15 \times \text{ndv} = 3,060$ (ndv = 204), with $p = 0.03$, and $a = 0.7$. A uniform initial probability was defined ($p_{0,i} = 1/\text{nd}$) = 1/6, $\forall i = 1, \ldots, 204$ in the base run and all six sensitivity analyses, assuming that initially all diameters have the same probability to be selected. The number of iterations until convergence was on average approximately 50, with an average of 147.23 non-dominated solutions at the best approximated Pareto front of a single run. The best Pareto front constructed using all 30 base runs contained 189 solutions, out of which 108 were also present at the merged approximated optimal
Table 1: Hanoi system—single-objective comparison results

<table>
<thead>
<tr>
<th>Link/Node (see Figure 1)</th>
<th>Fujisawa &amp; Khang (1991, Table 3)</th>
<th>Sonak &amp; Bhave (1993, p. 2442, third paragraph from top)</th>
<th>Elger et al. (1994, Table 4)</th>
<th>Current study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D$ (inch) $[L$ (m)] $P$ (m)</td>
<td>$D$ (inch) $[L$ (m)] $P$ (m)</td>
<td>$D$ (inch) $[L$ (m)] $P$ (m)</td>
<td>$D$ (inch) $[L$ (m)] $P$ (m)</td>
</tr>
<tr>
<td>1</td>
<td>40 (100)</td>
<td>100$^*$</td>
<td>40 (100)</td>
<td>40 (100)</td>
</tr>
<tr>
<td>2</td>
<td>40 (1,550)</td>
<td>97.14</td>
<td>40 (1,550)</td>
<td>40 (1,550)</td>
</tr>
<tr>
<td>3</td>
<td>40 (900)</td>
<td>61.67</td>
<td>40 (900)</td>
<td>40 (900)</td>
</tr>
<tr>
<td>4</td>
<td>40 (1,150)</td>
<td>57.09</td>
<td>40 (1,150)</td>
<td>40 (1,150)</td>
</tr>
<tr>
<td>5</td>
<td>40 (1,450)</td>
<td>51.42</td>
<td>40 (1,450)</td>
<td>40 (1,450)</td>
</tr>
<tr>
<td>6</td>
<td>40 (450)</td>
<td>45.47</td>
<td>40 (450)</td>
<td>40 (450)</td>
</tr>
<tr>
<td>7</td>
<td>30 (70); 40 (780)</td>
<td>44.08</td>
<td>40 (850)</td>
<td>40 (850)</td>
</tr>
<tr>
<td>8</td>
<td>30 (140); 40 (710)</td>
<td>42.04</td>
<td>40 (850)</td>
<td>40 (850)</td>
</tr>
<tr>
<td>9</td>
<td>30 (220); 40 (580)</td>
<td>40.10</td>
<td>40 (659.6); 30 (140.4)</td>
<td>40.26</td>
</tr>
<tr>
<td>10</td>
<td>24 (70); 30 (880)</td>
<td>38.36</td>
<td>30 (950)</td>
<td>30 (950)</td>
</tr>
<tr>
<td>11</td>
<td>24 (520); 30 (680)</td>
<td>36.58</td>
<td>24 (1,200)</td>
<td>24 (1,198.96); 30 (1.04)</td>
</tr>
<tr>
<td>12</td>
<td>20 (410); 24 (3,050)</td>
<td>34.44</td>
<td>24 (3,500)</td>
<td>24 (3,500)</td>
</tr>
<tr>
<td>13</td>
<td>16 (50); 20 (750)</td>
<td>NF 29.57</td>
<td>20 (639.5); 16 (160.54)</td>
<td>NF 29.50</td>
</tr>
<tr>
<td>14</td>
<td>12 (20); 16 (480)</td>
<td>35.21</td>
<td>16 (500)</td>
<td>12 (500)</td>
</tr>
<tr>
<td>15</td>
<td>12 (550)</td>
<td>34.10</td>
<td>12 (550)</td>
<td>12 (550)</td>
</tr>
<tr>
<td>16</td>
<td>20 (710); 24 (2,020)</td>
<td>33.50</td>
<td>12 (2,730)</td>
<td>NF 29.51</td>
</tr>
<tr>
<td>17</td>
<td>24 (1,170); 30 (580)</td>
<td>44.83</td>
<td>16 (1,682.8); 12 (67.2)</td>
<td>NF 29.51</td>
</tr>
<tr>
<td>18</td>
<td>24 (70); 30 (730)</td>
<td>54.23</td>
<td>20 (800)</td>
<td>24 (800)</td>
</tr>
<tr>
<td>19</td>
<td>24 (50); 30 (370)</td>
<td>59.18</td>
<td>20 (400)</td>
<td>24 (400)</td>
</tr>
<tr>
<td>20</td>
<td>30 (140); 40 (2,060)</td>
<td>52.15</td>
<td>40 (2,200)</td>
<td>40 (2,200)</td>
</tr>
<tr>
<td>21</td>
<td>16 (760); 20 (740)</td>
<td>33.49</td>
<td>20 (989.1); 16 (510.9)</td>
<td>34.69</td>
</tr>
<tr>
<td>22</td>
<td>12 (350); 16 (150)</td>
<td>NF 29.49</td>
<td>12 (500)</td>
<td>NF 29.52</td>
</tr>
<tr>
<td>23</td>
<td>30 (1,480); 40 (1,170)</td>
<td>42.41</td>
<td>40 (2,650)</td>
<td>43.94</td>
</tr>
<tr>
<td>24</td>
<td>20 (440); 24 (790)</td>
<td>35.77</td>
<td>30 (1,230)</td>
<td>38.16</td>
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<tr>
<td>25</td>
<td>16 (350); 20 (950)</td>
<td>30.71</td>
<td>30 (1,300)</td>
<td>34.42</td>
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<tr>
<td>26</td>
<td>12 (850)</td>
<td>31.27</td>
<td>20 (850)</td>
<td>30.05</td>
</tr>
<tr>
<td>27</td>
<td>20 (90); 24 (210)</td>
<td>31.86</td>
<td>16 (284.8); 12 (15.2)</td>
<td>NF 29.51</td>
</tr>
<tr>
<td>28</td>
<td>24 (650); 30 (120)</td>
<td>38.05</td>
<td>12 (750)</td>
<td>37.28</td>
</tr>
<tr>
<td>29</td>
<td>20 (830); 24 (670)</td>
<td>32.71</td>
<td>16 (1,500)</td>
<td>NF 29.51</td>
</tr>
<tr>
<td>30</td>
<td>16 (180); 20 (1,820)</td>
<td>NF 29.58</td>
<td>16 (968.6); 12 (1,031.4)</td>
<td>NF 29.51</td>
</tr>
<tr>
<td>31</td>
<td>16 (1,250); 20 (350)</td>
<td>NF 29.50</td>
<td>12 (1,600)</td>
<td>NF 29.72</td>
</tr>
<tr>
<td>32</td>
<td>12 (150)</td>
<td>NF 29.50</td>
<td>16 (150)</td>
<td>31.70</td>
</tr>
</tbody>
</table>
### Table 1  
(continued)

<table>
<thead>
<tr>
<th>Link/Node</th>
<th>Fujiwara &amp; Khang (1991, Table 3)</th>
<th>Sonak &amp; Bhave (1993, p. 2442, third paragraph from top)</th>
<th>Eiger et al. (1994, Table 4)</th>
<th>Current study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(see Figure 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>D (inch) [L (m)]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>P (m)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>12 (860) NA 16 (860) NA</td>
<td>16 (652.92); 20 (227.08) NA 16 (750.54); 20 (109.46) NA</td>
<td>16 (750.54); 20 (109.46) NA</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>20 (260); 24 (690) NA</td>
<td>24 (702.8); 20 (247.2) NA 24 (950) NA</td>
<td>24 (950) NA</td>
<td></td>
</tr>
<tr>
<td>Cost ($)</td>
<td>6,320,000</td>
<td>6,045,500</td>
<td>6,026,660</td>
<td></td>
</tr>
</tbody>
</table>

*Legend and comments: 100 = source total head of 100 m; 1 inch = 25.4 mm; NA = not applicable; D = pipe diameter (inch); L = pipe length (m); P = pressure (m); 30 = binding pressure constraint of 30 m; NF 29.5 = non feasible pressure less than 30 m; solution verification using EPANET 2.0.11@ http://www.epa.gov/nrmrl/wswrd/dw/epanet.html

### Table 2

<table>
<thead>
<tr>
<th>Study</th>
<th>Hazen-Williams formula*</th>
<th>Hazen-Williams coefficients</th>
<th>Optimal cost</th>
<th>Published solution</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quindry et al. (1981)</td>
<td>$h_f = (6.2 \times 10^{-4})^{-1.0.54}(Q/C)^{1.0.54}D^{-2.630.54}L$</td>
<td>$a = 1.155,883$, $b = 1/0.54$, $g = -2.63/0.54$</td>
<td>6,153,140</td>
<td>NA</td>
<td>Upper Hazen-Williams formula bound (Savic &amp; Walters 1997)</td>
</tr>
<tr>
<td>Fujiwara &amp; Khang (1991)</td>
<td>$h_f = 162.5(Q/C)^{1.85}D^{-4.87}L$ for $h_f$ (m); $Q$ (m$^3$/hr); $D$ (in); $L$ (m)</td>
<td>$a = 1.128,219$, $b = 1.85$, $g = -4.87$</td>
<td>6,027,806‡</td>
<td>6,320,000‡</td>
<td>Lower Hazen-Williams formula bound (Savic &amp; Walters 1997)</td>
</tr>
<tr>
<td>Sonak &amp; Bhave (1993)</td>
<td>$h_f = 162.5(Q/C)^{1.85}D^{-4.87}L$ for $h_f$ (m); $Q$ (m$^3$/hr); $D$ (in); $L$ (m)</td>
<td>$a = 1.128,219$, $b = 1.85$, $g = -4.87$</td>
<td>6,027,806‡</td>
<td>6,045,500‡</td>
<td>Same Hazen-Williams formula as in Fujiwara &amp; Khang (1991)</td>
</tr>
<tr>
<td>Eiger et al. (1994)</td>
<td>$h_f = \alpha(Q/C)^{1.852}D^{-4.87}L$</td>
<td>$a = NA$, $b = 1.852$, $g = -4.87$</td>
<td>6,026,660</td>
<td>NA</td>
<td>The Hazen-Williams $\alpha$ coefficient not published. Impossible to compare to GA-LP</td>
</tr>
<tr>
<td>Current study</td>
<td>$h_f = 4.727(Q/C)^{1.852}D^{-4.871}L$ for $h_f$ (ft); $Q$ (cfs); $D$ (ft); $L$ (ft)</td>
<td>$a = 1.134,391$, $b = 1.852$, $g = -4.871$</td>
<td>6,055,246</td>
<td>6,055,246</td>
<td>This study: Hazen-Williams formula used as in EPANET 2.00.11</td>
</tr>
</tbody>
</table>

$h_f = \alpha(Q/C)^{\beta}D^{\gamma}L$; where $h_f = \text{headloss}$; $Q = \text{flow}$; $C = \text{roughness coefficient}$; $D = \text{diameter}$; $L = \text{length}$; $a = \text{coefficient dependent on the units of} Q, D, \text{and} L; \beta, \gamma = \text{coefficients}$; NA = not available.


†Comparable solutions to Sonak & Bhave (1993).
Pareto front (i.e. BR & SA1-SA3) established using the solutions of all cases.

In sensitivity analysis 1—case L (SA1-L) the sample size was reduced to half of the base run size. The small sample size restricted the algorithm from properly spanning the objectives space. As a result, the average number of non-dominated solutions at a single run was the lowest (i.e. 133.70) among all cases with the highest standard deviation of 21.97. The number of non-dominated solutions at the merged approximated optimal Pareto front constructed using all solutions was also low (i.e. 29).

In SA1-U the sampling size was increased to five times the sampling size of the base run. As a result, the highest average number of non-dominated solutions at a single run with the lowest standard deviation (i.e. 182.03 and 10.08, respectively) was received. However, the number of solutions present at the merged approximated optimal Pareto front was not the highest among all cases. This is attributed to the value of the elite sample size parameter $\rho$ which remained unchanged. Increasing the sampling size while not reducing $\rho$ caused the sampling probability to be updated by a larger set of elite solutions of which some were not “elite solutions”. This caused the resulted number of solutions present at the merged approximated optimal Pareto front to be high, but not the highest.

In SA2-L $\rho$ was reduced to 0.01. This caused the elite sample to be too small and thus the sampling probability not to update properly. As a result, only one solution of this case was present at the merged approximated optimal Pareto front.

In SA2-U $\rho$ was increased to 0.04. This caused the elite sample to increase but also reduced the quality of the “elite” solutions employed for the sampling probability updating. As a result, the number of non-dominated solutions at the merged approximated optimal Pareto front was 51 compared to 108 in the base run.

Pareto optimal fronts for sensitivities of pressure and source head.

<table>
<thead>
<tr>
<th>Case</th>
<th>Sample size</th>
<th>$\rho$</th>
<th>$\alpha$</th>
<th>Average number of iterations to convergence (standard deviation)</th>
<th>Average number of non-dominated solutions (standard deviation)</th>
<th>Number of non-dominated solutions at best merged Pareto front</th>
<th>Number of non-dominated solutions at best merged Pareto front (BR &amp; SA1-SA3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR</td>
<td>3060</td>
<td>0.03</td>
<td>0.7</td>
<td>50.07 (7.57)</td>
<td>147.23 (17.28)</td>
<td>189</td>
<td>108</td>
</tr>
<tr>
<td>SA1-L</td>
<td>1530</td>
<td>0.03</td>
<td>0.7</td>
<td>48.10 (7.85)</td>
<td>133.70 (21.97)</td>
<td>175</td>
<td>29</td>
</tr>
<tr>
<td>SA1-U</td>
<td>15300</td>
<td>0.03</td>
<td>0.7</td>
<td>46.40 (7.89)</td>
<td>182.03 (10.08)</td>
<td>220</td>
<td>81</td>
</tr>
<tr>
<td>SA2-L</td>
<td>3060</td>
<td>0.01</td>
<td>0.7</td>
<td>40.37 (5.83)</td>
<td>136.07 (19.63)</td>
<td>149</td>
<td>1</td>
</tr>
<tr>
<td>SA2-U</td>
<td>3060</td>
<td>0.04</td>
<td>0.7</td>
<td>52.10 (10.2)</td>
<td>152.53 (20.30)</td>
<td>244</td>
<td>51</td>
</tr>
<tr>
<td>SA3-L</td>
<td>3060</td>
<td>0.03</td>
<td>0.6</td>
<td>56.37 (8.69)</td>
<td>160.07 (19.05)</td>
<td>225</td>
<td>152</td>
</tr>
<tr>
<td>SA3-U</td>
<td>3060</td>
<td>0.03</td>
<td>0.8</td>
<td>43.50 (7.15)</td>
<td>147.07 (18.10)</td>
<td>213</td>
<td>40</td>
</tr>
</tbody>
</table>

Note: BR = Base run; SA1-L = Sensitivity analysis 1—case L.

Figure 2 | Hanoi system—Pareto optimal fronts for the base run and sensitivity analyses.

Figure 3 | Hanoi system—Pareto fronts for sensitivities of pressure and source head.
In SA3-L the smoothing parameter $a$ was reduced to 0.6. This resulted in a larger portion of the probability vector of the previous iteration to be accounted for probability updating in the current iteration. This caused the algorithm to explore more solutions thus converging more slowly (i.e. the average number of iterations to convergence was 56.37—the highest among all cases), and yielding the smallest average Euclidean normalized generational distance to the best Pareto front (i.e. $8.98 \times 10^{-4}$). Also, as an outcome of that, the number of non-dominated solutions at the merged approximated optimal Pareto front was 152—the highest amongst all cases.

In SA3-U the smoothing parameter $a$ was increased to 0.8. This caused the algorithm to converge more quickly in comparison to SA3-U and to the base run, yielding 40 non-dominated solutions at the merged approximated optimal Pareto front.

Figure 2 describes the plots of the best Pareto fronts for all cases and the merged approximated optimal Pareto front. It can be seen from Figure 2 that all Pareto fronts are very close to each other. It should be noted that for zero maximum pressure deficit the best cost solution obtained was $6.22 \times 10^6$ which is higher than the best published single-objective cost solution of $6.08 \times 10^6$ ($\text{Perelman et al., 2008}$) for a non-split pipe diameter model With the option of split pipe diameters the best cost solution obtained was slightly improved to $6.05 \times 10^6$ (see Table 1).

Figure 3 presents sensitivity analyses for the minimum pressure constraint and for the total head at the source. Increasing the minimum pressure head for all nodes to 35 m (30 m at the base run) resulted the entire Pareto front to shift rightwards compared to the Pareto front of the base run (i.e. increasing all solutions cost). Increasing the total head at the source to 120 m (100 m at the base run), caused the Pareto front to shift leftwards with reduction of cost for all solutions.

CONCLUSIONS

The applications on the Hanoi system of single and multi-objective water distribution systems optimal design algorithms were presented. The single optimization model was able to provide improved results over previous published studies, while the multi-objective model produced robust and well explanatory outcomes for the base run and sensitivity analysis runs.

The single-objective optimization model provided the best published feasible split pipe solution for the Hanoi system. The algorithm is however limited with respect to system size and can handle only one loading gravitational systems. An extension of the single optimization model to multiple loadings and to systems of increased size requires further research. One of the possible directions for dealing with multiple loadings and increased size systems is to use an evolutionary algorithm for the optimal spanning trees selection (i.e. instead of scanning all trees and running an LP for each, employ a GA model for selecting an optimal tree for each loading).

Further research is also needed for testing the multi-objective model for: (1) more complex systems, (2) incorporation of different competing objectives such as reliability, redundancy, etc., (3) comparison to other evolutionary/analytical multi-objective schemes [i.e. others than NSGA-II for which a comparison (Perelman et al. 2008) for the New York Tunnels system (Schaake & Lai 1969) was performed], and (4) incorporation of water quality constraints and objectives.

As evolutionary computation is employed at both the single and multi-objective methodologies (i.e. GA and CE, respectively), further implementations will also require: 1. A much higher computational effort which could be addressed through parallel computation, and 2. Algorithm parameterization (e.g. population size, crossover probability, elite sample, etc.) which needs further exploration and research and probably the developments of additional enhanced calibration models.

ACKNOWLEDGEMENTS

This study was supported by the Technion Grand Water Research Institute (GWRI).

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