S and the Structure of the Upper Mantle

I. Lehmann


Summary

The European as well as the north-eastern American observations of S at small epicentral distances indicate the presence of a low velocity layer. In Europe its upper boundary seems to be at a depth of about 140 km. Since late S phases are observed at epicentral distances down to about 10° there is likely to be an abrupt increase of velocity (as well as of velocity gradient) at the lower boundary of the layer at about 220 km depth. Late S phases beyond 20° can be accounted for if a further strong increase of velocity gradient at a greater depth is assumed.

1. Introduction

It is well known that in shallow and not very deep earthquakes, S at epicentral distances smaller than about 25° causes difficulty. The "true" S is mostly weak or completely missing and a later, larger phase is often read for it. This seems to be true for most regions, and, using I.S.S. data, Lee (1938) demonstrated that at small epicentral distances and especially from about 5° to 14°, S travel times failed to concentrate on a mean value. Gutenberg, who always considered amplitude variation in relation to the shape of the time-curve, saw that both P and S were relatively weak at small epicentral distances, and in several publications he drew attention to the fact and suggested velocity functions that would account for it (Gutenberg 1926, 1959; for further references consult the latter publication). Wadati & others (1933) have remarked on the ambiguity of S and mentioned the presence of a large phase, seemingly not the true S. When the time-curve for S of a surface shock was constructed the curve from about 16° onwards was first obtained by reduction of the travel times of a deep focus shock and it was afterwards extended to small epicentral distances on the assumption that P and S travel times were always in the same ratio. It was a similar though more refined procedure that Jeffreys (1939) used when it had been found that the time-curve could not be constructed directly from observations.

Explosion results showed that the Jeffreys–Bullen (J.B.) transmission times for longitudinal waves were in error at small epicentral distances and European travel times have been revised in recent years (Jeffreys 1952, 1954, 1958). The P curve was found to be very nearly straight up to 15°. From this distance onwards the time-curve had a considerable bend. The upper part of the time-curve is found to be explicable as that of a wave refracted in a surface of discontinuity placed at a depth of 220 km or thereabouts (Lehmann 1959).
When a revision of the $S$ transmission times for short epicentral distances is to be undertaken we have to consider that the velocity of $S$ like that of $P$ has originally been taken too small just below the Mohorovičić discontinuity. From the J.B. tables the initial velocity of the direct wave $Sd$ was found to be $4.35 \text{ km/s}$ while recent explosion work has yielded velocities of about $4.7 \text{ km/s}$. The results have no great certainty and the velocity may not be quite so high, but there is no doubt about its being greater than the J.B. value.

2. N. American observations

In north-eastern America $Sd$ was found to be better recorded than in Europe and perhaps better than in most parts of the world (Lehmann 1955). In the few larger earthquakes that occurred in north-eastern America $Sd$ was a clear and well-recorded phase and the time-curve could be represented by the straight line

$$t = a \Delta s + \Delta \times 24.07 \text{ s/deg}$$

with $a$ varying a little from one earthquake to another. At $14^\circ$ epicentral distance the time-curve broke off; although $S$ was clearly recorded at about this distance it did not appear at greater distances. From about $20^\circ$ onwards the $S$ curve was found to be a J.B. curve at its original height. At intermediate distances there were only few observations, but the travel times fitted on approximately to the J.B. curve that at $14^\circ$ is $13^\circ$ above the $Sd$ line as determined.

The break in the time-curve and the delay of the second branch against the first one indicates the presence of a low velocity layer. At the depth grazed by the ray emerging at $14^\circ$ there is a sudden or a strong gradual decrease of velocity with depth. When this was realized it seemed at first as if the structure in north-eastern America of the upper mantle must differ radically from the structure in other regions having a low velocity layer at a different depth. In the course of further studies it was found that this was not the case.

We may ask at what depth the low velocity layer of north-eastern America is likely to be. The observations at hand are not very numerous and it is not possible to construct a time-curve with the accuracy that would be needed if the velocity distribution were to be derived from it. The $Sd$ curve is approximately a straight line, but it probably has a slight curvature corresponding to some increase of velocity with depth since the phases are so well recorded. It was assumed tentatively that the low velocity layer began at $120 \text{ km depth}$. Then the ray emerging at $14^\circ$ had to have its deepest point at this depth and this would happen if the velocity increased downwards from the Mohorovičić discontinuity by about $0.05 \text{ km/s}$, from, say, $4.60$ to $4.65 \text{ km/s}$. The time-curve calculated on this assumption has a slight bend that would not be shown by earthquake observations. Whether this velocity increase would be strong enough to produce clear $S$ phases in earthquakes not excessively strong is a question it may be difficult to answer. If we have to assume the velocity gradient to be higher, the ray emerging at $14^\circ$ will be more strongly bent and go deeper down so that we shall have to take the low velocity layer at a greater depth. It was found subsequently that in Europe the low velocity layer had to be taken deeper down, at a depth of $140-150 \text{ km}$. If in north-eastern America it is at the same depth, the velocity increase above the layer will be greater.

A low velocity layer for $S$ is also indicated by surface wave observations as found at the Lamont Geological Observatory. During my stay there in the winter
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of 1958 J. Dorman and M. Landisman were calculating dispersion curves for Rayleigh and Love waves on various velocity assumptions trying to obtain agreement with those empirically determined. It was of course important that the velocity assumptions they made should yield also acceptable time-curves and therefore co-operation began, M. Landisman undertaking to calculate time-curves on velocity assumptions I suggested, on the Columbia University automatic calculator, IBM 650. This was an invaluable help to me for, although the computations are quite simple when the velocity law \( v = v_0 (r/r_0)^{-k} \) is adopted, yet they are highly time-consuming.

The velocity was taken to decrease abruptly at the upper boundary of the low velocity layer, at first assumed to be at 120 km depth. There is, as far as I can see, nothing to show whether the decrease is abrupt or gradual, so for simplicity the velocity was taken to be constant in the low velocity layer and to increase again abruptly at the lower boundary at 220 km depth where a discontinuous increase of the \( P \) velocity also has been taken to occur.

The ray for which \( dt/d\Delta = r_1/v_1 \) for the upper boundary grazes this boundary. With \( dt/d\Delta \) decreasing the rays enter the layer and they are totally reflected at the lower boundary until \( dt/d\Delta \) assumes the value \( r_1/v_1 \sin i = r_2/v_2 \) of the lower layer. For smaller \( dt/d\Delta \) they pass into the layer. The "first" totally reflected ray emerges at a greater epicentral distance than the "last" direct ray as appears from Figure 1.

![Fig. 1.—Ray paths and time-curves in presence of low velocity layer.](https://academic.oup.com/gsmnras/article-abstract/4/1/124/653772)

The time-curve of the reflected wave is retrograde and it is some way above the time-curve of the direct wave, the distance between the two lines depending on the thickness of the low velocity layer and the velocity in it. The smallest distance at which we have a totally reflected wave depends also on the velocity below the discontinuity. The refracted wave, \( S_r \), begins to emerge at this distance, but it carries very little energy at first. With increasing velocity in the lower layer the intensity of the ray increases and so the \( S_r \) phase increases with distance.

A velocity distribution of the kind here described was considered by Dorman & others (1960) and depicted in their Figure 2. The crust was taken to be 35 km thick and the velocity in it 3.55 km/s. In the uppermost mantle, down to 120 km depth, the velocity increases from 4.60 to 4.65 km/s. In the low velocity layer the velocity is 4.30 km/s and at the lower boundary it increases to 4.70 km/s. It increases uniformly down to 410 km depth where there is an abrupt
increase from 4·95 to 5·15 km/s. It again increases uniformly down to 600 km where it is 5·65 km/s or practically the same as the J.B. value. On the whole there is no great departure from the J.B. velocities below 220 km depth.

The $S_d$ line calculated on the above assumptions has the slope 23·8 s/deg. Originally the north-eastern American $S_d$ travel times were fitted to a line of slope 24·8 s/deg, but they have no great precision and they fit on to the new line as well or better. The calculated $S_r$ travel times are 16 s delayed against $S_d$ at 14°. Around 20° the time-curve has the same shape as the J.B. curve but it is slightly below.

Rayleigh wave dispersion curves calculated on the above assumptions were found to be in good agreement with observations (loc. cit.). Love wave dispersion curves were calculated on similar assumptions by Landisman (unpublished) and by Jobert (1960), but here the agreement with observations was not quite so good.

3. European observations

It is north-eastern American observations of $S$ that have been considered so far. In Europe we have many more observations of $S$ at short epicentral distances, but the so-called true $S$ is usually weak even in the stronger shocks and it is difficult to obtain good observations of it. Most European shocks are not large. Occasionally large shocks occur in Italy and also in Greece, but the Greek surface shocks do not seem to give useful records of $P$ and $S$. Galanopoulus (1953) has remarked on the fact that both $P$ and $S$ are marked by groups of oscillations increasing from small and indefinite beginnings. I have made a study of the European records of one such shock and I found it difficult to fix on corresponding onsets in the records of the different observatories.
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In spite of the fact that improved data have accumulated since the time when the J.B. travel times were determined it may still be impossible to construct an $Sd$ time-curve with any accuracy from the observations of shallow shocks. The $S$ travel times of individual shocks usually scatter a great deal, and since the time-curve as we have it is likely to be in error, we cannot pick out the $Sd$ phases by comparison with it. It is only when the earliest readings of a number of stations within a certain distance range are sufficiently consistent to indicate a time-curve clearly that the phases with some confidence can be identified as $Sd$, and this does not seem to happen very often.

Searching the I.S.S. I found one earthquake in which unusually many of the $S$ travel times reported seemed to be those of $Sd$, and this is the earthquake of 1950 September 5 in the Gran Sasso in Central Italy. There are 37 reporting stations at distances between $2^\circ$ and $10^\circ$ epicentral distance. The $P$ and $S$ travel times of 27 of these together with some for greater distances are tabulated in Table I. Some of the stations omitted have either abnormally large negative $S$ residuals or no $S$; others have erroneous $P$ readings and this casts a doubt on the reliability of their $S$. There are no useful $S$ readings at distances greater than that of Moscow ($20^\circ 4$). Some of the stations have two $S$ readings; those in brackets are in the I.S.S. tabulated as additional readings. The time of occurrence has been taken 2 s later than for the I.S.S.

The residuals of $P$ are with reference to the tables I have calculated (Lehmann 1959, p. 389); from $22^\circ$ onwards the travel times are the same as the J.B. times with 3 s subtracted from them. For $S$ the O–C$_1$ are the residuals against the J.B. $S - 3s$. In the first column we have placed the residuals of all the "early" readings, regardless of whether in the I.S.S. they have been tabulated as $S$ or as additional readings. At the smallest distances these residuals scatter a great deal more than at greater distances, possibly because surface waves interfere and make it difficult to pick the $S$ phase. From $6^\circ$ to $10^\circ$ the residuals are consistently negative. The first of the O–C$_2$ columns contains the residuals of the same travel times against 

$$t = 14s + \Delta^\circ \times 23.8s/\text{deg}.$$  

They are small and have no systematic trend and the line will be taken to represent the $Sd$ time-curve. A line of the same slope was found to fit also the $Sd$ of north-eastern America. It corresponds to a velocity of about 4.63 km/s in the uppermost mantle, and this velocity is considerably greater than that found from the J.B. tables.

Beyond $10^\circ$ we have no travel times fitting on to our $Sd$ line so there is a shadow for the $Sd$ waves of this earthquake beginning at $10^\circ$. But we cannot conclude that in other earthquakes there will also be a shadow beginning at this distance. The Gran Sasso earthquake is not large, and the $Sd$ waves fade out with distance. We shall see presently that in a very large earthquake $Sd$ waves were recorded at greater epicentral distances. The Copenhagen record from an epicentral distance of $13^\circ$–$1$ is the only one I have seen and it is rather small. There is some movement at about the time when $S$ should appear, but there is no clear $S$ phase; the Copenhagen bulletin has no $S$. When for north-eastern America the $Sd$ curve was taken to break off at $14^\circ$ it was because $Sd$ was a clear phase up to that distance and then appeared no more.

It seems necessary to mention here that Di Filippo & Marcelli (1951) made a study of the Gran Sasso earthquake and that they found the $Sd$ travel times for distances up to $13^\circ$ to fit on to a straight line of slope 25.31 s/deg or a slope much
greater than the one here found and greater even than the mean slope of the J.B. S-curve. They determine a slightly different epicentre but that does not alter the travel times greatly. In Table 1 the negative residuals against the J.B. times indicating a smaller slope are at distances from 6° to 10°; 13 travel times have large negative residuals. In the study mentioned 8 of these were not considered and for 4 of the others the authors, reading the records themselves, found different travel times. They continued their line to 13° including a reading at De Bilt and also one at Copenhagen where I fail to see an S.

We shall maintain here that an Sd line of slope of about 23·8 s/deg has been determined and in other shocks we shall find Sd travel times fitting on to the line. It seems, however, to be a rare occurrence that an Sd line is clearly indicated in any one shock.

In Table 1 most of the S travel times that are not Sd times have positive residuals against the J.B. S-3s. They scatter greatly and evidently are not all travel times of one and the same wave. From 13° onwards, however, the scatter is small and a time-curve is indicated that at 20° merges into the J.B. S-3s. Beyond 20° S of this shock is poorly recorded, but we expect the European S-curve to continue as the J.B. S-3s so that, with P being the J.B. P-3s, the S-P interval is preserved. For north-eastern America the S curve was some seconds higher from around 20° onwards and the S-P exceeded the J.B. S-P.

But here as for north-eastern America the Sd curve is a separate branch not connecting with the rest of the S-curve. If prolonged beyond 10° it intersects the J.B. S—3s curve at 21°. At smaller distances the other S phases are greatly delayed against Sd as shown by the residuals in the second O–C2 column of Table 1. A structure accounting for north-eastern American travel times has been indicated and it appears that a similar structure will account for Sd and also for S as observed from about 13° onwards, the waves associated with the upper branch of the time-curve being explainable as waves refracted in a discontinuity surface at depth about 220 km, and the delay being brought about by a low velocity layer above.

Some of the observations at distances smaller than 13° are possibly of the refracted wave Sr, but all the observations are not of this phase. It has been mentioned that surface waves possibly interfere with Sd at the smallest distances, and at somewhat greater distances we can expect higher mode surface waves to arrive later, but not very much later, than Sd; with their relatively short period their appearance may be similar to that of S (see Lehmann & Ewing 1960). The O–C3 of Table 1 are residuals against \( t = \Delta \times 27·8 s/\text{deg} \) corresponding to surface velocity of 4 km/s which is the velocity of second mode surface waves of period 11 s—12 s. Many of the travel times fit on very well to this line, but we should have to examine the records to see whether the onsets read mark beginnings of surface wave trains of the appropriate period.

The readings of some of the observatories bear witness of the difficulty there sometimes is in fixing on an S-phase. Stuttgart (not in the table) at an epicentral distance of 6°9 reads S at 2 min 30 s (residual —32 s), and there are additional readings at 2 min 46 s, 54 s, 3 min 00 s, 06 s, 18 s. The time of Sd should be 2 min 58 s. Warsaw at 10°9 also has several readings.

Three observatories, Jena, Collmberg and Bucharest, have read phases not much later than Sd, two of them in addition to Sd. They probably mark an increase of the Sd oscillations. On 1951 August 8 there was another shock from the same epicentre. In this there were several such slightly late Sd readings with no
Table I

Earthquake of 1950 September 5, 4 h 9 m 2 s. Epicentre 42°6 N, 13°5 E

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<td>13</td>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>Moscow</td>
<td>20.4</td>
<td>4</td>
<td>38</td>
<td>1</td>
<td>82</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note.—For explanation of residuals, see text.
preceding $Sd$ to fit our line, and they prevented the $Sd$ line as we have determined it from being clearly apparent.

An earthquake has been found in which $Sd$ phases are clearly recorded out to an epicentral distance of nearly $19^\circ$ and this is the large Azores earthquake of 1931 May 20, magnitude 7.1 according to Gutenberg & Richter (1949). Its epicentre is taken at $37^\circ.6$ N $16^\circ.0$ W between the Portuguese coast and the Azores. In a study of this earthquake early $S$ phases were read that could not be explained at the time (Lehmann 1934, p. 33). The travel times are now found to fit in to our $Sd$ line of slope $23.8$ s/deg. The readings as they were given in the earlier publication and their residuals against our line are tabulated in Table 2. The $Sd$ phases are small but quite clear as seen in the Paris record of Figure 2 from an epicentral distance of $17^\circ.5$. With constant velocity in the uppermost mantle the ray to $19^\circ$ would have its deepest point at about $115$ km depth; with slightly decreasing velocity it would not go quite so deep. The structure in the epicentral region is not known but it is perhaps likely to be intermediate between continental and oceanic and to have the low velocity layer at a smaller depth than under the continents. In addition to $Sd$ phases late $S$ phases have also been recorded in the Azores shock.

The earthquake of 1933 April 23 in the Dodecanese had only few observations at short epicentral distances but $Sd$ was read at 4 stations at distances smaller than $10^\circ$. All other $S$ phases at distances up to $20^\circ$ were late, and late $S$ phases were also recorded beyond $20^\circ$ as found in a previous study of this earthquake (Lehmann 1953).

The Italian earthquake of 1933 July 23 has also been considered. It is a very much larger earthquake than the 1950 one, but there are in the I.S.S. only 8 readings of $Sd$ that fit on to our line. The greatest epicentral distance at which $Sd$ has been reported is $13^\circ.0$ (Hamburg). Some of the stations have no $S$ but most of them have late readings. The residuals of these scatter far more than in the other earthquakes considered. This and the lack of $Sd$ readings may partly be due to the fact that instrumental equipment was poorer in 1930 than it was in 1950, but the $S$ phase may also be less well developed and not so clear. At Copenhagen at a distance of $14^\circ.7$ where the late $S$ usually has begun to increase the phase is quite small. Much larger second mode surface waves arrive about $20$ s later on the E component record and somewhat later on N (see Lehmann & Ewing 1960, Figure 6).
Fig. 3.—4 shallow earthquakes. Observed and calculated travel times of $S$. 
The time-curves of Figure 3 were calculated on velocity assumptions similar to those on p. 126. The $Sd$ velocity was taken a little smaller, and the slope of the $Sd$ line is 24°. In the low velocity layer the velocity is as before 4:10 km/s, but at the 220 km discontinuity it increases to 4:75 km/s and at 410 km the velocity increase is from 4:90 to 5:05 km/s. At 600 km the velocity is as before 5:65 km/s. Above the $Sd$ line are the time-curves of the waves reflected on and refracted in the two discontinuity surfaces. At distances up to nearly 20° the lowest of the curves is the $Sr_1$ time-curve of the wave refracted in the 220 km discontinuity and it continues beyond 20° with no great curvature. The $Sr_2$ time-curve of the wave refracted in the 410 km discontinuity crosses it between 19° and 20°; beyond 20° it is identical with the J.B. $S-3$ s curve. To the left there is the line of slope 27:8 s/deg corresponding to surface velocity 4 km/s. The times of the figure are travel times minus $10\Delta s$.

The points plotted in the figure are from our four earthquakes. Open circles, squares, dots and crosses are from the 1950, 1931, 1933 and 1930 earthquakes respectively. The points of the late Azores phases have not been plotted, but with few exceptions they would have been very close to points in the figure.

There are many points on or near the $Sd$ line, most of them from the 1950 earthquake. The squares of the Azores earthquake are a little below the line and, as we have seen, they fit on to a line of slope 23:8 s/deg. It seemed at first as if the points of the other earthquakes fitted better to a line of slope 24:0 s/deg, but it was found later not to be so, and the slope 23:8 s/deg was adopted.

Several points are close to the line of slope 27:8 s/deg, and these may be due to second mode surface waves.

For distances smaller than 20° there are many points on or close to the $Sr_1$ curve and there are also some beyond. Most of the corresponding phases have been reported as $S$. The points follow the curve closely enough to make it seem safe to conclude that they are explainable on the kind of assumption made; they can be taken to be due to waves refracted in a layer of high velocity at about 220 km depth and to be delayed by a low velocity layer above. There are points on the $Sr_1$ curve from about 10° onwards. The appearance of refracted (or reflected) waves at so short an epicentral distance indicates that the velocity increase at 220 km depth is discontinuous. A gradual increase could also produce a retrograde branch of the time-curve, but this could not go so far back. There is a considerable scatter of points between 10° and 12° but it is due chiefly to the 1930 earthquake the $S$ phases of which do not seem to be very clear.

A second, deeper discontinuity in the velocity was assumed because late $S$ phases often are observed beyond 20°. This was noticed by Jeffreys & Bullen (1935, p. 41). It has been pointed out in some of my earlier studies and it was found that straight lines of slope of about 21 s/deg could be fitted to the travel time points of the late phases for distances both smaller and greater than 20° (Lehmann 1934, Figures 3 and 4; 1953, Figure 4). However, no structure was found that would account for these lines. Now, taking the velocity gradient below the 220 km discontinuity to be small enough for the $Sr_1$ curve to continue beyond 20° with no great curvature, we obtain an explanation of the late phases at those distances. But then we cannot account for the phases associated with the strongly bending time-curve merging into the J.B. $S-3$ s without assuming that at some depth there is a strong increase of velocity gradient and, perhaps, of the velocity itself.
In a recent report on the results obtained for $P$ and $S$ velocities in the upper mantle (Lehmann 1960) a deeper discontinuity was not mentioned. This was because, on re-examining some records, I did not feel convinced that two distinct phases were actually present and we see also that the points plotted in Figure 2 scatter greatly. However, late phases are often read, but no time-curve has been determined and further investigation is needed.

This also applies to distances smaller than $20^\circ$. Here most of the $S$ phases reported are late phases. For these no time-curve has been determined and it would probably be found impossible to determine time-curves for them with any accuracy using I.S.S. data. Those we have considered may be said to confirm in a general way that the time-curves of the phases are of the type here proposed, but they do not determine the time-curves in detail. In any one earthquake the residuals scatter a great deal and they also differ from one earthquake to another. This may partly be due to the fact that no special attention has been directed towards these phases. The routine observer will be looking for an $S$ phase and some will fix on the first trace of a phase that could be $S$ and neglect the subsequent movement while others will expect $S$ to be a more pronounced phase and prefer to read the later, larger phase for $S$. No care is probably taken to distinguish between the onset of a body wave and that of a train of short-period surface waves, and the distinction is, in fact, by no means always clear. Special studies of the $Sr$ phases are highly needed.

In Japan special studies have been made of $P$ and $S$ phases as recorded at epicentral distances from about $10^\circ$ to a little beyond $25^\circ$ (Nishimura & others 1958). Travel times, amplitudes and periods were measured in the records of 23 observatories all equipped with instruments of the same type. An $Sd$ phase distinguishing itself by a period smaller than 2 s was from $13^\circ$ onwards followed by a much larger $Sr_1$ phase of greater period. The two time-curves intersected at $18^\circ$. The amplitude of $Sr_1$ decreased with distance and was quite small at about $19^\circ$ from which distance onwards it was followed by an $Sr_2$ phase. The time-curves of the two $Sr$ phases intersected at about $24^\circ$. The amplitude of $Sr_2$ was also large where the phase first appeared and it decreased with distance.

Although we do not possess such precise results for Europe we can say definitely that the picture here given does not cover European findings. Our $Sr_1$ phases are not large at the smallest distances and they increase with distance. As to $Sr_2$ our information is vague, but very large late $S$ phases do not seem to appear around $20^\circ$ whereas late phases sometimes are very large at somewhat greater distances (see Lehmann 1953, p. 480, Figure 3). Our $Sd$ line intersects the $Sr$ curves at a distance greater than $20^\circ$.

For Europe results of far greater precision than those we possess are required if we are to determine the velocity distribution in the upper mantle with any precision. Travel times should be determined but we should know also how the periods and amplitudes of the different waves vary. Such knowledge, however, is not easily obtained for Europe where instrumental equipment is not very uniform and most horizontal instruments have no particular good response to $S$ waves at short epicentral distances. We have the short-period vertical seismographs for $P$, but no special effort has been made to obtain good records of $S$ at small distances. The 200 kg Wiechert horizontal seismograph used in Japan would not have sufficient magnification for European shocks, but the $Lg$ seismograph having its maximum magnification for waves of period $3\text{ s}–6\text{ s}$ would undoubtedly produce far better observations of $S$ than those now obtainable.
In the Copenhagen records of the earthquake of 1936 October 18 in the Italian Alps, we have an example of the influence of seismograph characteristics on the $S$ records obtained. $Sd$ is barely visible in the records of the long-period instruments (see Lehmann & Ewing 1960, p. 12 and Figure 4), whereas it is a distinct phase in the record of a Wood–Anderson instrument operating at the time. Its constants were not accurately known, but it is apparent in the record that it responded very well to waves of short period. A clear and relatively strong phase appears 21 s later, also in the records of the other instruments, and this is the phase that was reported as $S$. It seems to be the $Sr$ phase here appearing at so short an epicentral distance.

4. Deep focus earthquakes

So far I have considered only shallow shocks. On my velocity assumptions the travel times of deep shocks having their foci below the 220 km discontinuity will be practically the same as those of the J.B. tables, for below the discontinuity the velocity distribution is only slightly altered and the mean velocity in the upper layers is nearly the same. But the travel times of the earthquakes originating in the upper layers will be more or less affected. It is therefore of interest to see whether or not observed travel times of shocks having their foci in the upper layers are in accordance with the travel times calculated on our velocity assumptions.

In a recent investigation the travel times of the $P$ waves of six Rumanian earthquakes having a common focus at 130 km depth were considered (Lehmann 1959). For 31 observatories at distances up to 24° the $P$ travel times of the various shocks were found to be in excellent agreement and well-determined means of the travel times of individual observatories were used.

The $S$ travel times of the shocks are not in such good agreement but scatter a great deal. $S$ observations usually have smaller accuracy than those of $P$ because the movement does not start from rest and, indeed, often emerges from a strong background movement. This, however, was not the only reason why the observations scattered. We know that there is often more than one $S$ phase observed at distances up to and a little beyond 20°, and it was evident that it was not always the same phase that had been selected and reported as $S$. The I.S.S. travel times of some of the observatories did not all concentrate on one and the same mean value but indicated that in some of the earthquakes one phase had been read while another phase had been read in the others. It was not always easy, however, to separate the phases when only the I.S.S. travel times were considered since the observations of each of them scattered. It was found necessary to examine the records of one of the shocks to see what phases were present.

The records of 24 European stations of the earthquake of 1940 October 22 were used. In several of the records more than one $S$ phase was very clearly present. Often the mean of the I.S.S. travel times was in fair agreement with the travel time of one of these phases, but sometimes, when the I.S.S. travel times scattered too much for a mean to be formed, they were close enough to the travel times of different phases in the 1940 records to indicate that these were the phases read. Using the I.S.S. data for all the earthquakes and in addition my own readings of 1940 October 22, I determined the travel times of individual stations. Some of these were quite well determined means while others were less good; they are marked in Figure 4 by filled-out and open circles respectively. Some observations were not obtained in more than one shock; the corresponding travel times are marked by crosses.
The time-curves of Figure 4 were calculated on velocity assumptions differing somewhat from those used for surface shocks. The velocity in the uppermost mantle is 4.63 km/s and the low velocity layer is taken to begin at 150 km depth, but it extends as before to 220 km depth where the velocity increases abruptly. The velocity in the layer is only 4.10 km/s and so the delay of the $Sr$ phases is approximately the same as before. We have again a second discontinuity at 410 km depth.

![Graph showing travel times of S waves](https://academic.oup.com/gsmnras/article-abstract/4/1/124/653772)

**Fig. 4.**--6 Rumanian earthquakes, depth 130 km. Observed and calculated travel times of $S$.

The slope of the $Sd$ line for a surface focus will be 23.8 s/deg. For 130 km depth the slope of the $Sd$ line is 23.4 s/deg at the inflexion point at 10° epicentral distance, and the mean slope is approximately the same from about 5° to 15°. There are a few well-determined points and some more doubtful ones on the $Sd$ line of Figure 4. The two points on its upper end are those of Helsinki and Pulkovo at epicentral distances 14°.4 and 14°.2. I have seen the Helsinki records of three of the shocks and have found the phase to be clear and its amplitude surprisingly large. It is obviously the $Sd$ phase that has been recorded and this phase could not be recorded at a distance greater than that of the inflection point, 10°, if the low velocity layer began at the depth of focus. That is why the low velocity layer has now been taken to begin at 150 km depth, but it could perhaps have been taken a little higher up.

Most of our points are grouped around the $Sr_1$ curve above the $Sd$ line. The two $Sr$ curves intersect at about 17° and also beyond this distance there are points close to the $Sr_1$ curve seemingly confirming that it continues with no great curvature. There are no points on the $Sr_2$ curve beyond 17° in the figure, but the
travel time points for greater distances fit on to its continuation. We have some well-determined points on the lower part of the $S_{r_2}$ curve; the corresponding phases are very clear in the records examined.

Usually we do not for $P$ observe double phases around 20° corresponding to two different $P_r$ curves. In the Azores earthquake, however, a second phase following closely upon the first one was read in 7 records at distances from 20°.4 to 21°.3 and the phase was interpreted as $P_d$ (Lehmann 1934, p. 19). It now seems as if it may be $P_{r_1}$ while the earlier phases belong to a $P_{r_2}$ branch that continues beyond.

The travel times of $P$ were found to be consistent with a structure that had no low velocity layer, and although it is physically possible for the velocity of $S$ to decrease while that of $P$ does not, yet it seems more likely that the $P$ velocities are somewhat affected in the layer in which the $S$ velocities are low. The velocity distribution for $P$ was therefore altered so as to include a layer of slightly smaller velocity extending from 150 to 220 km depth, and it was found possible to do this without altering the travel times appreciably.

5. Conclusion

In conclusion it can be said that a low velocity layer in the mantle is strongly indicated by observations of $S$ phases at epicentral distances up to about 25°. In Europe the upper boundary of this layer is at a depth greater than the focal depth of the Rumanian earthquakes considered, and this depth is not likely to be smaller than 130 km. While no indication of the character of the upper boundary of the low velocity layer has been obtained, it is found that at the lower boundary, taken to be at 220 km depth, there is likely to be a discontinuous increase of velocity, for late $S$ phases with travel times as those calculated on this assumption appear at small epicentral distances and their amplitudes increase with distance. The presence of a deeper discontinuity is indicated. Special studies of the $S$ phases are required if progress is to be made on these lines. More adequate instrumentation for the recording of $S$ at small epicentral distances is needed.

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