The attenuation of first sound in liquid helium, near the λ-point, is expected to be understood from the dynamical scaling theories. These theories were originally proposed to explain the attenuation of second sound at the λ-point, and later were extended to spin waves at magnetic transitions. Recently, Barmatz and Rudnick explained their experimental results assuming that the "critical frequency" for second sound also gives the leading contribution to the damping of first sound. This idea may be the same as the one proposed by Tani and Mori to explain an anomalous behavior of the attenuation of sound wave near the magnetic transition point. The thermal fluctuations of spins perturb the sound waves due to a distortion of the exchange integral.

In this note, we reproduce Barmatz and Rudnick's results in order to obtain the...
coupling between first and second sounds. The second sound is a "critical" mode. Therefore, our idea is a natural application of Tani and Mori's idea to liquid helium.

In a previous work, we have shown that a set of collective variables \( \{n_k, Q_k, \dot{n}_k, \dot{Q}_k\} \) is a convenient set to study properties of sound propagation in superfluid helium, where \( n_k \) is the Fourier component of the local particle density, and \( Q_k = H_k - \hbar n_k \). Here \( H_k \) is the Fourier component of the local Hamiltonian density. \( \dot{A} \) denotes a time derivative of \( A \).

It is easy to show that a random force \( K \), acting upon the first sound, is given by

\[
K = K^{(1)} + K^{(2)},
\]

\[
K^{(1)} = \dot{n}_k + \Omega^1_k(n_k),
\]

\[
K^{(2)} = \Omega^2_k(Q_k, \dot{n}_k) Q_k,
\]

\[
(A, B) = \int d\lambda \langle e^{i\lambda A} e^{-i\lambda B} \rangle - \beta \langle A \rangle \langle B \rangle,
\]

\[
\langle A \rangle = \text{Tr}(\rho A), \quad \beta = e^{-\beta H}/\text{Tr} e^{-\beta H},
\]

where \( \Omega^1_k \) is the frequency of first sound. \( K^{(2)} \) contains the variable which produces the second mode, and therefore, may contribute to critical anomalies.

According to Mori's general expression for the attenuation constant of first sound, we have the anomalous part of it:

\[
\frac{d\alpha_1}{d\omega^2} = \frac{A}{c^2} \lim_{t \to +0} \Xi_k(\epsilon),
\]

where

\[
\Xi_k(\epsilon) = \int_0^\omega dt e^{-\omega t} \langle Q_k(t), \dot{Q}_k(\epsilon) \rangle \langle Q_k, \dot{Q}_k(\epsilon) \rangle / \langle Q_k, \dot{Q}_k(\epsilon) \rangle.
\]

\[
\lambda_0(\epsilon) = \frac{\Omega^2_k(\epsilon)}{\Omega^2_1 + \lambda_1},
\]

\[
\Omega^2_1(\epsilon)
\]

In the region of local hydrodynamics we obtain

\[
\lambda_0 \approx \frac{\kappa}{\rho C_p}, \quad \text{(for He I)}
\]

\[
\lambda_1 \approx k^2 D_T, \quad \text{(for He II)}
\]

where

\[
D_T = \frac{1}{\rho} \left( \frac{\rho_n}{\rho} \right) \left( 4 - \frac{3}{\gamma} + \frac{\rho_n}{\rho} \frac{m \kappa}{C_p} + \rho^2 \zeta_3 - 2 \rho \zeta_1 \right).
\]

The result (12) fully agrees with that obtained by Kadanoff and Swift and (13) agrees with that obtained by Ferrell et al.
