and discuss the comparison between theoretical and experimental results.

## Summary

In the present study, a theoretical model of the bump foil strip deformation in compliant foil bearings and dampers was developed. The model considers the friction forces between bumps and the housing or the top foil, local interaction forces, variable load distribution, and bump geometries. A comprehensive computer program was developed to determine deflections, stiffness, displacement, and reacting and friction forces for each bump in a bump foil strip. In addition to variable geometric parameters, the program enables the computation of equivalent friction coefficients and overall stiffness of a bump foil strip under various load distributions.

The stiffness and deflection of the bump predicted by the model follow the trend of early published experimental data. The bumps near the fixed end have a much higher stiffness (lower deflection) than the bumps near the free end. Whereas higher friction coefficients tend to increase stiffness and may pin down bumps near the fixed end, lower coefficients make bumps softer and cause higher deflections in both vertical and horizontal directions. An increase in the friction coefficient between the top foil and the bump is a more effective method of achieving both Coulomb damping and higher stiffness. Increases in bump thickness or height and decreases in length cause increased stiffness. In addition to bump geometric parameters that increase stiffness, the load distribution profile also greatly influences bump stiffness.

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## APPENDIX

The following equations are for the constants of integration $C_1$, $C_2$, $C_3$, and $C_4$.

$$C_1 = F^*_{Li}[3/4\sin\theta_0] + H^*_{Li}[3/4\cos\theta_0] + S^*_{Li}\sin\theta_0$$

$$\begin{aligned}C_2 = 1/4\{ & F^*_{Li}[2-3\cos\theta_0+\sec\theta_0-\theta_0\csc\theta_0] \\ & + H^*_{Li}[2\cot\theta_0 - 3\cos\theta_0\cot\theta_0 + \csc\theta_0 - \theta_0\sec\theta_0] \\ & + F^*_{Ri}[-2+\sec\theta_0+\theta_0\csc\theta_0] + H^*_{Ri}[2\csc\theta_0 - 2\cot\theta_0 + \theta_0\sec\theta_0] \\ & - S^*_{Li}[2\sec\theta_0\cos2\theta_0] - S^*_{Ri}[2\sec\theta_0]\}\end{aligned}$$

$$\begin{aligned}C_3 = 1/4\{ & F^*_{Li}[2\sin\theta_0 - 2\sin2\theta_0 + 2\theta_0\cos\theta_0] \\ & + H^*_{Li}[4\cos\theta_0 - 4\cos^2\theta_0 - 2\theta_0\sin\theta_0] \\ & + F^*_{Ri}[2\sin2\theta_0 - \sin\theta_0 + 2\theta_0\cos\theta_0] \\ & + H^*_{Ri}[4\cos^2\theta_0 - \cos\theta_0 + 2\theta_0\sin\theta_0] + S^*_{Li}[4\sin\theta_0]\}\end{aligned}$$

$$\begin{aligned}C_4 = 1/8\{ & F^*_{Li}[4-2\sec\theta_0-2\theta_0\csc\theta_0]\cos2\theta_0 \\ & + H^*_{Li}[4\cot\theta_0 - 4\csc\theta_0 + 2\theta_0\sec\theta_0]\cos2\theta_0 \\ & + F^*_{Ri}[3\sec\theta_0 - 4\theta_0\csc\theta_0 - \cos2\theta_0(4-\sec\theta_0+2\theta_0\csc\theta_0)] \\ & + H^*_{Ri}[3\csc\theta_0 - 4\theta_0\sec\theta_0 + \cos2\theta_0(\sec\theta_0 - 4\cot\theta_0 - 2\theta_0\sec\theta_0) \\ & - S^*_{Li}(4\sec\theta_0\cos2\theta_0) - S^*_{Ri}(4\sec\theta_0)\}\end{aligned}$$

## DISCUSSION

A. Gu[1]

The authors are to be congratulated for their excellent analysis on the structural stiffness, including friction effects, of a particular type of foil bearing. It is very interesting to see the kinematics of the bump foil movement under various foil geometry and friction values. The authors conclude that a high friction between the top foil and the bump foil is desired to achieve Coulomb damping. According to the authors, under high $\eta$ and high $(\eta-\mu)$ values, the contacts between the top foil and the bump are fixed and the bump ends remain free, providing Coulomb damping. For moderate values of $\eta$ the top center is not fixed and there is relative motion between the top foil and the bump, (see Fig. 8 of the paper) which can also provide Coulomb damping. Have the authors compared the relative effectiveness of the rubbing motions between the top foil and the bump foil, and between the bump foil and the housing for producing Coulomb damping?

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### Authors' Closure

We thank Dr. Gu for his thoughtful comments and questions.

In this paper, the load is assumed to be decreasing from the fixed and toward the free end, i.e., the bump foil strip under triangular load. Under this load condition, we obtained Fig. 8 and the conclusions. However, in hydrodynamic lubrication problems with a circular or tapered pad geometry, the load profile may not represent real bearing load profiles. It is obvious that the load profile will affect the behavior of the bump foil strip, therefore, we may not reach the same conclusion for different bearing configurations and load conditions. In the future papers, we will discuss the effects of rubbing motion for producing Coulomb damping in a journal bearing/damper.

  