The body force, F_ω, due to rotation ω in the radial direction at $\varphi = 0$, as shown in Fig. 2, can be found to be

$$F_\omega = \int_\varphi \int_z \int_r \frac{\gamma r^2 \omega^2}{g} \cos\varphi \, dr \, dz \, d\varphi \qquad (35a)$$

and the radial moment, M_ω, (at $\varphi = 0$) about the φ-axis, due to rotation ω, is

$$M_\omega = - \int_\varphi \int_z \int_r \frac{\gamma r^2 \omega^2}{g} z \cos\varphi \, dr \, dz \, d\varphi \qquad (35b)$$

For the cover disk, change the subscript h to c, index 3 to 1, and index 4 to 2, and set $E_b = \gamma_b = 0$.

From (28), ϵ_{r_0} and α_0 at any r-surface can be expressed in terms of u_0, α, F_r, and M_r at that r-surface,

$$\begin{Bmatrix} \epsilon_{r0} \\ \alpha_0 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \end{bmatrix} \begin{Bmatrix} u_0 \\ \alpha \\ F_r \\ M_r \end{Bmatrix} \qquad (36)$$

where

$$C_{11} = (R_{\alpha_0} M_u - M_{\alpha_0} R_u)/D \qquad (37a)$$

$$C_{12} = (R_{\alpha_0} M_\alpha - M_{\alpha_0} R_\alpha)/D \qquad (37b)$$

$$C_{13} = M_{\alpha_0}/D \qquad (37c)$$

$$C_{14} = -R_{\alpha_0}/D \qquad (37d)$$

$$C_{21} = (M_\epsilon R_u - R_\epsilon M_u)/D \qquad (37e)$$

$$C_{22} = (M_\epsilon R_\alpha - R_\epsilon M_\alpha)/D \qquad (37f)$$

$$C_{23} = -M_\epsilon/D, \qquad C_{24} = R_\epsilon/D \qquad (37g)$$

$$D = R_\epsilon M_{\alpha_0} - M_\epsilon R_{\alpha_0} \qquad (37h)$$

Equations (36) and (37) are good both for the hub disk with blades attached and for the cover disk. For the former case, the subscript h is applied; and for the latter, subscript c is applied.

Note that all those coefficients of equations (29), (33), and (37) are functions only of dimensions and physical properties of an impeller. They can be calculated readily and remain constant once the dimensions and the physical properties of an element are given; similarly for F_ω and M_ω of equations (35).

DISCUSSION

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The analysis and digital-computer program developed by the author is a significant contribution to the stress analysis of a complex rotating element. The computer program described in the paper has been used by the Product Engineering Department of the author's company to analyze shrouded (cover disk) centrifugal impellers as shown in Fig. 1.

Since there is a continuous demand for higher volumetric flow and higher head (pressure-ratio) centrifugal compressors possessing high efficiencies, the conventional stress analysis methods neglecting the effects of bending moments and the stress analysis of unshrouded radial flow impellers such as reported in reference [1] are not adequate to determine the stresses in a highly loaded shrouded impeller. A very realistic evaluation of the tangential and radial stresses is required for a shrouded impeller to determine when the shroud design must be abandoned and the unshrouded design adopted.

Impeller performance and impeller cost are directly related to the impeller design. Consequently, the limitation of a shrouded impeller becomes a primary factor in selecting a centrifugal compressor for a specific application. The limitation of the shrouded impeller design is fundamentally a function of the stresses encountered within the impeller due to the added centrifugal load of the shroud. Since the limitation of an impeller depends on the stresses within the impeller, the significance of the author's analysis and computer program are obvious.

Although the calculating procedure and computer program reported in the author's paper pertain to the stress analysis of a shrouded centrifugal impeller, the program has been applied to unshrouded impellers and unsymmetrical rotating elements.

Author's Closure

The author wishes to thank Mr. Vuksta for his kind comments.

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