4 Conclusions

On the basis of the free boundary problem of a starved tapered land thrust bearing, the two dimensional computations performed provide the following conclusions:

1 For any values of nondimensional load and L/R_2 ratio, with increased starvation, there is a continuous increase in maximum pressure and temperatures; and a continuous decrease in torque and extent of film.

2 The values of β_S which were obtained initially by the Newton-Raphson iterational method, are of the order of tapered portions of the thrust pad β, is a weak function of $\bar{h}_2$ and L/R_2 ratio, and it is a strong function of $\hat{Q}_S$. These useful theoretical results can be employed as initial guess values for the start of the fluid film θ_1 for given values of the index of starvation, $\hat{Q}_s$.

3 The values of inlet flow $\bar{Q}_1$ is a linear function of the degree of starvation $\hat{Q}_s$ and $\bar{h}_2$ but a weak function of L/R_2 and almost independent of β.

Acknowledgments

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DISCUSSION

A. Z. Szeri[1]

This paper contains a number of fairly severe assumptions and I would like to ask the authors to discuss these. In particular, I have in mind the following:

(1) Across the fluid film the variation of lubricant viscosity is neglected. This cannot be done without penalty, as has been shown by a long line of investigators (for example Boucompain, Fillon and Freene in a paper, 85-Trib-2, in this conference.) To quote from their conclusions: "the temperature gradients across and along the fluid film are important;" recognizing the above, Boucompain et al. obtained excellent agreement with experimental data. Equation 5 of the paper under discussion was first suggested by Professor Constantinescu in the later 1960s. Is this still the best we can do?

(2) The lubricant film does not transfer heat to the bearing surfaces in this analysis (i.e., the thermal boundary conditions are adiabatic). This assumption necessarily leads to a temperature profile which increases monotonically away from the bearing inlet and leads to a maximum film temperature that is far too high. We may again quote from the paper of Boucompain et al. Their Fig. 12 shows an actual temperature profile which reaches its maximum around $\theta = 220°$ or just about at the cavitation interface.

(3) The leading edge of the starved lubricant film is placed at some θ_1 = constant in this analysis. The authors do not provide a detailed study of the position of this upstream film boundary nor the associated pressure boundary conditions there. Do they have any reason to assume that θ is a constant? Furthermore is it possible for θ_1 to depend on the spatial coordinates (r, z)?

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M. J. Braun[2]

The authors are to be commended for an ingenious and rather elegant treatment of their energy equation. However, it is in connection with their treatment of the energy equation that I will address my first comments. I shall begin with the following equation

$$\frac{D}{Dt}(\rho u) = -P\nabla \cdot \mathbf{V} + \nabla \cdot (k\nabla \mathbf{T}) + \Phi \qquad (22)$$

In the above equation u is internal energy, V is the velocity and Φ is the energy dissipation. Equation (22) can now be written in terms of enthalpies.

$$\rho\frac{Dh}{Dt} - \rho\frac{D}{Dt}\left(\frac{P}{\rho}\right) = -P\nabla \cdot \mathbf{V} + \nabla \cdot (k\nabla T) + \Phi \qquad (23)$$

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If the lubricant properties are functions of temperature and pressure, then

$$\rho\,\frac{D}{Dt}\left(\frac{P}{\rho}\right)=P\rho\,\frac{D}{Dt}\,\frac{1}{\rho}+\frac{DP}{Dt}\tag{24}$$

and the continuity equation is

$$-\frac{D\rho}{Dt}=\rho\,\nabla\cdot\mathbf{V}\tag{25}$$

Then by substituting equation (24) into equation (23) and using the Maxwell relationships, equation (23) becomes

$$\frac{D}{Dt}(\rho C_P T)=\left.\frac{\partial\,\ln(1/\rho)}{\partial\,\ln\,T}\right)_P\frac{DP}{Dt}+\nabla\cdot k\nabla T+\Phi\tag{26}$$

Equation (26) appears in a much simpler form if, $\nabla\cdot\mathbf{V}=0$ and that form is

$$\rho C_P\,\frac{DT}{Dt}=\nabla\cdot k\nabla T+\Phi\tag{27}$$

Equation (3) in the paper being discussed corresponds to equation (27). The left-hand side corresponds to $\rho C_P V\cdot\nabla T$, and the conduction has been eliminated. The right-hand side is globally presented by out term Φ.

I now question if the elimination of the term

$$\left.\frac{\partial\,\ln\,1/\rho}{\partial\,\ln\,T}\right)_P\frac{DP}{Dt}$$

is inconsistent with the recognition that the lubricant properties are variable and the continuity is not simply $\nabla\cdot\mathbf{V}=0$ but rather $\rho\nabla\cdot\mathbf{V}\ =\ -\mathbf{D}\rho/Dt$.

In a nutshell, for the variable properties case, I believe the continuity equation is not fully accounted for in the energy equation, as it is presented in equation (3) of the paper. In addition, I suggest that the Reynolds equation is not fully coupled with the energy equation since the influence of pressure on the fluid properties is not accounted for in the equations for properties variation. *Would the authors offer any additional insight concerning these remarks.*

My second comment refers to the conduction terms which have been eliminated. From numerical experiments which Braun, Mullen, and Hendricks [19] have performed, it appeared that for a thin film of lubricant the conduction into the metal shaft is rather significant, and the shaft acts practically as a heat sink.

Differences in the lubricant temperature are shown in Fig. 16 (i.e., for a journal bearing). This figure corresponds to the cases when the heat transfer-coefficient $h\rightarrow\infty$ at the solid-liquid interface. Figure 17 shows the result for $h=0$. These results show the difference between the adiabatic and isothermal bearing case. Figure 18 shows a parametric study of fluid energy gain and this probably dramatizes more than Figs. 16 and 17 the necessity of accounting for the heat transfer to the bearing pad and the shaft, for, realistic evaluation of density, dynamic viscosity and load carrying capacity variation. *While I do recognize the assumption of the adiabatic behavior the authors made, I would like to ask how does this assumption square with experiments which the authors may have performed or might be reported in the literature.*

Finally, I would like to mention the work of Bonneau and Frene [20], treating an isoviscous flow at the inlet of a starved contact. They have partitioned the inlet region in three zones and I quote:

Zone 1: "The fluid layer on the moving plane is not disturbed by the meniscus proximity. The velocity field is uniform and equal to U.

Zone 2: "The flow is two-dimensional. This zone is the actual inlet zone and its boundaries are unknown.

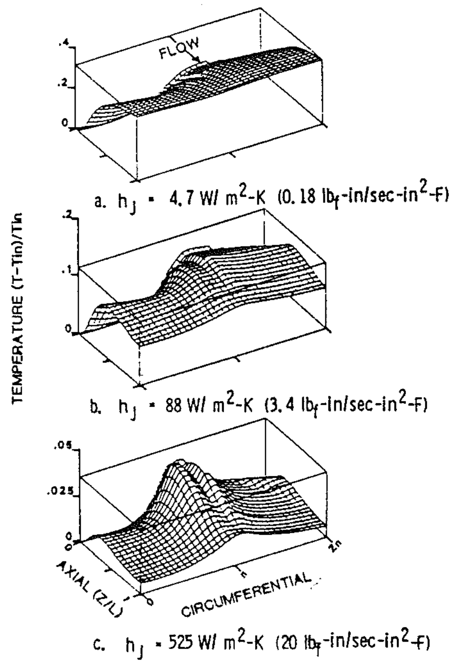

Fig. 16(*a*) Variation of axial and circumferential temperature profile with changes in heat transfer coefficient to the journal for a tilt angle of 0.4 T_{in} = 38 C and rpm + 1000

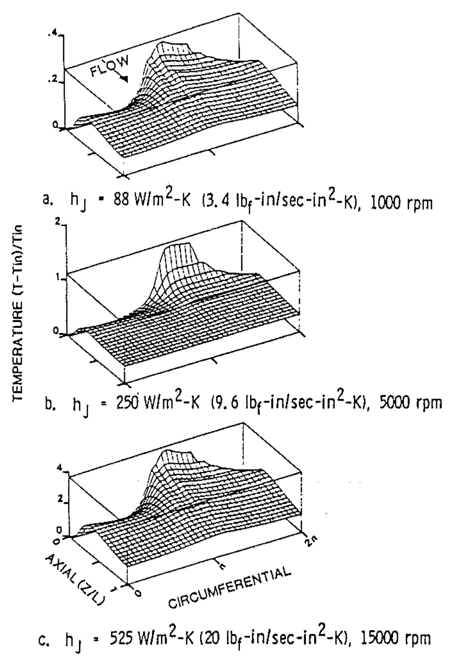

Fig. 16(*b*) Variation of axial and circumferential temperature profile with angular velocity and heat transfer coefficient to the journal for a tilt angle of 0.48, T_{in} = 38 C

Zone 3: "The y velocity component v tends toward zero and the pressure p depends on x only. Reynolds equation can be used to describe this zone."

The treatment of the various zones is rather interesting and uses a simplified form of the Navier-Stokes equation for zone 2 and Reynold's equation in zone 3. Have the authors thought to incorporate such an analysis to treat the various zones of their geometric configuration?

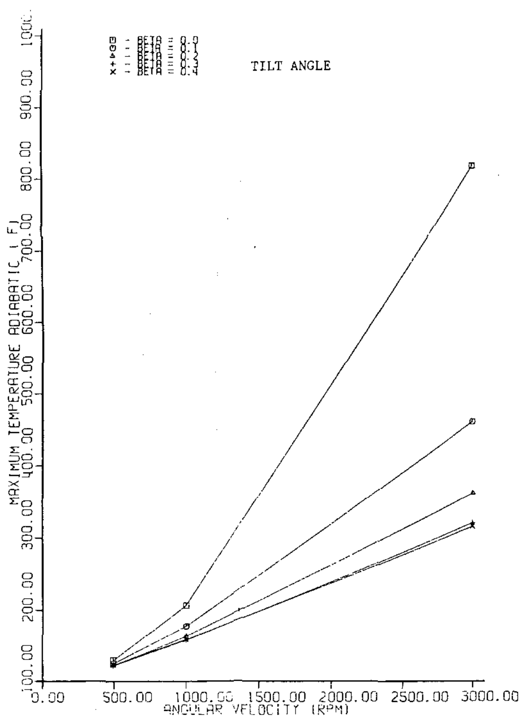

Fig. 17 Maximum temperature in a hydrodynamic bearing under adiabatic conditions (radius = 1.5 in. radial clearance c = 0.003 in.)

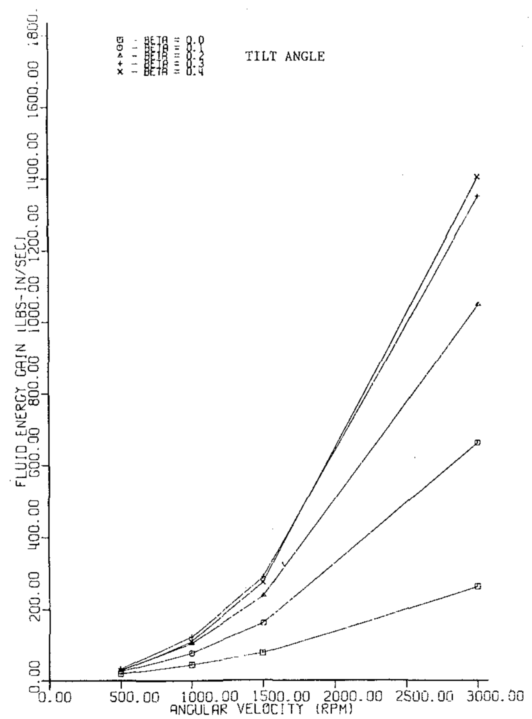

Fig. 18 Fluid energy gain in a hydrodynamic bearing under adiabatic conditions (R = 1.5 m, c = 0.003 in.)

R. C. Hendricks[3]

While the concept of transformations which lead to the elimination of variable property effects are commonly used in heat transfer and fluid mechanics (i.e., Goldmann's integral properties, and flat boundary layer problems), the use of these methods here to decouple the energy and Reynolds equations appears novel. Although I do not fully understand the author's methodology and results (e.g., I have not programmed or parameterized their results) it appears that the problem as formed may already be decoupled. Perhaps the authors could comment on the coupling problem and say a little more about the use of 'common' transformations of heat transfer and boundary layers relative to their normal transform.

As a second comment, inclusion of variable properties implies the compressible problem and properties which are functions of two parameters for single phase fluids, (e.g., pressure and temperature). Perhaps the authors may wish to explain more fully their reduction to one parameter (e.g., temperature).

Additional References

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Authors' Closure

The analysis described in this paper concentrates on the effects of inlet starvation as well as on the coupling between the Reynolds and energy equations that is brought about by the dependence of fluid viscosity on the temperature of an adiabatic film. For this reason, the analysis assumed that:

1. The pressure and temperature fields are two-dimensional.
2. The lubricant is incompressible.
3. The viscosity is only a function of the temperature.
4. The lubricant does not conduct any heat, neither to the walls nor to the surrounding fluid.

These assumptions were used in order to develop an analytical tool capable of evaluating bearing performance (including conservative maximum temperature) efficiently while concentrating on the governing phenomena for oil-ring fed bearings.

We agree with Professor Szeri that the assumption of adiabatic bearing surfaces produces higher temperatures. Comparison with experiments performed on tapered-land thrust bearing pads, reported in reference [21], show fairly good agreement, although the analysis consistently produced higher temperatures. Since the film thickness never increases for tapered-land thrust bearing pads, no cavitation is possible and comparison with a cavitated journal bearing film is not appropriate.

The boundary condition applied to the upstream boundary is given in equation (8) by: $P(Z, \theta_1) = 0$ where θ_1 is determined by the condition that the total flow into the film is equal to the amount supplied, as given by equation (10). Our analysis has assumed that θ_1 is independent of the radial coordinate. A more exact boundary condition would allow θ_1 to be a function of r, as suggested by Professor Szeri. In this case the distribution of circumferential flux (flow per unit length), given by the integrand in equation (10), would be prescribed. However, this boundary condition is much more difficult to apply, not only from a mathematical point of view, but also because the radial distribution of flux is unknown, particularly in oil-ring fed bearings. Furthermore, as is discussed in references (2), (3), and (17), this distribution does not have a significant effect on the bearing load capacity provided that conservation of the total flow is satisfied.

The work of Bonneau and Frene, although very interesting, is not only isoviscous but also two-dimensional (i.e., the two

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dimensions being along and across the film). Incorporation of their zone 2 in the present analysis would involve solving the Navier-Stokes equation in three-dimensions. This is something we are not currently attempting to do.

We appreciate Professor Braun's derivation the more general form of the energy equation given in (26). Of course, for the incompressible fluid, the density is constant so that $D\rho/dt = 0$. Therefore, when the thermal conduction term is eliminated, the energy equation is fully accounted for in our equation (3).

With regards to Dr. Hendricks' comment, the transformation used in the present analysis does not fully uncouple the Reynold's and energy equations. It only eliminates the first order coupling which is due to the presence of the viscosity in the first term of the right-hand side of equation (3). After the transformation, the viscosity continues to appear in the pressure gradient terms, which are of lesser importance.

Additional Reference

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