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Rod Cross

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Comment on "Bouncing on a slope" [Am. J. Phys. 89, 143–146 (2021)]

Rod Cross
School of Physics, University of Sydney, Sydney, Australia

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I once read that a theoretical physicist is someone who will approximate a four-legged table as one with either one or two legs to simplify the problem and will then conclude that it is unstable. I was reminded of that quote when I read the paper by Rostamian *et al.* in this journal.[1] They considered a point mass projectile bouncing up an incline, assuming that the projectile and the surface were both ideal rather than physical objects. It is standard if not universal practice to make simplifying assumptions when developing a physical model, but sometimes it can be taken too far. I tried the experiment using a real ball on a real surface and found that none of their assumptions or conclusions were valid. The time intervals between successive bounces were not equal, the rebound velocity was not equal for all impacts, the angle of incidence to the normal was not equal to the angle of reflection, the friction force was not negligible, the downhill path did not coincide with the uphill path, the ball rolled rather than bounced down the incline, the spin of the ball played a major role, and the number of uphill bounces could not be made arbitrary large. The maximum number of uphill bounces I observed was five. I think that purely mathematical papers published in *The American Journal of Physics* should come with a warning that any resemblance to the real world is likely to be coincidental.

[1] R. Rostamian, A. M. Soane, and J. M. Tavares, "Bouncing on a slope," Am. J. Phys. **89**, 143–146 (2021).

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Response to Rod Cross's Letter

Rouben Rostamian
Department of Mathematics and Statistics, University of Maryland, Baltimore County (UMBC), Baltimore, Maryland 21250

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One is never certain who, if anyone, will read an article that one publishes. I am happy to see that Rod Cross not only has read our paper but also was interested enough to perform experiments to test its conclusions. He reports that his experiments have failed to reproduce those conclusions. Of course! The paper's opening paragraph sets the stage by describing explicitly and precisely the assumptions built into the analysis: point-mass projectile, no air resistance, no energy loss, and no surface friction. From what Dr. Cross has described, his equipment certainly does not meet those criteria. Any resemblance of his experimental observations to what is described in that article is entirely coincidental.

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Breakdown of a misinterpretation of Noether's theorem

Kirk T. McDonald[a)]
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

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A recent paper by Lemos titled,[1] "Breakdown of the connection between symmetries and conservation laws for semiholonomic systems," may unintentionally lead the reader to suppose that Noether's theorem suffers a "breakdown" in the example of a mass that slides without friction inside a cylinder that rolls without slipping on a horizontal plane.

In classical mechanics, Noether's theorem[2] is a restatement of an insight of Lagrange that if the Lagrangian L of a system is invariant under coordinate q (that is, independent of q), then the CANONICAL (or generalized) momentum $p_q = \partial L/\partial \dot{q}$ is a constant of the motion (i.e., a conserved quantity). Unfortunately, this theorem is often misinterpreted/oversimplified to mean that if the Lagrangian of a system of total mass M is independent of the spatial coordinate x, then the total LINEAR momentum $M\dot{x}$ is conserved. However, the linear momentum is conserved only if $p_x = \partial L/\partial \dot{x} = M\dot{x}$.

In the example of Lemos, the Lagrangian is independent of the horizontal coordinate x of the center of the cylinder, but $p_x = \partial L/\partial \dot{x} \neq M\dot{x}$. Although p_x is a constant of the motion in this example, Lemos's paper suggests that this is a breakdown of Noether's theorem because p_x does not equal $M\dot{x}$. This inference is a disservice to Lagrange and to Noether, as a conserved momentum related to an invariance/symmetry does exist in Lemos's example, exactly in accordance with Noether's theorem.

Lemos cited Noether's theorem in the Introduction to his paper, and immediately afterwards wrote,[1,3] "The conservation of linear momentum, angular momentum, and energy for many-particle systems is associated with invariance of the action under translations, rotations, and time displacements, respectively." In that context, readers might assume that the quotation *is* Noether's theorem, although it is rather only an important special case of it. Lemos then went on to show that this statement does *not* hold for his example of the rolling cylinder, because, as is shown in his textbook,[4] this statement holds only for "proper" holonomic mechanical systems (as well as for ones with no constraints), but not for semiholonomic systems or for nonholonomic systems.[5]

According to the narrow but popular view of the association/connection between conservation laws and invariance stated in the above quote from Lemos's paper, its title, "Breakdown of the connection between symmetries and conservation laws for semiholonomic systems," is valid. But, the wording of Lemos's paper may lead readers to think that Noether's theorem suffers a breakdown for semiholonomic systems. This letter affirms that the full power and validity of Noether's theorem, for any and all systems describable by a Lagrangian, is unaffected by Lemos's result.

[a)]Electronic mail: kirkmcd@princeton.edu, ORCID: 0000-0002-3502-7380.

[1]N. A. Lemos, Am. J. Phys. **90**, 221–224 (2022).

[2]See, for example, Sec. 13.7 of Ref. 1 of Lemos's paper, H. Goldstein, C. P. Poole, Jr., and J. L. Safko, *Classical Mechanics*, 3rd ed. (Addison Wesley, San Francisco, 2001). Noether wrote about "invariance" rather than "symmetry" (mainly in the context of general relativity), although the term symmetry is now popularly associated with her theorem.

[3]For systems with constraints, the term "action" (used, but not defined in Lemos's paper) can be construed to mean (in the context of Lemos's paper) the "extended" Lagrangian $\mathcal{L}$ defined in his Eq. (25), which includes the constraints via a Lagrange multiplier.

[4]The only previous demonstration of this fact may be that on p. 69 of Lemos's book, *Analytical Mechanics* (Cambridge U. P., Cambridge, 2018).

[5]The terms "holonomic" and "semiholonomic" are discussed on p. 264 of Ref. 9 of Lemos's paper, J. G. Papastavridis, *Analytical Mechanics* (World Scientific, Singapore, 2014). Holonomic systems have constraints of the form $f_i(\{q_j\}, t)$. Semiholonomic systems have velocity-dependent constraints of the form $g_i(\{q_j, \dot{q}_j\}, t)$ that can be integrated to the holonomic form, but which include constants that depend on the initial conditions. Hence, semiholonomic systems are a subset of holonomic ones. A distinct subset of holonomic systems is sometimes called proper, for which constants in the constraints of these holonomic subsystems are independent of the initial conditions. The term "holonomous" = integral (ὅλος) laws (νομός) was introduced by H. Hertz in Sec. 123, p. 80 of *The Principles of Mechanics* (Macmillan, New York, 1899); see also p. 91 of the original German edition (Barth, Leipzig, 1894).

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Talking ’bout misinterpretation

Nivaldo A. Lemos[a)]
Instituto de Física, Universidade Federal Fluminense, Campus da Praia Vermelha, Niterói 24210-340, RJ, Brazil

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I’m afraid my paper[1] has been misunderstood by Professor K. T. McDonald. He puts words in my mouth that I never said. The phrase “breakdown of Noether’s theorem” is never to be found or suggested in my paper. Nor do I ascribe this supposed breakdown of Noether’s theorem to the fact that p_x does not equal $M\dot{x}$ in my example. In the Introduction, I say that “Noether’s theorem establishes the most general correspondence between invariance under continuous transformations and constants of the motion.” Next, I mention that conservation of linear momentum, angular momentum, and energy are associated with “invariance under translations, rotations, and time displacements, respectively.” These results are, obviously, particular cases of Noether’s theorem because translations, rotations, and time displacements are not the most general continuous transformations. In no way whatsoever have I suggested that these particular results *are* Noether’s theorem. McDonald has misread my paper.

Let $L = T - V$ be a standard Lagrangian, where V does not depend on velocities. When all constraints are holonomic, invariance of both the Lagrangian and the constraints under translations implies conservation of the total linear momentum, which turns out to coincide with the corresponding total canonical momentum.[2] Velocity-dependent integrable constraints, which are said to be semiholonomic, seem completely equivalent to holonomic constraints because, in their integrated form, they restrict coordinates alone. So, it came to me as a surprise that semiholonomic constraints are not equivalent to holonomic constraints as regards the connection between symmetries and conservation laws. My example shows that the Lagrangian is invariant under translations but the corresponding conserved canonical momentum is not the total linear momentum, which is not conserved. In short, contrary to expectations, the conserved quantity associated with translation invariance is not the total linear momentum.

According to McDonald, I misinterpret Noether’s theorem by stating or intimating that it provides a connection between symmetries and conservation laws *only* for standard Lagrangians and holonomic systems. But this is nowhere to be found in my paper. The connection between translation invariance and conservation of the total linear momentum for a holonomic system with a standard Lagrangian is a *particular* case of Noether’s theorem because, for transformations that leave time unchanged, the invariance of the Lagrangian is equivalent to the invariance of the action. Never do I hint that the case of standard Lagrangian and holonomic constraints is the only instance of a connection between symmetries and conservation laws.

Finally, McDonald incorrectly states that “semiholonomic systems are a subset of holonomic ones.” If this were true, one would face the contradiction of a result that holds for all members of a set A but does not hold for members of a subset of A. It’s just the other way around, holonomic systems are a subset of semiholonomic systems.

I find McDonald’s criticism unwarranted because it rests on a misinterpretation of my paper.

[a)]Electronic mail: nivaldolemos@id.uff.br, ORCID: 0000-0002-2386-1247.

[1]N. A. Lemos, “Breakdown of the connection between symmetries and conservation laws for semiholonomic systems,” Am. J. Phys. **90**, 221–224 (2022).
[2]N. A. Lemos, *Analytical Mechanics* (Cambridge U. P., Cambridge, 2018), Chap. 2.

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