APPLICATION OF DYNAMIC PROGRAMMING MODELS
FOR OPTIMUM FARM AND HOME PLANS*

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Farm and home planning by the Extension services has now become
well established in most states. Most plans are made for the farm by
budgeting techniques, with reference usually made for some point of time
in the future. Ordinarily, the farm plan relates indirectly to the home plan.
Farmers have become increasingly interested in these plans, and they have
been productive and useful. They need even wider use and development in
this day of growing commercialism in farming.

However, availability of recent empirical tools may extend the amount
of farm planning which can be done to make plans more realistic. Plans can
be made more realistic by incorporating time and greater farm-household
interdependence. Farm and home planners know that both are important
but have been handicapped in incorporating them into actual empirical
plans because of the amount of computation involved. However, linear pro­
gramming procedures and high speed computers promise to help solve this
problem. Instead of a plan for some point of time in the future, we may
readily solve for the best plan in a series of years; with the optimum for any
one year depending on the optimum in other years, on the availability of
and returns on capital in other years, on the need for household consump­
tion at different points in time, etc. Based on resource supplies and optimum
use in previous years, these procedures also can specify the plans for transi­
tional years required to get to "the future point in time," as most budgets
are made up.

This presentation is an application of dynamic linear programming. The
model involves solving optimum plans for a series of years where produc­
tivity of resources in the farm business are related to expenditure needs of
the farm family. We present only one of the simpler problems being solved,
in order to make presentation less complex. We first outline the general
algebraic model; next, we show, by means of a simple example, the structure
of coefficients used. Finally, we present results from one of the least com­
plex of several programs computed for an actual Iowa farm family. The
model used is dynamic only in a Hicksian sense where inputs and outputs
are dated. It is not dynamic in the sense of variability in prices and co­
efficients, although the latter has some limited possibilities in computation.¹
Too, we recognize that farmers often would change the sequence of plans,
after realizing the outcome of plans in a particular year.

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¹ Earl O. Heady and Wilfred Candler, Linear Programming Methods, Ames: Iowa State
A Dynamic Model

A dynamic linear programming model can be developed by merely expanding the ordinary simplex model. The concept for a dynamic model is to identify, in a Hicksian sense, each coefficient with a particular time period. Thus, complete identification of any coefficient in the programming matrix refers to row, column and year. For example, a_{ij}, the input-output coefficient in the ordinary static model, refers to the amount of the i-th resource or restraint used per unit of the j-th activity in a single year or time period. In the dynamic model, this notation is supplemented by a superscript, k, which denotes a particular year. Each coefficient is now identified as a_{ik}; the amount of the i-th resource used per unit of the j-th activity in the k-th year.

Following this notation, each alternative production process for any one year ordinarily is expressed as x_{j}. To identify the activity x_{j} with a particular year, in our dynamic model, a superscript k is added to give x_{jk} (j = 1, \ldots, n, k = 1, \ldots, t). Likewise, resource supplies or restraints are indicated by s_{i}(i = 1, \ldots, m) and becomes s_{ik} when reference is made to the i-th resource supply in the k-th year. Unit returns to activities are denoted as c_{j}, to indicate the return for the j-th activity in the k-th year. In terms of these notations, the first dynamic programming equation is expressed as:

\[
\begin{align*}
\text{s}_{1} & \geq a_{11}x_{1} + a_{12}x_{2} + \cdots + a_{1j}x_{j} + \cdots + a_{1n}x_{n} + a_{12}x_{2} + \cdots + a_{1j}x_{j} + \cdots + a_{1n}x_{n} + a_{12}x_{2} + \cdots + a_{1j}x_{j} + \cdots + a_{1n}x_{n} + \cdots \\
& + a_{11}x_{1} + \cdots + a_{1n}x_{n} + a_{11}x_{1} + a_{12}x_{2} + \cdots + a_{1j}x_{j} + \cdots + a_{1k}x_{k} + \cdots \\
& + a_{1n}x_{n} + \cdots + a_{11}x_{1} + a_{12}x_{2} + \cdots + a_{1j}x_{j} + \cdots + a_{1n}x_{n}.
\end{align*}
\]

This equation is complete for the first resource supply in the first year (s_{i}^{1}). However, when k \neq 1, all a_{ik}^{k}(k \neq 1) are equal to zero, except those representing inter-year capital flows, because activities for year 2 and beyond will not use resource supplies from year 1(s_{i}^{1}). Therefore, the relevant terms (those without zero coefficients) of the above equation become:

\[
\begin{align*}
\text{s}_{1} & \geq a_{11}x_{1} + a_{12}x_{2} + \cdots + a_{1j}x_{j} + \cdots + a_{1n}x_{n}.
\end{align*}
\]

Exception to the above equation occurs for the s_{ik}^{k}(k = 2, 3, \ldots, t) that represent transfer of net income from one year to operating capital of the next year for years 2 through t. The supply of operating capital is increased each year by the difference between (a) the net income of the previous year and (b) fixed costs and household withdrawals of the previous year. This process becomes automatic in the programming operation by including two conditions which represent (1) withdrawal of funds for fixed costs and household expenditures and (2) transfer of capital between years.
representing the magnitude of annual fixed costs and household expenditures is entered in the resource vector (Pₐ column) to permit a withdrawal activity to enter the plan at this exact level. This activity is "forced" into the plan at this level for each year by assigning an artificially large cᵢ value (+m value) to it. "False profit" so accumulated in the plan is subtracted out of the final program.

The capital transfer between years might be accomplished by several methods. Here we accomplish this as follows:

Any activity produced in the k-th year has a positive coefficient in the capital equation for year k but has a negative coefficient in the capital equation for year k+1. In simplex calculation, for example, one unit (acre) of corn may require $20 of operating capital in year 1 and yield a net return (Cᵢ value) of $35 in year 1. A unit of this activity produced in year 1 will add $20+$35 to the supply of operating capital in year 2, if the farmer is operating from his own funds and need not repay a loan with the $20. Since $35 is added to the capital supply of the next year, −55 becomes the coefficient in the column for the corn activity and the row for capital supply in year 2. Algebraically, the total supply of operating capital so accumulated in year k is:

\[
S_k = \sum_{i=1}^{k} (C_i - a_{ij}x_j) - a_{jn}x_n
\]

where capital is the first resource supply (s₁) and xₙ is the fixed cost-family living activity.

In terms of the programming model, the set of equations for year 1 can now be expressed as:

\[
\begin{align*}
S_1 & \geq a_{11}x_1 + a_{12}x_2 + \cdots + a_{1j}x_j + \cdots + a_{1n}x_n \\
S_2 & \geq a_{21}x_1 + a_{22}x_2 + \cdots + a_{2j}x_j + \cdots + a_{2n}x_n \\
\vdots & \vdots \\
S_{m} & \geq a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mj}x_j + \cdots + a_{mn}x_n
\end{align*}
\]

where s₁ refers to the capital supply in year 1, s₂ \cdots sₘ represent other resource restrictions, s₁ is the fixed cost, including household consumption, for year 1 and all aₘᵢ are zero for j \neq n.

Remaining equations for years 2 through t are somewhat enlarged in the

\footnote{It should be noted here that only operating capital is transferred between years. If some portion of the capital requirement includes investment, only the part used for operating costs is transferable.}
s^k_1 row because of the capital transfer process. For example, in year k (k ≠ 1) these equations become:

\[
\begin{align*}
    s^k_1 & \geq k^{-1} - a_{11} x_1 - a_{12} x_2 - \cdots - a_{1j} x_j - \cdots - a_{1n} x_n \\
    s^k_2 & \geq k^{-1} + a_{11} x_1 + a_{12} x_2 + \cdots + a_{1j} x_j + \cdots + a_{1n} x_n \\
    \vdots &= \cdots \\
    s^k_m & \geq k^{-1} + a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mj} x_j + \cdots + a_{mn} x_n
\end{align*}
\]

where \( s^k_1 \) refers to the supply of capital, \( s^k_2 \cdots s^k_m \cdots s^k_{m-1} \) represent other types of resources which do not have inter-year transfers and \( s^k_m \) is the total fixed cost, including household consumption, and all \( a^k_{ij} \) (in the equation for \( s^k_j \)) entries, indicating additions to capital from the previous year’s production, are negative. Again, all \( a^k_{ij} \) for \( j \neq n \) are zero.

As in the ordinary static model, \( x^k \geq 0 \) and the profit function

\[
Z = c_{11} x_1 + c_{12} x_2 + \cdots + c_{1j} x_j + \cdots + c_{1n} x_n + c_{21} x_1 + c_{22} x_2 + \cdots + c_{2j} x_j + \cdots + c_{2n} x_n + \cdots + c_{n1} x_1 + c_{n2} x_2 + \cdots + c_{nj} x_j + \cdots + c_{nn} x_n
\]

is maximized. In this, \( Z \) is the maximum present value of future incomes, under the constraint that certain fixed costs and family living expenses of each year are “just exactly met.” Hence, each element \( c^k_j \) is a discounted quantity defined as \( c^k_j = c^0_j / (1 + r)^k \) where \( c^0_j \) is the nondiscounted net revenue of the \( j \)-th activity in the \( k \)-th year. Although it is not done in this study, the objective in many studies might be maximization of capital values, with activities arranged to express values of resources at the end of the relevant period, depending on the optimum program selected.

**Numerical Illustration**

A dynamic linear programming model has been outlined algebraically in the above section. This model is numerically illustrated in Table 1 by using an oversimplified farm situation. As the problem is set up in Table 1, there are two production alternatives (hogs and beef) and two resource supplies (capital and labor) for each year. In addition, a fixed cost-family living activity and a corresponding restraint are included for each year. The identity matrix for disposal activities is not included in Table 1.
**DYNAMIC PROGRAMMING MODELS**

**TABLE 1. NUMERICAL TABLEAU FOR DYNAMIC LINEAR PROGRAMMING**

<table>
<thead>
<tr>
<th>Resource supplies</th>
<th>Cj</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>60</td>
<td>36</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>P0</td>
<td>P1</td>
<td>P2</td>
<td>P3</td>
</tr>
<tr>
<td>Capital</td>
<td>7,000</td>
<td>150</td>
<td>-210</td>
<td>-156</td>
</tr>
<tr>
<td>Labor</td>
<td>720</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fixed costs*</td>
<td>4,000</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Capital</td>
<td>750</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Labor</td>
<td>3,500</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fixed costs*</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* Includes annual withdrawal for family living plus fixed annual business costs.

For simplicity, the undiscounted returns \( (c_j^k) \) per unit of hog and beef production are identical in all three years. For actual situations, price cycles might be predicted, with net return altered accordingly. At a 6 per cent interest rate, the discounted return per unit of hog production is

\[
\frac{$60}{(1 + .06)^2} = $53.40
\]

in year 2, and

\[
\frac{$60}{(1 + .06)^3} = $50.38
\]

in year 3. Likewise, the discounted returns per unit of beef production in years 2 and 3 are $32.04 and $30.23, respectively.

The fixed cost-family living activity for this example has an arbitrary \( c_j \) value of 500 in all years. The important thing in selecting this \( c_j \) value is that it must be large enough to "force in" the fixed cost-family living activity. That is, an activity is "forced in" when its' \( c_j \) value gives an "artificially higher return" on capital than other activities.

Many variations in production alternatives and resource supplies can be used for different years in a dynamic programming model: Family labor supplies may increase in future years as children progress in age; the level of household expenditures and business fixed costs may change from year to year, depending on the family budget; levels of soil fertility may be increased; etc. The example of Table 1 has variations in both annual labor supplies and fixed costs. Although identical production alternatives are shown for each of the three years, an empirical model may allow new pro-
duction alternatives when the operator's equity has increased or he has accumulated sufficient capital.

Parallel to the algebraic illustration explained previously, the supplies of operating capital for years 2 and beyond are determined by the magnitude of production in prior years and the amount of fixed expenditures for the year concerned. For the example in Table 1, the amount of operating capital in year 2 is: number of hog units produced in year 1 times $210, plus number of beef units produced in year 1 times $156, minus $4,000—the amount of fixed expenditures in year 2. The $210 forthcoming from each unit of hog production is equal to the capital requirement ($150) plus the return ($60) in year 1, since the farmer uses his own funds. Likewise, a unit of beef in year 1 contributes $120+$36 = $156 of capital to year 2.

Actually, the method for determining future capital supplies may be arbitrarily chosen by the programmer. The method described above is used in the problem and solution which follows. One exception to the exact routine described occurs when the initial capital requirement includes investment in equipment, breeding stock, etc.: the capital requirement for initial production then is necessarily higher than for production in following periods. Since depreciation and replacement are annual costs, the return or $c_{ij}^k$ value remains the same for the investment period and following periods; only the capital requirement or cash outlay is increased when the investment is made. Likewise, capital used for investment in one year is not transferred to the capital supply for following years.

An Empirical Application to an Iowa Farm

This section is devoted to the application and results of a dynamic linear programming model applied to an Iowa farm. The 300-acre central Iowa farm is tenant-operated with a 50-50 livestock-share lease. A brief summary of the restrictions and production alternatives for the case farm are given in Table 2. Input-output data for crop and livestock enterprises were developed from detailed cost account records and feed and labor records kept by the operator, and by consultation with the farm operator. All planning restraints, except the capital supply, listed in the first column of Table 2 were repeated in equations of each year; the supply of operating capital varied from year to year according to the procedure outlined earlier.

Household expenditures

In addition to the information in Table 2 and corresponding input-output data, projected living costs for the case farm family were needed for the programming model used. The planning period chosen for analysis of the case farm was eight years. Hence, estimates of total annual living costs were required for eight years. To facilitate detailed analysis of household ex-
penditures, they are itemized into 10 categories as shown in Table 3. These individual cost items were estimated by the farm family with the aid of detailed budgetary forms and the counsel of a home economist. The family devoted much thought to these. In making up these estimates, the family projected planned expenditures for such items as an additional baby, a set of encyclopedias, a bicycle, as well as conventional outlays. With a categorized list of living costs, adjustments within the budget are more easily identified. That is, if living costs are spontaneously decreased or increased, because of changes in expected farm income, the per dollar marginal returns in the farm business may be useful in determining the exact place to make cost changes in the household budget. This routine permits a comparison of satisfactions between profits which are measured quantitatively and certain intangibles associated with family preferences.

Table 3. Projected Living Costs for the Case Farm Family

<table>
<thead>
<tr>
<th>Item</th>
<th>Living costs in dollars for year:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Food purchased</td>
<td>1,170</td>
</tr>
<tr>
<td>Clothing and personals</td>
<td>271</td>
</tr>
<tr>
<td>Household operation</td>
<td>529</td>
</tr>
<tr>
<td>Repairs and furnishings</td>
<td>135</td>
</tr>
<tr>
<td>Health</td>
<td>739</td>
</tr>
<tr>
<td>Recreation</td>
<td>30</td>
</tr>
<tr>
<td>Education</td>
<td>32</td>
</tr>
<tr>
<td>Giving</td>
<td>185</td>
</tr>
<tr>
<td>Auto–family use</td>
<td>125</td>
</tr>
<tr>
<td>Income and social security taxes</td>
<td>458</td>
</tr>
<tr>
<td>Total</td>
<td>3,600</td>
</tr>
</tbody>
</table>
Other costs

The "fixed expenditure" activity referred to earlier in the dynamic programming model includes machinery depreciation, taxes, insurance and added investments, in addition to living costs. The sum of these costs for each year is the fixed amount of funds subtracted from the potential annual supplies of operating capital for the farm business.

In Table 4, total annual fixed expenditures are shown for the eight-year period. The figures under household expenditures are transferred directly from Table 3. Additional fixed expenditures are as indicated in the third column of Table 4. The "added investments" portion of fixed costs refers to purchases that do not replace any item in the current inventory. Instead, these investments are "added to" the inventory of depreciable goods already on the farm.

### Table 4. Summary of Projected Annual Fixed Costs for the Case Farm

<table>
<thead>
<tr>
<th>Year number</th>
<th>Total household expenditures</th>
<th>Total expenses for machinery depreciation, taxes, insurance and added investments</th>
<th>Total fixed expenditures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>$3,600</td>
<td>$2,536</td>
<td>$6,136</td>
</tr>
<tr>
<td>Year 2</td>
<td>4,087</td>
<td>1,733</td>
<td>5,820</td>
</tr>
<tr>
<td>Year 3</td>
<td>4,187</td>
<td>2,836</td>
<td>7,023</td>
</tr>
<tr>
<td>Year 4</td>
<td>4,412</td>
<td>3,536</td>
<td>7,948</td>
</tr>
<tr>
<td>Year 5</td>
<td>4,257</td>
<td>3,036</td>
<td>7,293</td>
</tr>
<tr>
<td>Year 6</td>
<td>4,393</td>
<td>2,036</td>
<td>6,429</td>
</tr>
<tr>
<td>Year 7</td>
<td>4,443</td>
<td>2,036</td>
<td>6,479</td>
</tr>
<tr>
<td>Year 8</td>
<td>4,443</td>
<td>2,036</td>
<td>6,479</td>
</tr>
</tbody>
</table>

Case farm results

Within the planning boundaries just mentioned for the case farm, optimum farm plans for eight successive years are shown in Table 5. These plans were computed by use of the dynamic linear programming model outlined in an earlier section.

The amount of expected operating capital for each of the eight years is shown in the second column; the third column gives the optimum farm plan and corresponding returns are shown in the fourth column. Total fixed expenditures, as illustrated in Table 4, are listed in the fifth column; the difference between farm returns and total fixed charges—the amount added to operating capital in the coming year—is indicated in the sixth column.

Having established the magnitude of operating capital in year 1 and the amounts of total annual fixed charges (column 5 in Table 5), optimum farm plans, return figures and supplies of operating capital for years 2 through 8 are all generated by the dynamic programming process. That is, the optimum farm plans for year 1 and successive years actually show the most
### Table 5. Annual Expansion of a Dynamic Farm Plan by Investing Cumulating Returns

<table>
<thead>
<tr>
<th>Year</th>
<th>Cumulative operating capital</th>
<th>Farm plan</th>
<th>Return</th>
<th>Fixed(d) expenditures</th>
<th>Return minus fixed expenditures (Col. 4 minus Col. 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$4,813</td>
<td>Crops 45 hog litters</td>
<td>$6,822</td>
<td>$6,136</td>
<td>$686</td>
</tr>
<tr>
<td>2</td>
<td>5,499</td>
<td>Crops 56 hog litters</td>
<td>7,246</td>
<td>5,820</td>
<td>1,426</td>
</tr>
<tr>
<td>3</td>
<td>6,925</td>
<td>Crops 80 hog litters</td>
<td>8,058</td>
<td>7,023</td>
<td>1,065</td>
</tr>
<tr>
<td>4</td>
<td>7,990</td>
<td>Crops 80 hog litters 17 long-fed steers</td>
<td>8,433</td>
<td>7,948</td>
<td>485</td>
</tr>
<tr>
<td>5</td>
<td>8,475</td>
<td>Crops 80 hog litters 23 long-fed steers</td>
<td>8,540</td>
<td>7,293</td>
<td>1,247</td>
</tr>
<tr>
<td>6</td>
<td>9,722(e)</td>
<td>Crops 80 hog litters 48 short-fed heifers</td>
<td>8,892</td>
<td>6,429</td>
<td>2,463</td>
</tr>
<tr>
<td>7</td>
<td>12,185(e)</td>
<td>Crops 80 hog litters 48 short-fed heifers</td>
<td>8,892</td>
<td>6,479</td>
<td>2,413</td>
</tr>
<tr>
<td>8</td>
<td>14,508(e)</td>
<td>Crops 80 hog litters 48 short-fed heifers</td>
<td>8,892</td>
<td>6,479</td>
<td>2,413</td>
</tr>
</tbody>
</table>

\(a\) All capital and return figures are for tenant only, but the farm plan indicates the total production for the farm.

\(b\) Operating capital does not include investment in machinery.

\(c\) The long-run cropping plan is established as 115 acres of CCOMM and 157 acres of CCSb.

\(d\) Fixed expenditures including living expenses, machinery depreciation, taxes, insurance and added investments.

\(e\) The amount of operating capital required for the plan in these years is $9,579. Hence, the amount of capital exceeding $9,579 represents surplus or unused capital.

**Profitable sequence for investing increasing amounts of operating capital in the farm business.** The exact amounts of operating capital for years 2 through 8 and corresponding plans are dependent on the original supply of operating capital, expected returns in each year and total annual fixed expenditures.

**Interpreting farm plans**

The amount of operating capital in year 1 is most profitably invested in
crops and 45 hog litters. (Several crops are produced, but we do not go into the details of their “entry” into the farm plan because emphasis in this paper is elsewhere. Similarly, we do not indicate the basis of selection of livestock activities from the numerous ones used.) Remaining funds, after sufficient operating capital has been allocated to crops, give highest returns when used for hog production. Accordingly, increases in operating capital in years 2 and 3 are used for increased hog production. However, buildings and managerial ability restrict hog production to 80 litters. When the amount of available operating capital exceeds the capital requirements for crops and 80 hog litters, the farm plan is expanded by including long-fed steers (the optimum plan for year 4). Additional funds at this stage are used for increased numbers of steers. This particular expansion is retarded in year 6 when the supply of home-grown feeds becomes limiting. Consequently, short-fed heifers replace long-fed steers in year 6, because heifer production gives higher returns to feed as compared to steers. With a combination of crops, 80 hog litters and 48 short-fed heifers, $9,579 of operating capital is required and other scarce resources on the farm are optimally utilized. Hence, an increase in operating capital cannot increase returns in years 7 and 8, and the optimum farm plan remains unchanged, given the short-run restraints imposed in the programming model. In fact, there was already a surplus capital of $143 in year 6 which could not be profitably used in the farm business. The surplus increased in years 7 and 8.

As the data of Table 5 indicate, the plan has become “stable” at the end of six years. Hence, only this number of years would have been required for this problem. However, the number of years needed to arrive at a plan which becomes “stable” for additional years will depend on several factors: family goals, consumption withdrawals, capital availability, opportunities for lifting the short-run restrictions on resources and planning, etc. In the case presented here, we have not allowed for “increasing the length of run” for several restrictions, although we have done so for capital.

Analyzing household expenditures

The long-range farm planning in Table 5 establishes important guides for appraising household expenditures for the same period. If returns to capital are seemingly high when used for the farm business, the family may prefer to cut household expenditures to a minimum. As the level of operating capital is increased and per dollar returns decrease, the farm family may get more satisfaction from increasing household costs and allocating less funds to operating capital for farm business. Regardless of what decisions are made, a more critical analysis is possible by formulating a household budget and deriving long-range farm plans.

The optimum long-run cropping plan is established as 115 acres of the CCOMM rotation and 157 acres of the CCSb rotation. All crops are fertilized at the highest rate considered.
This method of analyzing farm and home plans may be illustrated for any one of the years in the planning period. For example, education costs for year 4 in Table 3 are increased by $200 over year 3. The case farm family earmarked this increased expenditure for a set of encyclopedias. At the time the budget was made, the parents thought their children should have access to encyclopedias four years hence. Hence, their household budget included this purchase in year 4. Now, by re-examining Table 5, one can determine the returns to added operating capital in year 5. That is, the dollars and cents sacrifice of buying encyclopedias in year 4 can be estimated by computing the potential return of this investment if it had been used for the farm business in year 5.

The addition to operating capital in year 5 is $485. This added investment increased returns $107 ($8,540 - $8,433 = $107). The average per dollar returns for this added investment is about $.22 (107/485 = .2206). Consequently, returns could be increased $44 ($200 x .22 = $44) in year 5 by allocating the $200 for encyclopedias to the farm business. Similarly, one can compute the potential returns in the farm business from reallocating other expenditures in Table 3. The critical decision for the family is whether or not they get more satisfaction from extra farm profits than less intangible gain from certain household expenditures.

Use in extension

The complete process of developing optimum farm plans, projecting a household budget and making applied analysis as illustrated above, presents a supplementary tool for farm and home planning. Programming farm plans, of course, requires both trained help and mechanical aid. Too, formulating a household budget requires the counseling aid of technicians in that particular field. In fact, home accounts are as important as farm records when making a complete appraisal of the farm and home unit.

Although the majority of farms maintain some sort of records (for income tax purposes if nothing else), the exact form of data required for linear programming is seldom available. Most cost accounts refer to aggregate expenditures such as fuel, seed, fertilizer, feed, etc., and there is no indication of per unit input-output data on an enterprise basis. The seasoned specialist may find adequate information in conventional records for sufficient guidance to some families. But even then, a long-run planning horizon incorporating specific figures in the projected plans would certainly facilitate the specialist in his job and the family in identifying their values and goals.

\* At first, one may argue that per dollar returns of increased capital in year 5 would be equal to the marginal returns of increased capital in year 6. However, the optimum plan in year 5 includes the same production until operating capital is greater than $8,765. This capital level gives an optimum plan with 25 long-fed steers. As operating capital exceeds $8,765, heifers replace steers. Hence, $200 added to operating capital in year 5 would give a total supply of $8,765 and per dollar marginal returns are as already indicated for year 5.
Further Possibilities in Farm and Home Planning

The potential use of applying dynamic linear programming techniques to the individual farm and home unit depends mainly on data availability and types of situations to be analyzed. In our example, we have only "one way dependence," since the optimum farm plan depends on the level of family living expenditures and fixed costs required in each year. Still keeping the approach simple, we could have incorporated "two way dependence" into the model; with the best combination farm and home plan, one where choice on the home side would depend on opportunities on the farm side and vice versa. These methods will be outlined elsewhere.