A Model of CP Violation

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A model of CP violation is proposed. In this model both CP-conserving and CP-violating nonleptonic decays are attributed to the same primary interaction, and they occur, respectively, in the second and third orders of the primary interaction. This model predicts (1) $3 \times 10^{-4} > |\gamma_{01}| > 1 \times 10^{-3}$, and (2) $|D| = |\mu_{01}/e| = 2 \times 10^{-22}$ cm for the neutron.

§ 1. Introduction

The discovery of the decay mode $K_L^0 \to \pi^+ + \pi^-$ by Cristenson, Cronin, Fitch and Turlayl led to the conclusion that CP is violated, and the magnitude of CP violation is usually represented by parameters $\gamma_{+-}$ and $\gamma_{00}$, both being of the order of $10^{-3}$. Various models have been proposed to account for the new decay modes. For instance, some authors suggested the possibility of attributing CP violation to the electromagnetic interactions, and others proposed the superweak interaction model. In most of them, however, new parameters are introduced and are adjusted to fit the observed decay rates for $K_L^0 \to 2\pi$.

It seems to be rather unnatural, however, that such an extremely weak interaction should exist solely to cause CP-violating decays without affecting other processes. For this reason we proposed a new model of nonleptonic decays based on an assumption that both CP-conserving and CP-violating decays can be attributed to the same primary interaction. This idea necessarily conflicts with the widely accepted current-current interaction model of nonleptonic decays.

We first observe that the amplitudes of typical CP-conserving nonleptonic decays are of the order of $10^{-4}$, whereas those of CP-violating decays, namely, $K_L^0 \to 2\pi$, are of the order of $10^{-5}$. This fact suggests that CP-conserving and CP-violating decays might represent, respectively, the second and third order processes of the same interaction that is odd under CP and is of the order of $10^{-3}$ in amplitude. In this article we take this possibility for granted, then this interaction must violate $P$, $C$ and CP, separately, since both $P$ and $C$ are violated in nonleptonic decays.

Two questions are immediately raised. The first one is why we have not observed large CP-violating effects corresponding to the first order interaction of the order of $f \sim 10^{-5}$ in amplitude, and the second one is what the possible form of this new interaction could be. It is one of the purposes of this paper to answer these questions.
In §2 we first propose the primary interaction responsible for nonleptonic decays. This interaction is determined in such a way that it has no observable manifestations in the first order. Therefore, the second order gives the lowest non-vanishing $S$ matrix elements. A brief discussion of the second order $S$ matrix elements is given in §3. The primary interaction necessarily violates conservation of strangeness, and consequently there arises the possibility characteristic of the present model that strangeness might change by two units in the lowest observable order in contradiction to the experimentally established selection rule $|\Delta S|=0,1$. In §4 we show that these unwanted processes can be forbidden by introducing a new set of constraints which has rather interesting intuitive interpretations.

The second order $S$ matrix describes the $CP$-conserving nonleptonic decays. In §5 we study the $S$-wave hyperon decays and determine the coupling constant of our new interaction by comparing theory with experiment. The overall agreement between theory and experiment is reasonable. In §6, on the basis of the present model we prove the octet dominance in the $P$-wave hyperon decays and suggest the origin of the Lee-Sugawara sum rule.

Having determined the coupling constant in §5 we proceed to the $CP$-violating third order. In §7 we study the decay amplitudes for $K_2^0\rightarrow2\pi$ and predict $|\varepsilon'|=0.4\times10^{-3}$, where $3\varepsilon'=\eta_{\pi^+}\!-\!\eta_{\pi^0}$. In §8 we evaluate the electric dipole moment of the neutron and get $|\mu_d/e|=2\times10^{-22}$ cm.

Some topics related to the text are discussed in the appendices.

§2. The primary interaction

As we have argued in the Introduction the primary interaction violates $P$, $C$ and $CP$ separately and is of the order of $10^{-3}$ in amplitude. Parity violation of the order of $10^{-1}$ in amplitude is, however, experimentally excluded. Therefore, the first order transitions must be forbidden. This requirement leads to the condition

$$\int d^4xH(x)=0,$$  \hspace{1cm} (2.1)

where we have employed the Heisenberg representation defined with reference to the strong and electromagnetic interactions. The model presented here may be classified as an intermediate-weak-interaction model. Similar models have been proposed by Okubo$^8$ and by Fronsdal,$^6$ but the condition (2.1) or its modification is a common feature to all the intermediate-weak-interaction models.

Equation (2.1) is not the only condition that should be imposed on the Hamiltonian. The first order transitions should be forbidden even in the presence of an external electromagnetic field, since otherwise we should observe too large a static electric dipole moment of the neutron. This leads to the second condition
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\[ \int d^4x \int d^4y T(H(x), j_\alpha(y)) A_\alpha^{\text{ext}}(y) = 0. \] (2.2)

The solution of Eq. (2.1) is given by

\[ H = f \partial_\lambda K_\lambda, \] (2.3)

that is, Eq. (2.1) is satisfied automatically if \( H \) is given by a divergence of a current. It should be emphasized here, however, that the Hamiltonian or the negative of the Lagrangian becomes equal to this form as a consequence of field equations for strong and electromagnetic interactions. As is well known a divergence in the Lagrangian in the real Heisenberg representation has no physical consequences whatsoever, and in order to avoid misunderstanding on this point Appendix A has been added.

If we employ (2.3) as the Hamiltonian, the condition (2.2) reduces to

\[ \left[ \int_{x_0} d^3x K_\alpha(x), j_\alpha(y) \right] = 0. \] (2.4)

There are various possible choices of the current \( K_\lambda \) for satisfying this commutation relation. As we shall show in the Appendix D, \( K_\lambda \) must be a linear combination of the neutral densities in the algebra of currents. This still does not determine \( K_\lambda \) uniquely and we need an additional assumption. For this purpose it is instructive to recall the more or less established Cabibbo form of the semileptonic decay interaction:

\[ H_{\alpha} = \frac{G_F}{\sqrt{2}} (J_\alpha j_\alpha^\dagger + J_\alpha^\dagger j_\alpha), \] (2.5)

where \( j_\alpha \) denotes the leptonic current, and \( J_\alpha \) is the hadronic current given, in the conventional notation, by

\[ J_\alpha = \cos \theta (\mathcal{F}_{\alpha}^{(t)} - i\mathcal{F}_{\alpha}^{(t)}) + \sin \theta (\mathcal{F}_{\alpha}^{(t)} - i\mathcal{F}_{\alpha}^{(t)}), \] (2.6)

with

\[ \mathcal{F}_{\alpha}^{(t)} = \mathcal{F}_{\alpha}^{t_\alpha} \pm \mathcal{F}_{\alpha}^{t_\alpha}. \] (2.7)

In order to determine the structure of the neutral current \( K_\lambda \) more precisely we introduce a new postulate.

**Postulate**

The space integrals of \( J_\alpha, J_\alpha^\dagger \) and \( K_\alpha \), namely,

\[ \int d^3x J_\alpha(x), \int d^3x J_\alpha^\dagger(x) \quad \text{and} \quad \int d^3x K_\alpha(x) \]

form an \( SU(2) \) algebra.

Then, on the basis of the algebra of currents the neutral current \( K_\lambda \) is uniquely determined and is given by:

\[ H_{\alpha} = \frac{G_F}{\sqrt{2}} (J_\alpha j_\alpha^\dagger + J_\alpha^\dagger j_\alpha), \] (2.5)
It is worth mentioning that the above postulate automatically leads to the conclusion that \( \partial_\lambda K_\lambda \) is odd under \( CP \) in agreement with our assumption made in §1. In the following sections we shall approximate \( K_\lambda \) by dropping the sin\( ^2 \theta \) term and also replacing \( \cos \theta \) by unity, that is,

\[
K_\lambda \approx \mathcal{F}_{kl}^{(+)} + \beta \mathcal{F}_{kl}^{(+)}
\]

with

\[
\beta \approx -\frac{1}{2} \tan 2\theta \approx -\frac{2}{7}.
\]

Let us now define \( K'_\lambda \) by replacing \( \mathcal{F}_{kl}^{(+)} \) and \( \mathcal{F}_{kl}^{(+)} \) in (2·8) by \( \mathcal{F}_{kl}^{(\ast)} \) and \( \mathcal{F}_{kl}^{(\ast)} \), respectively, then

\[
\partial_\lambda K_\lambda = \partial_\lambda K'_\lambda,
\]

since

\[
\partial_\lambda \mathcal{F}_{kl}^{(\ast)} = \partial_\lambda \mathcal{F}_{kl}^{(\ast)} = 0.
\]

Rigorously speaking, \( K_\lambda \) in (2·4) has to be replaced by \( K'_\lambda \) although Eq. (2·4) holds for both \( K_\lambda \) and \( K'_\lambda \). Equation (2·4) guarantees that processes of the first order in \( H \) and also of the first order in the external electromagnetic field are forbidden, but it is not difficult to prove that this is also the case for the higher order corrections due to the external field.

§ 3. Second order processes

The second order \( S \) matrix is given by

\[
S^{(2)} = \frac{1}{2} (-i f)^{\ast} \int d^4 x \int d^4 y T(\partial_\lambda K_\lambda(x), \partial_\lambda K'_\lambda(y))
\]

\[
= -\frac{1}{2} f^{\ast} \int d^4 x \int d^4 y T(\partial_\lambda K'_\lambda(x), \partial_\lambda K'_\lambda(y)).
\]

We introduce an effective second order Hamiltonian by

\[
S^{(2)} = -i \int d^4 x H^{(2)}(x),
\]

then the approximate form of \( H^{(2)}(x) \) is given, on the basis of the approximate expression (2·9) for \( K \), by

\[
H^{(2)} = \frac{i}{2} f \left[ \int_{y=x} d^3 y K'_\lambda(y), \partial_\lambda K'_\lambda(x) \right] \]
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\[
\frac{i}{2} f^a [F_s^a, \partial_s \mathcal{F}_5^b] + i f^a \beta [F_0^a, \partial_s \mathcal{F}_5^b] + \frac{i}{2} f^a \beta [F_{\pm}^a, \partial_s \mathcal{F}_5^b],
\]

(3.3)

where

\[
F_s^a = \int d^4x \mathcal{F}^b_{50}(x), \text{ etc.},
\]

and use has been made of the equality

\[
\int d^4x [F_s^a(x_0), \partial_s \mathcal{F}^b_{\pm}(x)] = \int d^4x [F_0^a(x_0), \partial_s \mathcal{F}^b_{\pm}(x)]
\]

in writing down the second term in (3.3). The first term in (3.3) conserves C, P and strangeness and is not subject to any experimental test. The second term obeys the selection rule \(\Delta S = \pm 1\) and is fully responsible for the strangeness-changing nonleptonic decays. The last term, however, can generally cause transitions obeying either \(\Delta S = 0\) or \(\pm 2\). The strangeness-conserving part of the last term is considered to be responsible for parity violation in nuclear forces. The corresponding coupling constant \(f^a\beta\) is of the order of \(0.5 \times 10^{-6}\), as we shall see later, and is in conformity with the experimentally observed order of magnitude of the parity mixing in nuclear forces.\(^3\)

§ 4. The selection rule \(\Delta S = 0, 1\) in the second order

As we have seen in the previous section there is a term in the effective second order Hamiltonian which might change strangeness by two units. The observed value of \(K_s - K_L\) mass difference definitely excludes transitions changing strangeness by two units so that we have to require that the last term in (3.3) should commute with strangeness or \(F_\nu\). This requirement leads to

\[
[F_s, [F_0^a, \partial_s \mathcal{F}^b_{\pm}(K^0)]] = 0,
\]

(4.1)

or

\[
[F_0^a, \partial_s \mathcal{F}^b_{\pm} + [F_0^a, \partial_s \mathcal{F}^b_{\pm}] = 0.
\]

(4.2)

This relation is equivalent to

\[
[F_{\pm}^a, \partial_s \mathcal{F}_{\lambda}^{(\pm)}(K^0)] = 0,
\]

(4.3)

where

\[
F_{\pm}^a(K^0) = (F_0^{(\pm)} - iF_{\pm}^{(\pm)}) / \sqrt{2}, \text{ etc.}
\]

We shall make the condition (4.3) stronger by requiring

\[
[F_{\pm}^a(K^0), \partial_s \mathcal{F}_{\lambda}^{(\pm)}(K^0)] = 0,
\]

(4.4)
In the spirit of the vector dominance model we represent and for this reason we shall assume the vector-meson-exchange dominance. Then are very similar to those for the S-wave pion-nucleon scattering at low energies, therefore, we study the CP-conserving nonleptonic decays of hadrons.

The first thing that we have to do is to show that these constraints are consistent with the current commutation relations. In order to prove the consistency between them it is sufficient to construct one model satisfying both conditions. This subject is picked out in Appendix B. Furthermore, these constraints have a very intuitive physical interpretation. \( \partial_\lambda \mathcal{F}^{(\pm)}(K^0) \) vanishes in the exact chiral \( SU(3) \times SU(3) \) symmetry limit. Hence, it is not difficult to guess that they dictate the pattern of symmetry breaking without reference to the Hamiltonian formalism. This statement is thoroughly illustrated in Appendix C. Once these constraints are assumed, the selection rule \( |\Delta S|=0,1 \) is valid also in the third order as we shall see in Appendix D.

§ 5. S-wave hyperon decays

We have introduced a new interaction (2·3) and have explicitly assumed that this interaction is fully responsible for the CP-conserving nonleptonic decays. Thus, we can determine the coupling constant \( f \) from the CP-conserving nonleptonic decays without invoking the CP-violating \( K_L \) decays. The amplitudes for the latter processes can be evaluated, at least in principle, from the Hamiltonian (2·3) once \( f \) is determined. This is probably one of the most important features of the present model. In §1 it has been suggested that the coupling constant \( f \) be of the order of \( 10^{-3} \) on the basis of the above reasoning, so that it has to be verified that \( f \) is really of this desired order of magnitude. In this section, therefore, we study the CP-conserving nonleptonic decays of hadrons.

The S-wave decays of hadrons are relatively well understood, so that we shall utilize the S-wave decays to determine \( f \). The S-matrix corresponding to hadron decays obeying \( |\Delta S|=\pm1 \) in the order \( f^2 \) is given by the second term in (3·3). If we confine ourselves to the S-wave decays, the corresponding S matrix is obtained from the second term in (3·3) by replacing \( F_6^{(+)\pm} \) by \( F_6 \). As has been emphasized in previous papers, the Feynman diagrams for these processes are very similar to those for the S-wave pion-nucleon scattering at low energies, and for this reason we shall assume the vector-meson-exchange dominance. Then the next question is how to realize this mechanism within the framework of the present model.

The coupling of the octet vector mesons to the octet baryons and the octet pseudoscalar mesons is represented, in the conventional matrix notation, by

\[
H_V = \frac{i}{\sqrt{2}} f_V (\langle \bar{T}_V \gamma_\lambda V_\lambda - \bar{T}_V \gamma_\lambda \gamma_5 V_\lambda \rangle \\
+ \langle \gamma_\lambda \gamma_5 V_\lambda - \gamma_\lambda \gamma_5 V_\lambda \rangle).
\]

In the spirit of the vector dominance model we represent \( \partial_\lambda \mathcal{F}^{(\pm)}_\lambda \) by an effective interaction causing transitions between vector mesons and pseudoscalar mesons.
For this purpose, we treat $\partial_\lambda \mathcal{F}_{\pi}^a$ as a spurion by contracting this expression with $\mathcal{P}$ in (5·1). Then, we may replace $\partial_\lambda \mathcal{F}_{\pi}^a$ by an effective interaction of the form

$$\partial_\lambda \mathcal{F}_{\pi}^a \rightarrow -\frac{1}{2} f_\pi \left( \frac{M}{G} \right) \left( \frac{g_A}{g_\rho} \right) \text{Tr}(C V_a[\partial_\lambda \mathcal{P}, \lambda_t]), \quad (5·2)$$

where the proper proportionality constant has been determined with reference to the Goldberger-Treiman relation. $g_A$ and $g_\rho$ represent the Gamow-Teller and Fermi coupling constants, respectively. $M$ denotes the nucleon mass and $G$ is the pion-nucleon coupling constant.

The commutator $[F_\mu, \partial_\lambda \mathcal{F}_{\pi}^a]$ may be replaced by

$$[F_\mu, \partial_\lambda \mathcal{F}_{\pi}^a] \rightarrow \frac{i}{2} \left( -\frac{1}{2} f_\pi \left( \frac{M}{G} \right) \left( \frac{g_A}{g_\rho} \right) \text{Tr}(C V_a[\partial_\lambda \mathcal{P}, \lambda_t]) \right). \quad (5·3)$$

Then, by combining (5·3) with (5·1) we can evaluate the matrix elements for the decays $B_i \rightarrow B_f + \pi, \; K_i^0 \rightarrow 2\pi$.

For instance, the decay amplitude for $B_i \rightarrow B_f + \pi$ is given by

$$S(B_i \rightarrow B_f + \pi, \; S\text{-wave}) = -(2\pi)^4 \delta^4(P_f - P_i) \left( -\frac{1}{4} \right) f_\pi \left( \frac{g_A}{g_\rho} \right) \left( \frac{f_\pi^2}{2G} \right) \times \frac{(M_i - M_f) M_{\pi} u(B_f) u(B_i)}{\mu_*^2} \frac{A_{11}}{\sqrt{2q_0}}, \quad (5·5)$$

where $\mu_*$ denotes the $K^*$ meson mass. The factor $A$ depends on the decay mode and is given by

$$A(A^0 \rightarrow p + \pi^-) = \sqrt{3},$$
$$A(\Sigma^- \rightarrow A^0 + \pi^-) = \sqrt{3},$$
$$A(\Sigma^+ \rightarrow n + \pi^+) = 0,$$
$$A(\Sigma^- \rightarrow n + \pi^-) = \sqrt{2},$$
$$A(\Sigma^+ \rightarrow p + \pi^0) = -1, \quad (5·6)$$

and the selection rule $|\Delta I| = 1/2$ is obeyed in this approximation. It is also worth mentioning that the Lee-Sugawara sum rule\(^{19}\) holds in this model as a consequence of the Gell-Mann-Okubo mass formula for the baryon octet. The coupling constant $f$ can be determined by fitting the theoretical amplitude $A^0 \rightarrow p + \pi^-$ to the experimental one, and we obtain, by assuming $f_\pi^2/4\pi = 2.5$, the result

$$|f| \approx 3.0 \times 10^{-3}. \quad (5·7)$$

The overall agreement between theory and experiment for other modes of decay is reasonably good.
Similarly, we can evaluate the decay width for $K_1^0 \rightarrow 2\pi$ in the corresponding approximation, and the result is given by

$$
\Gamma(K_1^0 \rightarrow 2\pi) = \frac{3}{2} \Gamma(K_1^0 \rightarrow \pi^+ + \pi^-)
$$

$$
\approx \frac{3}{2} \Gamma(K_1^0 \rightarrow \pi^+ + \pi^-)
$$

$$
= \frac{3}{128} \beta^2 f^2 \frac{g_s}{g_v} \left( \frac{f_{\pi}}{4\pi} \right)^2 (\mu_K^2 - 4\mu_\pi^2)^{3/2}
$$

$$
\times \left( \frac{M}{\mu_K} \right)^2 \left( \frac{\mu_K^2 - \mu_\pi^2}{\mu_\pi^2} \right)^3.
$$

(5.8)

It should be mentioned that the decay rate of the physical $K_1$ meson is well approximated by that of the mathematically defined $K_1$ meson. Their definitions will be given at the end of this section.

The coupling constant determined from (5.8) is reasonably close to (5.7) and is given by

$$
|f| \simeq 2.6 \times 10^{-3}.
$$

(5.9)

The small difference between (5.7) and (5.9) may be attributed to the difference in the final state interactions. Since, however, our main interests are the $K$ meson decays we shall employ the latter value (5.9) in the following calculations. It should also be mentioned that the coupling constant $f$ has been determined by using the approximate form (2.9), and it is more precise to say that Eqs. (5.7) and (5.9) represent the numerical value of $|f| \cos 2\theta$. Since the constant $f$ appears always to be multiplied by $\cos 2\theta$, this introduces no serious error in the following calculations.

Finally, we mention the phase conventions adopted throughout this paper. The field operators of $K_1$ and $K_i$ are defined by

$$
\Phi_i = \frac{i}{\sqrt{2}} (\Phi - \Phi^*) , \quad \Phi_i = \frac{1}{\sqrt{2}} (\Phi + \Phi^*),
$$

(5.10)

where $\Phi$ denotes the field operator of the $K^0$ meson. This convention is obviously different from the one adopted by Wu and Yang.\textsuperscript{20} We have adopted such a convention for the convenience of incorporating the algebra of currents in our scheme. This difference, however, simply introduces an extra factor $i$ in $\gamma$'s and in related quantities. In order to make our convention as close to the conventional one as possible, we shall make the following observation:

The strong interactions are invariant under the simultaneous phase transformation

$$
\phi_a \rightarrow \phi_a \exp(i\alpha S_a),
$$

(5.11)

where $S_a$ is the strangeness of the quantum of the $\phi_a$ field. For the neutral $K$ meson fields we have
In terms of $\Phi_1$ and $\Phi_2$ this transformation is represented by

$$
\begin{align*}
\Phi_1 &\rightarrow \Phi_1 \cos \alpha - \Phi_2 \sin \alpha , \\
\Phi_2 &\rightarrow \Phi_1 \sin \alpha + \Phi_2 \cos \alpha .
\end{align*}
$$

Making use of the fact that weak interactions are no longer invariant under this phase transformation we shall choose a convenient phase $\alpha$ in such a way that we have, after the transformation,

$$
M(K^2_2 \rightarrow 2\pi, I=0) = 0 .
$$

Then we have

$$
\begin{align*}
|K_1\rangle &= |K_1\rangle + \frac{M_{21}}{M_{11} - M_{21}} |K_2\rangle , \\
|K_2\rangle &= |K_2\rangle + \frac{M_{21}}{M_{22} - M_{21}} |K_1\rangle ,
\end{align*}
$$

provided that the off-diagonal mass matrix elements $M_{12} = M_{21}$ are much smaller than $M_{11} - M_{21}$, and the parameter $\varepsilon^{12}$ is introduced by

$$
i\varepsilon = \frac{M_{12}}{M_{22} - M_{21}} .
$$

### § 6. P-wave hyperon decays

On the basis of the vector-meson-exchange-dominance model we have established the octet dominance and the selection rule $|M|=1/2$ for the $S$-wave decays and obtained a reasonable agreement with experiment. In this section we prove the octet dominance for the $P$-wave decays.

The parity-conserving $P$-wave decays of hyperons are described by

$$
H^{(s)}_{ps} = i f_+ \beta [F^i_\alpha, \partial_\alpha X_{2i}],
$$

and we prove that this Hamiltonian transforms as the sixth component of an octet in the exact $SU(3)$ symmetry limit.

From the Lorentz invariance of the commutation relation

$$
[F^i_\alpha , F^j_\alpha ] = i f_{ijkl} F^k ,
$$

it follows, by using the same technique as the one shown in Appendix D, that

$$
[F^i_\alpha , \partial_\alpha X_{j\alpha} ] + [\partial_\alpha X_{i\alpha} , F^j_\alpha ] = i f_{ijkl} \partial_\alpha X_{k\alpha} ,
$$

so that the following expressions must be symmetric in $i$ and $j$:

$$
i[F^i_\alpha , \partial_\alpha X_{j\alpha} ] + \frac{1}{2} f_{ijkl} \partial_\alpha X_{k\alpha} = i[F^i_\alpha , \partial_\alpha X_{i\alpha} ] + \frac{1}{2} f_{ijkl} \partial_\alpha X_{k\alpha} .
$$

In the exact $SU(3)$ limit $\partial_\alpha X_{k\alpha}$ vanishes, and the above expressions may be expressed as the sum of three representations $1+8_s+27$, namely,

$$
\begin{align*}
\Phi \rightarrow \Phi e^{i\alpha} , & \quad \Phi^\dagger \rightarrow \Phi^\dagger e^{-i\alpha} .
\end{align*}
$$
where \( k = 1, \ldots, 8 \), and \( \alpha = 1, \ldots, 27 \). In the present model we have a set of constraints to forbid transitions changing strangeness by two units. One of them is given by

\[
[F^8(K^0), \partial_x F^8_x(K^0)] = 0. \tag{6·6} \text{ or } (B·5)
\]

Combining (6·5) with (6·6) we find that

\[
S_{\alpha'}^{(\alpha')} = 0, \tag{6·7}
\]

where \( \alpha' \) is a member of the 27-plet with \( Y = 2 \), \( I = 1 \), \( \bar{I} = -1 \). Thus we may conclude that all the members of the 27-plet in (6·5) are equal to zero, and we have established the desired relation

\[
i[F^8_x, \partial_x F^8_x] = \delta_{ij} S^{(0)} + d_{ijk} S^{(0)}_k, \tag{6·8}
\]

We can now conclude that the Hamiltonian (6·1) transforms as the sixth component of an octet since \( d_{ijk} \neq 0 \). This completes the proof of the octet dominance in the \( P \)-wave decays of hyperons in the exact \( SU(3) \) limit.

Next we try to understand the Lee-Sugawara sum rule for the \( P \)-wave decays of hyperons. Obviously it is not possible to derive this sum rule without introducing an additional assumption, so that we start from the relation (B·15)

\[
S^{(\alpha')} = i[F^8, \mathcal{O}], \tag{6·9} \text{ or } (B·15)
\]

where \( \mathcal{O} \) is an \( SU(3) \)-invariant pseudoscalar. From this relation we immediately get

\[
[F^8_x, S^{(\alpha')}_j] = i[F^8_x, [F^8_j, \mathcal{O}]] \\
= i[F^8_x, [F^8_j, \mathcal{O}]] + i[[F^8_x, F^8_j], \mathcal{O}] \\
= [F^8_x, S^{(\alpha')}_j] - f_{ijk} [F^8_k, \mathcal{O}].
\]

Since \( \mathcal{O} \) is \( SU(3) \)-invariant the second term vanishes, and we get a symmetry relation

\[
[F^8_x, S^{(\alpha')}_j] = [F^8_j, S^{(\alpha')}]. \tag{6·10}
\]

We can show that the Lee-Sugawara sum rule follows from (6·10).

First, we write the phenomenological Hamiltonian for the \( P \)-wave decays of hyperons as

\[
S^{(\alpha')} = M_1 \text{ Tr}(\overline{\lambda}_2 \rho) + M_2 \text{ Tr}(\overline{\lambda}_3 \rho) + M_3 \text{ Tr}(\overline{\lambda}_4 \rho) \\
+ M_4 \text{ Tr}(\overline{\lambda}_5 \rho) + M_5 \text{ Tr}(\overline{\lambda}_6 \rho) + M_6 \text{ Tr}(\overline{\lambda}_7 \rho) \\
+ M_7 \text{ Tr}(\overline{\lambda}_8 \rho) + M_8 \text{ Tr}(\overline{\lambda}_9 \rho) + M_9 \text{ Tr}(\overline{\lambda}_10 \rho) \\
+ M_{10} \text{ Tr}(\overline{\lambda}_11 \rho), \tag{6·11}
\]

where we have suppressed \( i\gamma_5 \) common to all the terms in (6·11). Also, the
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real Hamiltonian is obtained by putting \( j=6 \), but here we need this general form. Next, we study the matrix elements of the symmetry condition (6·10) between two single-baryon states. Then, at the phenomenological level we may replace the commutator between \( F_i^b \) and a pseudoscalar field by its vacuum expectation value, namely

\[
[F_i^b, \varphi_j] \rightarrow [F_i^b, \varphi_j]_0
\]

\[
= - \int d^4x \langle T(\partial_x \mathcal{T}^{\pi_8}_4(x), \varphi_j(0)) \rangle_0
\]

\[
= -i \delta_{ij} \left( \frac{M}{G} \right) \left( \frac{g_4}{g_\pi} \right) + \cdots , \tag{6\cdot12}
\]

where the pole-dominance has been assumed. In this approximation \([F_i^b, S_j^{(8)}] \) may be obtained by replacing \( \mathcal{Q} \) in (6·11) by \( \lambda_i \). Then the symmetry condition leads to the following equalities:

\[
M_1 = M_3 , \quad M_3 = M_6 , \quad M_7 = M_6 \tag{6\cdot13}
\]

and

\[
M_5 = M_4 . \tag{6\cdot14}
\]

The equalities (6·13) are also the consequences of the CP invariance of \( H^{(8)} \), so that Eq. (6·14) is really the consequence of the symmetry condition. Now, from the decay amplitudes given by the right-hand side of (6·11) we find by putting \( j=6 \) the relation

\[
2 \mathcal{M} (\Xi^0 \rightarrow \Lambda^0 + \pi^0) - \mathcal{M} (\Lambda^0 \rightarrow p + 
\]

\[ \sqrt{3} \mathcal{M} (\Sigma^+ \rightarrow p + \pi^0) \]

\[ = (2/\sqrt{6}) (M_4 - M_6) . \tag{6\cdot15}
\]

Hence, the Lee-Sugawara sum rule results as a consequence of the octet dominance and the symmetry condition. In this way we have succeeded in explaining the essential features of the P-wave hyperon decays starting from a small number of assumptions. It is still difficult to understand the P-wave decays dynamically,\(^3\) since it is partly a problem of strong interactions.

\section{7. CP-violating decays}

The \( S \) matrix for the decay \( K_1^0 \rightarrow 2\pi \) is given by

\[
S^{(8)}(K_1^0 \rightarrow 2\pi) = \frac{i}{2} f^3 \beta \int d^4x \int d^4y \int d^4z \times <2\pi, \text{out}| T(\partial_x \mathcal{T}^{\pi_8}_4(x), \partial_y \mathcal{T}^{\pi_8}_4(y), \partial_z \mathcal{T}^{\pi_8}_4(z)) | K_1^0> . \tag{7\cdot1}
\]

There is another term proportional to (D·1), but the corresponding coupling constant is \( f^3 \beta^3 \sim 0.08 f^3 \beta \) so that this term has been neglected in (7·1). It should also be mentioned that it is not trivial to evaluate the decay amplitudes
for $K_L^0 \to 2\pi$ from those for $K_i^0 \to 2\pi$.

We can easily rewrite (7·1) as

$$S^{(0)}(K_i^0 \to 2\pi) = -i \int d^4x \langle 2\pi, \text{out}|H^{(0)}(x)|K_i^0\rangle,$$

(7·2)

where

$$H^{(0)} = -f^\beta F^{(0)} [F^\beta, \partial \mathcal{L}_{\text{int}}].$$

In evaluating the matrix element of $H^{(0)}$ it is very useful to recall that the parity-conserving part of the second order nonleptonic decay interaction is represented by the following effective Hamiltonian:

$$H^{(0)}_{\text{eff}} = i f^\beta [F^\beta, \partial \mathcal{L}_{\text{int}}].$$

(7·3)

Then we may write (7·2) as

$$H^{(0)} = i f [F^\beta, H^{(0)}_{\text{eff}}].$$

(7·4)

However, the choice of the effective Hamiltonian is by no means unique. For instance, we could choose

$$H^{(0)'} = i f^\beta [F^\beta, \partial \mathcal{L}_{\text{int}}]$$

(7·5)

without changing the second order $S$ matrix. In this case we get

$$H^{(0)} = i f [F^\beta, H^{(0)'}] - \frac{i}{2} f^\beta [F^\beta, \partial \mathcal{L}_{\text{int}}].$$

(7·6)

In evaluating the relevant matrix elements, however, we can show that the above ambiguity remains only in the $I=0$ final state, and that we have a rather reliable method of evaluating the matrix elements for the $I=2$ final state. In order to prove the first statement we introduce an effective Hamiltonian for the second order process $K_i^0 \to 2\pi$ by

$$H^{(0)}_{\text{eff}} = i f^\beta [F^\beta, \partial \mathcal{L}_{\text{int}}].$$

(7·7)

Then the relevant matrix element of the additional term in (7·6) is given by

$$- \frac{i}{2} f^\beta \langle 2\pi, \text{out}|[F^\beta, \partial \mathcal{L}_{\text{int}}]|K_i^0\rangle$$

$$= \frac{i}{2} f^\beta \langle 2\pi, \text{out}|[F^\beta, \partial \mathcal{L}_{\text{int}}]|K_i^0\rangle$$

$$= f \langle 2\pi, \text{out}|H^{(0)}_{\text{eff}}|K_i^0\rangle.$$  

(7·8)

Since $K_i^0$ decays dominantly into the $I=0$ final two-pion state, we have established the first statement.

Next, in order to evaluate the relevant matrix element of (7·4) or the first term in (7·6) we replace $H^{(0)}_{\text{eff}}$ or $H^{(0)'}_{\text{eff}}$, by a phenomenological interaction for $K_i^0 \to 3\pi$, namely,
\[ H_{\text{ph}}^{(3)} \rightarrow g_{\Phi} \varphi_{3} (\varphi_{1}^{2} + \varphi_{2}^{2} + \varphi_{3}^{2}). \] (7.9)

Then, by using the soft pion technique (7·4) or the first term in (7·6) can be replaced by

\[ i f[F^2, H_{\text{ph}}^{(3)}] \rightarrow f \left( \frac{g_{A}}{g_{F}} \right) \frac{M}{G} g_{F} (\varphi_{1} + \varphi_{2} + 3 \varphi_{3}). \] (7·10)

It is now easy to evaluate the relevant matrix elements. If we also express \( H_{\text{ph}}^{(3)} \) by an effective phenomenological Hamiltonian for \( K_{L} \rightarrow 2\pi_{0} \),

\[ H_{\text{ph}}^{(3)} \rightarrow g_{\Phi} \varphi_{3} (\varphi_{1}^{2} + \varphi_{2}^{2} + \varphi_{3}^{2}), \] (7·11)

we easily get the following results:

\[
\begin{align*}
\mathcal{M}(K_{S} \rightarrow \pi^{0} + \pi^{0}) & = f \left( \frac{g_{A}}{g_{F}} \right) \frac{M}{G} (g_{s}), \\
\mathcal{M}(K_{L} \rightarrow \pi^{0} + \pi^{0}) & = 3f \left( \frac{g_{A}}{g_{F}} \right) \frac{M}{G} (g_{s}).
\end{align*}
\] (7·12)

(7·13)

Unfortunately we cannot identify them with \( \eta_{+} \) and \( \eta_{0} \), respectively, for two reasons. First, \( \eta \)'s are defined with reference to the physical particles \( K_{S} \) and \( K_{L} \) rather than to the mathematically defined particles \( K_{1} \) and \( K_{3} \). Secondly, as we have mentioned, there is an ambiguity regarding the evaluation of the decay amplitude into the \( I=0 \) final state. Taking (5·14) into account, however, we may conclude that the decay amplitude for \( K_{L} \rightarrow 2\pi_{0}, I=2 \) is still free from both objections. Therefore, when we parametrize \( \eta \)'s as

\[
\begin{align*}
\eta_{+} & = \varepsilon + \varepsilon', \\
\eta_{0} & = \varepsilon - 2\varepsilon',
\end{align*}
\] (7·14)

we can evaluate \( \varepsilon' \) in a rather reliable way apart from its phase.

\[
|\varepsilon'| = \frac{1}{3} \left| \frac{\mathcal{M}(K_{S} \rightarrow \pi^{0} + \pi^{0})}{\mathcal{M}(K_{L} \rightarrow 2\pi_{0})} - \frac{\mathcal{M}(K_{S} \rightarrow 2\pi_{0})}{\mathcal{M}(K_{L} \rightarrow 2\pi_{0})} \right|
\]

\[
= 2 \left( \frac{g_{A}}{g_{F}} \right) \frac{M}{G} (g_{s}) f.
\]

\[
\approx 0.4 \times 10^{-3}.
\] (7·15)

In the present calculation no final state interactions have been introduced, but the above result should give the right order of magnitude for \( \varepsilon' \).

If we set \( |\eta_{+}| = 2 \times 10^{-3} \) for convenience,\(^{14}\) we get triangular inequalities for \( |\eta_{0}| \) as

\[
3 \times 10^{-3} \geq |\eta_{0}| \geq 1 \times 10^{-4}.
\] (7·16)

Next, we discuss implications of \( CP \) violation in semileptonic decays. The radiative correction to \( H_{41} \) in (2·5) due to the interaction (2·3) is given by
Of these three form factors one survives only when both parity and time-reversal are violated, and for we have
\[ j_\mu \rightarrow j_\mu - i f \int_{a_\mu=0} d^3 x K_\mu(x) \]
Thus, this correction effectively modifies the Cabibbo current \( J_\lambda \) as
\[ J_\lambda \rightarrow J_\lambda - i f \int_{a_\mu=0} d^3 x K_\mu(x) \]
\[ = (1 - i f) (\mathcal{F}_{11}^{(4)} - i \mathcal{F}_{12}^{(4)}) \cos \theta + (1 - i b f) (\mathcal{F}_{11}^{(4)} - i \mathcal{F}_{12}^{(4)}) \sin \theta, \] (7.18)
where
\[ a \equiv 1, \quad b \equiv \frac{1}{2}. \] (7.19)
This means that the effects of the CP-violating radiative correction to the semileptonic interaction can be absorbed by the phases of the overall coupling constants for both strangeness-conserving and -changing parts, separately. Hence, the effects of CP violation in semileptonic decays are not observable. The second order correction is again CP-conserving and the third order correction is too small to be observed. There is, however, an exceptional case. That is the semileptonic decay of the \( K_L \) meson. In this case, the effects of CP violation enter in this process through the mass matrix, namely,
\[ \Gamma(K_L^0 \rightarrow \pi^- + l^+ + \nu) - \Gamma(K_L^0 \rightarrow \pi^+ + l^- + \bar{\nu}) = 2 \text{Re} \varepsilon. \] (7.20)
Experiments show that Re \( \varepsilon \) is not zero revealing CP violation.\(^\text{15} \)

\section*{§ 8. The electric dipole moment of the neutron}
As has been pointed out by Landau\(^\text{40} \), the static electric dipole moment vanishes identically unless both space-reflection and time-reversal invariances are violated. Since both of them are now known to be violated we may expect a non-vanishing value for the electric dipole moment, say, of the neutron, and we shall evaluate it on the basis of the present model.

The electromagnetic form factors of the neutron are introduced by
\[ \langle n' | j_\mu'(0) | n \rangle \]
\[ = \bar{u}(n') [i T_{\mu\nu} F_1(q^2) - i \sigma_{\mu\nu} q \cdot F_2(q^2) - \gamma_\mu \sigma_{\mu\nu} q \cdot F_3(q^2)] u(n) \] (8.1)
in obvious notation. \( j_\mu' \) represents the current operator including the radiative corrections due to weak interactions, and \( q = n' - n \) denotes the momentum transfer. This is the most general form of the matrix element of the current operator \( j_\mu' \) between two neutron states that is consistent with both Lorentz invariance and conservation of charge. Of these three form factors \( F_3, F_2 \) and \( F_3 \), the last one survives only when both parity and time-reversal are violated, and for \( q^2 = 0 \) we have
\[ F_1(0) = 0, \quad F_1(0) = \mu_{\text{md}}, \quad F_1(0) = \mu_{\text{ed}}, \quad (8 \cdot 2) \]

where \( \mu_{\text{md}} \) and \( \mu_{\text{ed}} \) denote, respectively, the magnetic and electric dipole moments of the neutron. The electric dipole moment arises in the \( f^3 \) correction to the current operator. The first order correction vanishes identically as has already been shown in § 2, and the second order correction conserves \( CP \) so that it does not contribute to the electric dipole moment. The \( f^3 \) correction to the current operator \( j_\pi \) is then given by

\[
\langle n' | j_\pi^{(0)}(0) | n \rangle = \left( -\frac{i f^3}{3!} \right) \int d^4 x \int d^4 y \int d^4 z \langle n' | T(j_\pi(0), \partial_x K_\pi(x), \partial_y K_\pi(y), \partial_z K_\pi(z)) | n \rangle.
\]  

(8 \cdot 3)

Since we are interested only in the strangeness-conserving corrections to \( j_\pi \) and also since \( \beta^2 \approx 0.08 \) is small compared with unity, we may replace \( \partial_x K_\pi = \partial_x K_\pi' \) by \( \partial_x \mathcal{F}_{\pi}^* \) in Eq. (8 \cdot 3). Then we get

\[
\langle n' | j_\pi^{(0)}(0) | n \rangle = i f^3 \int d^4 x \langle n' | T(j_\pi(0), P(x)) | n \rangle,
\]  

(8 \cdot 4)

where

\[
P = \frac{1}{3} [F_2^0, [F_2^0, \partial_x \mathcal{F}_{\pi}^*]].
\]  

(8 \cdot 5)

This is exactly of the same form as that of the matrix element for the process \( \gamma + n \rightarrow n + \pi^0 \) at zero energy since \( P(x) \) is a pseudoscalar field operator. Thus, on the basis of the Kroll-Ruderman theorem\(^{17}\) we obtain

\[
\frac{\mu_{\text{ed}}}{\mu_{\text{md}}} = 4 \lambda_0 f^3 \left( \frac{4 \pi}{G_F} \right) \left( \frac{M}{\mu} \right)^2 \left( \frac{g_A}{g_Y} \right)^3,
\]  

(8 \cdot 6)

where \( M \) and \( \mu \) denote the masses of the neutron and of the \( \pi^0 \) meson, respectively, and \( \lambda_0 \) is the matrix element of \( P \) between the vacuum and zero energy \( \pi^0 \) states. The renormalized coupling constant \( \lambda \) is defined as the value of the pion-pion scattering amplitude at the symmetrical point \( s = t = u = 4 \mu^2 / 3 \) when all four pions are on the mass shell, whereas \( \lambda_0 \) is the scattering amplitude for \( 2\pi^0 \rightarrow 2\pi^0 \) when all four pions carry zero energy-momentum. Some theories\(^{18}\) give \( \lambda_0 = -3 \lambda = 0.03 \), and we assume \( 4 \lambda_0 = 0.11 \) here. Then we find

\[
D = \frac{\mu_{\text{ed}}}{e} = \mp 2 \times 10^{-28} \text{ cm for } \lambda_0 f \geq 0.
\]  

(8 \cdot 7)

This result is still consistent with the presently available measurements of \( D \).\(^{19}\)

**Appendix A**

*Four divergence in the Lagrangian*

In the text we have introduced a four divergence as the interaction Lagrangian for nonleptonic decays. An objection usually raised against this proposal is that
a four divergence in a Lagrangian gives no physical effects whatsoever. The point that has to be stressed, however, is that the weak interaction Lagrangian becomes equal to a four divergence as a consequence of field equations for strong and electromagnetic interactions excluding the weak interaction itself. When we include the weak interaction, however, the weak interaction Lagrangian no longer becomes equal to a four divergence and it can give rise to observable effects. Also, it is clear from the above prescription that the first order weak interaction vanishes when we approximate field equations by omitting the weak interaction. Thus, we expect that the effects of this Lagrangian will start to show up from the second order in the weak interaction.

In what follows we demonstrate this assertion by a simple example. Let us consider a system consisting of a nucleon field and a neutral pseudoscalar field, and write the strong interaction Lagrangian as

\[ L = -\bar{\psi} (i\partial_t + M) \psi - \frac{1}{2} [ (\partial_x \phi)^2 + \mu^2 \phi^2 ] - G \bar{\psi} i\gamma_5 \psi \phi \]  

(A·1)

Now introduce

\[ K_\lambda = \bar{\psi} i\gamma_\lambda \gamma_5 \psi \]  

(A·2)

By using field equations derived from (A·1), we get

\[ \partial_\lambda K_\lambda = 2M \bar{\psi} i\gamma_\lambda \psi - 2G \bar{\psi} \phi \psi \phi \]  

Thus we take

\[ L_w = -2 f (M \bar{\psi} i\gamma_\lambda \psi - G \bar{\psi} \phi \psi \phi) \]  

(A·3)

as the weak interaction Lagrangian. Then, in the representation defined with reference to (A·1), we get

\[ L_w = -f \partial_\lambda K_\lambda \]  

(A·4)

Next, let us derive field equations from \( L + L_w \), for example,

\[ i\partial_t \psi + M(1 + 2i f \gamma_5) \psi + iG \gamma_5 (1 + 2i f \gamma_5) \psi \phi = 0 \]  

(A·5)

The field equations derived from (A·1) are obtained, obviously, by putting \( f = 0 \) in (A·5).

Finally, we show that the first order terms have no observable manifestations. Introduce a canonical transformation

\[ \psi' = e^{if \gamma_5 \phi} \]  

(A·6)

and get the field equation for \( \psi' \), namely,

\[ i\partial_t \psi' + M(1 + 2f^2 + \cdots) \psi' + iG \gamma_5 (1 + 2f^2 + \cdots) \psi' \phi = 0 \]  

(A·7)

It is also observed, to the order \( f^2 \), that the change of the effective Hamiltonian, after the canonical transformation (A·6), is given by

\[ H^{(2)}(x) = \frac{f^2}{2} \int d^4 y K_\lambda(y), \partial_\lambda K_\lambda(x) \]
A Model of CP Violation

\[ = 2f^2(M\psi\bar{\psi} + G\bar{\psi}i\tau\phi\psi) \]

It is clear that the Lagrangian (A·3) gives rise to observable effects in the order \( f^2 \) since the nucleon mass is shifted by \( A \cdot S \).

Appendix B

A dynamical quark model

In the text, we have introduced a set of constraints which is necessary for forbidding the unwanted transitions obeying \( \Delta S = \pm 2 \).

\[
[F^{(+)}(K^0), \partial_\mu \mathcal{F}^{(+)}_\mu(K^0)] = 0 , \quad (B\cdot1)
\]

\[
[F^{(+)}(K^0), \partial_\mu \mathcal{F}^{(-)}_\mu(K^0)] = 0 , \quad (B\cdot2)
\]

or

\[
[F(K^0), \partial_\mu \mathcal{F}^{(+)}_\mu(K^0)] = 0 , \quad (B\cdot3)
\]

\[
[F(K^0), \partial_\mu \mathcal{F}^{(-)}_\mu(K^0)] = 0 , \quad (B\cdot4)
\]

\[
[F^{(+)}(K^0), \partial_\mu \mathcal{F}^{(+)}_\mu(K^0)] = 0 . \quad (B\cdot5)
\]

Since the appearance of this set of constraints is a new feature characteristic of the present model, it is necessary to examine its consistency with the commutation relations in the algebra of currents.

In order to answer this question we have constructed a quark model with specific interactions. The commutation relations in the algebra of currents are satisfied by quark models with arbitrary but reasonable interactions, but the constraints involving divergences of current densities are sensitive to the dynamics of the system concerned. In what follows we shall give a simple example in which all the constraints above are satisfied:

\[
L = -\bar{q}(\gamma^\mu m)q + L_{\text{int}} , \quad (B\cdot6)
\]

where \( q \) denotes the quark field, and \( m \) is assumed to be of the form

\[
m = m_0 + m_q \lambda_5 . \quad (B\cdot7)
\]

The current densities are given by

\[
\mathcal{F}_{\mu} = \frac{1}{2} \bar{q}i\gamma_\mu \lambda q , \quad (B\cdot8)
\]

\[
\mathcal{F}^{\alpha}_{\mu} = \frac{1}{2} \bar{q}i\gamma_\mu \gamma_5 \lambda^{\alpha} q . \quad (B\cdot9)
\]

If \( L_{\text{int}} \) represents Fermi interactions, we can easily verify that any linear combination of the following Fermi interactions is consistent with the constraints (B·1) and (B·2):

\[
\bar{q}i\gamma_\mu \lambda q ,
\]

\[
\bar{q}i\gamma_\mu \gamma_5 \lambda q ,
\]

\[
\bar{q}i\gamma_\mu \gamma_5 \lambda_5 q ,
\]

\[
\bar{q}i\gamma_\mu \lambda_5 q .
\]
It should also be mentioned that introduction of the electromagnetic interactions and mass differences does not alter this conclusion.

In connection with the discussion of the P-wave hyperon decays, we have introduced another set of constraints. We show that they also result from the above model. In the limit of an exact $SU(3)$ symmetry, we keep only the first two interactions in (B·10) and replace $m$ by $m_0$. Then, we immediately get

$$\partial_{\mu}\mathcal{F}^\mu_{ia} = m_0i\gamma_5\lambda_iq.$$  \hspace{1cm} (B·11)

If we define $S^{(0)}$ and $S_k^{(8)}$ by

$$i[F^{\dagger}_i, \partial_{\lambda}\mathcal{L}^\lambda_{iu}] = \delta_{ij}S^{(0)} + d_{ijk}S_k^{(8)},$$  \hspace{1cm} (B·12)

we get, in the present model, the following results:

$$S^{(0)} = \frac{1}{2}m_0\bar{q}q,$$  \hspace{1cm} (B·13)

$$S_k^{(8)} = m_0\bar{q}\lambda_kq.$$  \hspace{1cm} (B·14)

Then, it is easy to derive the following relationship:

$$S_k^{(8)} = i[F^{\dagger}_k, \mathcal{O}],$$  \hspace{1cm} (B·15)

where

$$\mathcal{O} = m_0\bar{q}\gamma_5\lambda_iq.$$  \hspace{1cm} (B·16)

Equation (B·15) forms the basis of the Lee-Sugawara sum rule for the P-wave hyperon decays.

Appendix C

Contents of the constraints

In Appendix B we have demonstrated the consistency of the constraints (B·1) and (B·2) with the algebra of currents. In the present appendix we shall study their consequences to understand their physical significance. Perhaps one can say that these constraints dictate the pattern of the chiral $SU(3) \times SU(3)$ breaking. In particular, Eq. (B·3) dictates the pattern of $SU(3)$ breaking as we shall see below. In the conventional formulation of $SU(3)$ breaking one assumes that the total Hamiltonian can be split into two parts, one part being invariant under $SU(3)$ and the other transforming as the eighth component of an octet. The constraint (B·3) represents essentially the same content without reference to the Hamiltonian formalism as we shall see in what follows. Recently a similar viewpoint has been presented by Gell-Mann, Oakes and Renner.\(^{21}\)

(a) The Gell-Mann-Okubo mass formulas

Let us take the matrix element of Eq. (B·3) between $|\Sigma^\circ\rangle$ and $|n\rangle$ and evaluate this matrix element by keeping only $|\Sigma^\circ\rangle$ and $|A^\circ\rangle$ in the intermediate
states. Then, a vertex such as \( \langle \Sigma^0 | \partial_x \mathcal{F}_A (K^0) | n \rangle \) becomes essentially the product of the mass difference between the initial and final baryons and the corresponding \( SU(3) \) Clebsch-Gordan coefficient. The resulting relationship in this approximation reduces to the Gell-Mann-Okubo mass formula, namely,
\[
2m(N) + 2m(\Xi) = 3m(A) + m(\Sigma). \tag{C.1}
\]

Similarly, one can derive the \( SU(3) \) mass formulas for other octets as well as for decimets. In this kind of derivations the most important assumption made throughout this paper is that the \( SU(3) \) is broken mainly by the mass differences within the same multiplet rather than by coupling constants, and this assumption is supported by Kim's recent analysis of the kaon-nucleon scattering.\(^{22}\)

It is not surprising that we could derive Eq. (C.1) from Eq. (B.3). Fubini, Furlan and Rosetti\(^{20}\) assumed, instead of Eq. (B.3), the relation
\[
[F(K^+), \partial_x \mathcal{F}_A (K^+)] = 0 \tag{C.2}
\]
to derive the mass formula (C.1). Although (B.3) and (C.2) are mathematically equivalent in the absence of the electromagnetic interactions, only Eq. (B.3) remains valid in their presence since Eq. (C.2) is not a gauge invariant relationship.

(b) Magnetic moments of the neutral baryons

In order to verify the above statement we shall apply Eq. (B.3) in the presence of the electromagnetic interactions. We start from the following relation:
\[
\int d^4x T(\partial_x \mathcal{F}_A (x), \partial_x \mathcal{F}_B (y), j_i (z)) = 0, \tag{C.3}
\]
where \( \mathcal{F}_A \) is the abbreviation of \( \mathcal{F}_A (K^0) \) and \( j_i \) is the electric current. Equation (C.3) follows from Eq. (B.3) and the equal-time commutation relation
\[
[F(K^+), j_i (z)] = 0. \tag{C.4}
\]
Then, we get from Eq. (C.3) the relation
\[
\langle \Sigma^0 | \int d^4x T(\partial_x \mathcal{F}_A (x), \partial_x \mathcal{F}_B (y), j_i (z)) | n \rangle = 0. \tag{C.5}
\]
We evaluate this matrix element as in (a) and approximate the matrix element of \( j_i \) between single neutral baryon states by the corresponding static magnetic moment, that is,
\[
\langle B^0, p | j_i (0) | B^0, p \rangle = -i \bar{u}(p') \sigma_i (p' - p) u(p) \cdot \mu(B^0). \tag{C.6}
\]
Then we rediscover Okubo's sum rule\(^{20}\) for the magnetic moments of the neutral baryons:
\[
\mu_{\tau}(\Sigma^0 A^0) = \frac{1}{2\sqrt{3}} [\mu(\Sigma^0) + 3\mu(A^0) - 2\mu(\Sigma^0) - 2\mu(n)], \tag{C.7}
\]
which is known to be valid to first order in symmetry-breaking.

(c) The SU(6) mass formulas

We proceed to the discussion of the constraint \((B \cdot 4)\). Since the commutator in \((B \cdot 4)\) changes parity we shall consider a transition between a vector meson and a pseudoscalar meson, and for this purpose we need the matrix element of \(\partial_\lambda \mathcal{F}_\lambda^5\) between them.

The most general form of the matrix element of \(\mathcal{F}_\lambda^5\) is given by

\[
\langle p | \mathcal{F}_\lambda^5(0) | v \rangle = \frac{1}{\sqrt{2p_0} \sqrt{2\nu_0}} \left[ ae_\lambda + b (p + v)_\lambda (p_\mu e_\mu) + c (p - v)_\lambda (p_\mu e_\mu) \right],
\]

where \(p\) and \(v\) denote the four-momenta of the pseudoscalar and of the vector mesons, respectively. The coefficients \(a, b\) and \(c\) are form factors being functions of the momentum transfer squared \(s = (p - v)^2\), and \(e_\lambda\) denotes the polarization vector of the vector meson. Then, we get

\[
\langle p | \partial_\lambda \mathcal{F}_\lambda^5(0) | v \rangle = \frac{-i}{\sqrt{2p_0} \sqrt{2\nu_0}} (p_\mu e_\mu) (A + B (m_\rho^2 - m_\pi^2)),
\]

where

\[
A = a + sc, \quad B = b.
\]

Substituting the above expression in the equation

\[
\langle K^* | [F(K^*), \partial_\lambda \mathcal{F}_\lambda^5(K^*)] | K^{*0} \rangle = 0,
\]

and replacing the form factors \(A\) and \(B\) by appropriate SU(3) Clebsch-Gordan coefficients of the \(F\) type we get, after using the SU(3) mass formulas, the result

\[
m^3(K^*) - m^3(\rho) = m^3(K) - m^3(\pi),
\]

which is known to be a consequence of the SU(6) symmetry.25 We have reproduced this formula without invoking the SU(6) symmetry.

Similarly, we extend the above method to evaluating the matrix element of \(\mathcal{F}_\lambda^5\) between a decimet baryon resonance and an octet baryon. We start from

\[
\langle B | \mathcal{F}_\lambda^5(0) | D \rangle = f u(B) U_\lambda(D) + ig u(B) \gamma_5 B_\mu U_\mu(D),
\]

and a resulting equation

\[
\langle B | \partial_\lambda \mathcal{F}_\lambda^5(0) | D \rangle = -i g u(B) B_\mu U_\mu(D) (f + g (m_\rho - m_\pi)).
\]

We end up with a mass formula

\[
m(Y^*) - m(N^*) = m(\Xi) - m(\Sigma),
\]

which is also a consequence of the SU(6) symmetry.

In both cases the first form factors lead to identities, and it is the second form factors that leads to the mass formulas.
(d) The $f/d$ ratio

Finally, we study the manifestations of Eq. (B·5). We take the matrix element of (B·5) between $|\Xi^{0}\rangle$ and $|n\rangle$ and evaluate it by keeping only the single-baryon intermediate states excluding the deciment resonances. Then, in the limit of degenerate baryon masses this equation determines the $f/d$ ratio in the axial-vector current, namely,

$$f = \frac{1}{\sqrt{3}} = 0.58,$$

(C·15)

which is consistent with the experimental value of 0.59. 26)

Appendix D

The selection rule $|\Delta S| = 0, 1$ in the order $f^3$

The constraints (B·1) and (B·2) have been introduced in order to forbid transitions changing strangeness by two units in the order $f^2$. Since, however, the observed $K_S-K_L$ mass difference indicates that transitions obeying $|\Delta S|=2$ take place only in the second or higher order of the conventional weak interactions, they must occur in the order $f^3$ in the present scheme. This means that such transitions must be forbidden not only in the order $f^3$ but also in the order $f^3$. We prove in this appendix that it is really the case.

In what follows we use the approximate form of $K_S$, Eq. (2·9), but as one can easily verify all the arguments used in this appendix is valid for the exact form, (2·8), as well. When we expand the $S$ matrix in the order $f^3$ we find two dangerous terms that might lead to transitions obeying $|\Delta S|\geq1$. The first one is proportional to

$$\int d^4x \int d^4y \int d^4z T(\partial_x \mathcal{F}^{(+)\dagger}(x), \partial_y \mathcal{F}^{(+)\dagger}(y), \partial_z \mathcal{F}^{(+)\dagger}(z))$$

$$= -2 \int d^4y \int d^4z T([F^{(+)}(y), \partial_y \mathcal{F}^{(+)\dagger}(y)], \partial_z \mathcal{F}^{(+)\dagger}(z)).$$

(D·1)

The commutator in the $T$ product is strangeness-conserving as we have postulated in §4, so that this term obeys the selection rule $\Delta S=\pm1$. The second one is proportional to

$$\int d^4x \int d^4y \int d^4z T(\partial_x \mathcal{F}^{(-)\dagger}(x), \partial_y \mathcal{F}^{(-)\dagger}(y), \partial_z \mathcal{F}^{(-)\dagger}(z))$$

$$= -\int d^4y \int d^4z T([F^{(-)}(y), \partial_y \mathcal{F}^{(-)\dagger}(y)], \partial_z \mathcal{F}^{(-)\dagger}(z))$$

$$-\int d^4y \int d^4z T(\partial_x \mathcal{F}^{(-)\dagger}(y), [F^{(+)}(y), \partial_z \mathcal{F}^{(-)\dagger}(z)]).$$

(D·2)

Of the two terms in (D·2), the first one conserves strangeness and is consequently safe. The second one can be written as
\[ \int d^4z [F_0^{(+)}(z_0), [F_0^{(+)}(z_0), \partial_\mu F_0^{(+)}(z)]] = \frac{i}{2} F_1^{(+)} \]  

(D.3)

In order to prove that this term conserves strangeness we have to appeal to the Lorentz invariance of the commutation relations in the algebra of currents. We may write the commutation relation

\[ [F_0^{(+)}, F_1^{(+)}] = \frac{i}{2} F_2^{(+)} \]  

(D.4)

in a manifestly covariant form

\[ \left[ \int d\sigma \mathcal{F}_0^{(+)}(x), \int d\sigma \mathcal{F}_1^{(+)}(y) \right] = \frac{i}{2} \int d\sigma \mathcal{F}_2^{(+)}(z), \]  

(D.5)

where \( \sigma \) is an arbitrary space-like hypersurface. Taking the functional derivative of the above equation with respect to \( \sigma \), we find

\[ [\partial_\sigma \mathcal{F}_0^{(+)}, F_1^{(+)}] + [F_0^{(+)}, \partial_\sigma \mathcal{F}_2^{(+)}] = \frac{i}{2} \partial_\sigma \mathcal{F}_3^{(+)}. \]  

(D.6)

By combining (D.4) and (D.6) and the Jacobi identity, we get

\[ [F_0^{(+)} + F_1^{(+)} + \partial_\mu \mathcal{F}_3^{(+)}] = \frac{i}{2} \left( [F_0^{(+)}, \partial_\mu \mathcal{F}_1^{(+)}] + [F_1^{(+)}, \partial_\mu \mathcal{F}_2^{(+)}] \right) \]

\[ - [F_0^{(+)}, F_1^{(+)}] \]  

[D.7]

As we have seen in \( \S \) 4, the first term in the parentheses must vanish and the second term is strangeness-conserving.

Thus we have established the selection rule \(|\Delta S| < 2\) to the third order in \( f \). It is worthwhile to emphasize that this selection rule is a consequence of the assumption that the current \( K_\lambda \) is a linear superposition of the densities of the generators of the algebra of currents.

If this selection rule should fail in the order \( f^3 \), the branching ratio

\[ \frac{\Gamma(\Xi^- \rightarrow \eta + \pi^-)}{\Gamma(\Xi^- \rightarrow A^0 \pi^-)} \]  

(D.8)

would be of the order of \((\beta f)^3 \sim 10^{-6}\) rather than \((\beta f^3)^3 \sim 10^{-11}\) predicted by the present model. The present experimental upper limit of this ratio \( \text{[27]} \) is \( 10^{-8} \), which is still far from distinguishing between these two cases.

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