Magnetic Moments of Nucleons in Strong Coupling Meson Theory

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The implications of identifying the first excited vibrational state of static pion and nucleon system to the Roper resonance, for the isovector magnetic moments of nucleons are studied with a refined form of the Pauli and Dancoff strong coupling theory. A satisfactory result is obtained when we introduce a strong attractive pion-pion interaction through the reduced effective mass of bound pions.

Then we computed the ratio of magnetic moments of the neutron and proton for unitary irreducible representations with the Young tableaux \((N, 0, 0)\), where \(N=1, 3, 5\) and \(\infty\), of a group \(SU(4)\). It is 0, \(-2/3\), \(-16/19\) and \(-1\). Strong coupling theory and group theoretic argument for \(N=\infty\) give the same value \((-1)\).

§ 1. Introduction

In a previous paper\(^1\) we have shown that the Roper resonance \(T=1/2\) and \(J^p=1/2^+\) with resonant mass value 1470 MeV might be explained as the first excited vibrational state, by considering correctly the relevant Hamiltonian of the kinetic energies of bound meson fields for the static pion and nucleon system. With the refined form of the original Pauli and Dancoff scheme\(^2\) we study the implications of this new finding for the quantitative estimations of the electromagnetic properties of nucleons, especially, of their magnetic moments.

Following the aforementioned revised procedure we construct in § 3 the wave functions of physical nucleons. A typical difference in comparison with a previous paper\(^3\) is that we characterize the proton and neutron by \(T_3^p\) (the component of the total isotopic spin along the third axis of the space-fixed frame) instead of, by \(L_3^p\) or \(T_3^n\) (the component of the total angular momentum or total isotopic spin along the third axis of the body-fixed frame).

Then we compute (§§ 4 and 5) the expectation values of charge density and magnetic moments. We obtain the results exactly the same as the previous ones. The magnetic moment in the zeroth approximation with respect to nondiagonal operators \(\sigma_{l(\neq 0)}\) and \(\tau_{\alpha(\neq 0)}\) turns out to have an isovector part alone, so that \(\beta=\mu(n)/\mu(p)=-1\). This approximation which picks out states with \(\sigma_3=\tau_3=1\)
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and the conditions of $y=0$) and of no vibrational excited state, corresponds to a band of nucleon isobars, $(1/2, 1/2)$, $(3/2, 3/2)$, $(5/2, 5/2)$, ... in terms of $(J, T)$. We give the structure of the Lie algebra which produces such an infinite component representation. The underlying group is a noncompact and noninvariant group $[(SU(2))_R \otimes (SU(2))_T] \times T$, as given by Cook, Goebel and Sakita.6

In the next section (§ 6) we introduce a simple exponential form for the source function for computing the numerical values of isovector magnetic moments. We show that we can explain the observed isovector magnetic moments by introducing a strong attractive pion-pion interaction through a modification of the effective mass of bound pions.5

In summary we have five experimental data, (1) mass of $(3/2, 3/2)^+$-resonance (1240 MeV), (2) mass of Roper resonance (1470 MeV), (3) charge radii of nucleons $<r^2>_{\text{proton}}=0.62 F^2$ and $<r^2>_{\text{neutron}}=0$, (4) charge form factors $G_F(q^2)=0$ and $G_E(q^2)=(1+q^2/0.71)^{-1}$ for the region $q^2 \leq 3$ (GeV/c)$^2$, and (5) isovector magnetic moment $(2.36 e/2m_p)$.

These data are fitted rather qualitatively well by three parameters, (1) coupling constant $\beta=2.11$, (2) exponential source function with cutoff length $\Lambda=1.77 \times 0.210 F$ and (3) effective mass of bound pions $\kappa=0.174 \times 140$ MeV. More quantitatively satisfactory results will be obtained if we could choose a smaller value for $\Lambda$ which might need some modifications of explaining the mechanism of the Roper resonance.

Finally (§ 7) we add a consideration concerning the question raised by Bég, Lee and Pais.5 They argue that the $SU(6)$ theory predicts $\beta=-2/3$ independently of the magnitudes of coupling constants of strong interactions while according to the conventional field theory $\beta$ depends on coupling constants, that is, we should expect that $\mu(n)=0$ and $\mu(p)=e/2m_p$ for vanishing coupling constants. To answer partially to this question we compute $\beta$ for those supermultiplets with the Young tableaux $(N, 0, 0)$ where $N=1, 2, 3, 5$ and $0$ and dimensions 4, 20, 56 and $\infty$ of $SU(4)$ generated by fifteen operators $\frac{1}{2}\sigma_\alpha$, $\frac{1}{2}\tau_\alpha$ and $\frac{1}{2}\sigma_i\tau_\alpha$. We assume5 that total magnetic moments of nucleon isobars are ascribed to intrinsic magnetic moments of fundamental objects (baryonettes)5 which construct, in combination, nucleon isobars and of which baryon number $n_B=1/N$ and that the magnetic moment operator is given by $M=Q \otimes \sigma=(\frac{1}{2}n_B+\frac{1}{2}n_B) \otimes \sigma$, the same expression as that of baryonettes.5

8) Exactly speaking we showed5 only that the states with $y=1$ do not exist. To guarantee safely that the states with $y=0$ alone are picked out, we have to demand that the coupling constant is set to infinity, though the spectrum of nucleon isobars given in this theory is infinite already for finite values of coupling constant. Then at the same time no vibrational state is excited and each element with $J=T$ of an infinite band appears only once.

Erratum: The phrase, "as the underlying group for the spectrum generating algebra" on 37th line of p. 559 of reference 1) should be omitted. Also the phrase "the group for the spectrum generating algebra" on 29th and 30th lines of p. 569 of reference 1) should be read as "the approximate symmetry group".
We obtain $\beta=0$, $-2/3$, $-16/19$ and $-1$. For $N=\infty$, $M$ has an isovector part alone. This is the same situation as in the Pauli and Dancoff scheme where the expectation value of the intrinsic isoscalar magnetic moment vanishes. Owing to this very fact, for the same unitary irreducible representation $(0,0,0)$ of two different groups we have the same $\beta=-1$.

§ 2. Brief review of deriving the expressions for magnetic moments

We start from writing down the expression for magnetic moments in static pion and nucleon system:

$$H = \frac{1}{2} \sum_a \left( \pi_a - \frac{\mathbf{a}}{c^2} \nabla^2 \right) \mathbf{X}_a \frac{1}{4\pi} \int r \mathbf{a} \cdot \nabla U \, \mathrm{d}x. \quad (2.1)$$

$U(r)$ is a spherically symmetric source function normalized to unity,

$$\int U(r) \, \mathrm{d}x = 1. \quad (2.2)$$

The total magnetic moment $M_{12}$ consists of $M_{12}^{(\text{source})}$ (source contributions) and $M_{12}^{(\text{meson})}$ (meson contributions). Each part is given, respectively, by

$$M_{12}^{(\text{source})} = \frac{e}{2m} \frac{1}{2} (1 + \tau_3) \sigma_3 \quad (2.3)$$

and

$$M_{12}^{(\text{meson})} = \frac{e}{2} \int (x \tau_x S_b(x) - x_b S_x(x)) \, \mathrm{d}x. \quad (2.4)$$

The current density $S(x)$ is, in the approximation of omitting external meson field $\varphi_a'(x)$ and $\pi_a'(x)$, given by

$$S(x) = \left( \hat{\varphi}_1(x) \nabla \hat{\varphi}_1(x) - \hat{\varphi}_1(x) \nabla \hat{\varphi}_2(x) \right) + \sqrt{4\pi} \frac{f}{\kappa} \left( \hat{\varphi}_1(x) \tau_2 - \hat{\varphi}_2(x) \tau_1 \right) \sigma U(r). \quad (2.5)$$

Here the bound meson field $\hat{\varphi}_a(x)$ and its canonically conjugate momentum $\hat{p}_a(x)$ are defined as follows:

$$\hat{\varphi}_a(x) = \frac{1}{\sqrt{4\pi}} \sum_k \hat{\varphi}_{ak} \frac{\partial \xi(r)}{\partial x_k}, \quad (2.6)$$

and

$$\hat{p}_a(x) = \sqrt{4\pi} \sum_k \hat{p}_{ak} \frac{\partial U(r)}{\partial x_k}, \quad (2.7)$$

where

$$\xi(r) = \frac{X(r)}{I}, \quad (c^2 - \nabla^2) X(r) = 4\pi U(r) \quad \text{and} \quad I \theta_{ij} = \int \frac{\partial U(r)}{\partial x_i} \frac{\partial X(r)}{\partial x_j} \, \mathrm{d}x. \quad (2.8)$$
Now we insert the expressions for $\phi_a(x)$ and $\tilde{\phi}_a(x)$ given by Eqs. (2.5) and (2.6) to Eq. (2.4), and obtain the following equation for $M_{12}^{(\text{neon})}$:

$$
M_{12}^{(\text{neon})} = -\frac{e}{8\pi} \sum_{\text{ka}} \left( \phi_{2m} \phi_{1m} - \phi_{2m} \phi_{1k} \right) \int \left( x_1 \frac{\partial^2 \xi(r)}{\partial x_1 \partial x_m} - x_2 \frac{\partial^2 \xi(r)}{\partial x_1 \partial x_m} \right) \frac{\partial \xi(r)}{\partial x_k} \, dx
$$

$$
\quad + \frac{f}{\kappa} \sum_{\text{ka}} \left[ \left( \phi_{2m} \phi_{1m} - \phi_{2m} \phi_{1k} \right) x_1 \frac{\partial^2 \xi(r)}{\partial x_k} + \left( \phi_{2m} \phi_{1m} - \phi_{2m} \phi_{1k} \right) x_2 \frac{\partial \xi(r)}{\partial x_k} \right] U(r) \, dx.
$$

(2.9)

Then we apply the Pauli-Dancoff representation given by

$$
\phi_{aj} = \sum_{s} A_{r1} B_{rs} Q_{r}
$$

(2.10)
to the above expression. We have

$$
M_{12}^{(\text{neon})} = -\frac{e}{24\pi} \sum_{r,s} \left( A_{r1} A_{s2} - A_{r2} A_{s1} \right) \left( B_{r1} B_{s2} - B_{r2} B_{s1} \right) Q_{r} Q_{s} \int (\nabla^2 \xi(r))^2 \, dx
$$

$$
- \frac{e f}{6} \sum_{r} \left[ (A_{r1} B_{r2} - A_{r2} B_{r1}) \sigma_1 + (A_{r1} B_{r2} - A_{r2} B_{r1}) \sigma_2 \right] Q_{r}
$$

$$
\times \int \left( x \cdot \nabla \xi(r) \right) U(r) \, dx.
$$

(2.11)

Finally we perform two unitary transformations generated by $S_1$ and $S_2$ in accordance with the diagonalization of the interaction Hamiltonian. They are given by

$$
S_1 = \exp \left\{ i \psi \left( \frac{1}{2} \sigma_3 + L_3 \right) \right\} \exp \left\{ i \theta \left( \frac{1}{2} \sigma_3 + L_3 \right) \right\} \exp \left\{ i \phi \left( \frac{1}{2} \sigma_3 + L_3 \right) \right\}
$$

$$
\times \exp \left\{ i \phi' \left( \frac{1}{2} \tau_3 + T_3 \right) \right\} \exp \left\{ i \theta' \left( \frac{1}{2} \tau_3 + T_3 \right) \right\} \exp \left\{ i \phi' \left( \frac{1}{2} \tau_3 + T_3 \right) \right\}
$$

(2.12)

and

$$
S_2 = \frac{1}{\sqrt{2}} \left( \sigma_1 - i \tau_2 \right).
$$

(2.13)

We omit the nondiagonal terms with respect to operators $\sigma_{i(-3)}$ and $\tau_{a(-3)}$ and obtain the magnetic moment operators in the zeroth approximation:

$$
M_{12}^{(\text{neon})} = -\frac{e}{24\pi} \sum_{r,s} \left( A_{r1} A_{s2} - A_{r2} A_{s1} \right) \left( B_{r1} B_{s2} - B_{r2} B_{s1} \right) Q_{r} Q_{s} \int (\nabla^2 \xi(r))^2 \, dx
$$

$$
- \frac{e f}{6} \sum_{r} \left[ (A_{r1} B_{r2} - A_{r2} B_{r1}) \sigma_1 + (A_{r1} B_{r2} - A_{r2} B_{r1}) \sigma_2 \right] Q_{r}
$$

$$
\times \int \left( x \cdot \nabla \xi(r) \right) U(r) \, dx.
$$

(2.14)

Also for $M_{12}^{(\text{source})}$ we have

$$
M_{12}^{(\text{source})} = -\frac{e}{4m_p} \sum_{r} A_{r1} B_{r2}.
$$

(2.15)

Now that the magnetic moment operators have been given, we go over to eigenvectors of the ground states in the next section.
§ 3. Eigenvectors of ground states

The Hamiltonian with which we work to determine the ground states is given by

\[ H^0 = -\frac{N}{2} \sum_{r} \frac{\partial^2}{\partial Q_r^2} + \frac{1}{2I} \sum_{r} Q_r^2 - \frac{\tau_3}{\kappa} (\tau_2 Q_1 + \sigma_3 Q_2 + \sigma_5 Q_3)
\]

\[ + N \left\{ \frac{(L_{01}^3 + T_0^{3})^2}{(Q_1 - Q_2)^2} + \frac{1}{2} (1 - \sigma_3) - 1 + \frac{(L_{01}^3 + T_0^{3})^2}{(Q_2 - Q_3)^2} + \frac{1}{2} (1 - \tau_3) - 1 \right\}.
\]

Here the constant \( N \) is defined by

\[ N = \frac{4\pi}{3} \int (\nabla U(r))^2 dx.
\]

From this expression (3·1) we see that the operators which commute with each other and also with the Hamiltonian are \( \sigma_3, \tau_3, \sum_i (L_{0i}^3), \sum_i (T_i^3) \) and \( T_3^3 \). For states characterized by \( \sigma_3 = \tau_3 = 1 \), the average value of \( Q_r \) is a quantity of magnitude \( 1/f /\kappa \), so that the last three terms in Eq. (3·1) can be neglected. In this approximation we have

\[ H^0 = -\frac{N}{2} \sum_{r} \frac{\partial^2}{\partial Q_r^2} + \frac{1}{2I} \sum_{r} Q_r^2 - \frac{\tau_3}{\kappa} \sum_{r} Q_r + \frac{N}{8} \sum_{r} \frac{(L_{01}^3 + T_0^{3})^2}{(Q_1 - Q_2)^2}.
\]

Then we have another operator \( \sum_i (Y_i^3)^2 = \sum_i (L_{oi}^3 + T_i^3) \), which commutes with six aforementioned operators and the Hamiltonian (3·3).

The expressions for four operators \( L_{0i}^3, L_{0i}^3, L_{0i}^3 \) and \( \sum_i (L_{0i}^3)^2 \) are given by

\[ L_{0i}^3 = \sin \phi \left( \frac{1}{i} \frac{\partial}{\partial \phi} + \cos \phi \left( \frac{1}{i} \frac{\partial}{\partial \phi} - \frac{1}{i} \frac{\partial}{\partial \theta} \right) \right),
\]

\[ L_{0i}^3 = \cos \phi \left( \frac{1}{i} \frac{\partial}{\partial \phi} - \sin \phi \left( \frac{1}{i} \frac{\partial}{\partial \phi} + \frac{1}{i} \frac{\partial}{\partial \theta} \right) \right),
\]

\[ L_{0i}^3 = \frac{1}{i} \frac{\partial}{\partial \phi},
\]

\[ \sum_i (L_{0i}^3)^2 = - \frac{1}{\sin \phi} \frac{\partial}{\partial \phi} \left( \sin \phi \frac{\partial}{\partial \phi} \right) - \frac{1}{\sin \phi} \left( \frac{\partial^2}{\partial \theta^2} - 2 \cos \phi \frac{\partial^2}{\partial \phi \partial \theta} + \frac{\partial^2}{\partial \phi^2} \right).
\]

The expression for \( \sum_i (L_{0i}^3)^2 \) is just the kinetic energy of a symmetric top with three equal principal moments of inertia of magnitude 1/2. Therefore the eigenfunctions of three operators \( \sum_i (L_{0i}^3)^2, L_{0i}^3 \) and \( L_{0i}^3 \) are given by\(^{10}\)
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\[ u_{m_1, m_2, \tau}(\theta, \varphi, \psi) = N \left( \sin^3 \frac{\varphi}{2} \right)^{d/2} \left( \cos^3 \frac{\varphi}{2} \right)^{s/2} \Phi \left( -p, 1 + d + s + p, 1 + d, \sin^3 \frac{\varphi}{2} \right) \times \exp i (m_1 \theta + m_2 \phi). \]  

(3.5)

Here the eigenvalues of \( \sum \ell (L_\ell^3) \), \( L_3^0 \) and \( L_0^3 \) are denoted by \( l(l+1) \), \( m_1 \) and \( m_2 \), respectively, and

\[ d = |m_1 - m_2|, \]
\[ s = |m_1 + m_2|, \]
\[ l = p + m^*, \]

where

\[ m^* = |m_1| \quad \text{when } |m_1| \geq |m_2| \]
\[ = |m_2| \quad \text{when } |m_1| < |m_2| \]  

(3.6)

and

\[ p = 0, 1, 2, \ldots \]

The function \( \Phi \) is given by a hypergeometric series as follows:

\[ \Phi \left( -p, 1 + d + s + p, 1 + d, \sin^3 \frac{\varphi}{2} \right) = \sum_{r=0}^{\tau} (-1)^r \binom{\tau}{r} \frac{(d + s + p + r)!}{(d + s + p)! (d + r)!} \left( \sin^3 \frac{\varphi}{2} \right)^r. \]  

(3.7)

The first four functions of \( u_{m_1, m_2, \tau}(\theta, \varphi, \psi) \) are given in Table I. The corresponding eigenvalues and eigenfunctions of \( \sum \ell (T_\ell^3) \), \( T_3^0 \) and \( T_0^3 \) are denoted by \( t(t+1) \), \( n_1 \) and \( n_2 \), and \( u_{m_1, m_2, \tau}(\Theta, \Phi, \Psi) \).

<table>
<thead>
<tr>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>Wave function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td>1/2</td>
<td>( \frac{1}{2\pi} \cos \frac{\varphi}{2} \exp i (\theta + \phi)/2 )</td>
</tr>
<tr>
<td>1/2</td>
<td>-1/2</td>
<td>( \frac{1}{2\pi} \sin \frac{\varphi}{2} \exp i (\theta - \phi)/2 )</td>
</tr>
<tr>
<td>-1/2</td>
<td>1/2</td>
<td>( \frac{1}{2\pi} \sin \frac{\varphi}{2} \exp i (\theta + \phi)/2 )</td>
</tr>
<tr>
<td>-1/2</td>
<td>-1/2</td>
<td>( -\frac{1}{2\pi} \cos \frac{\varphi}{2} \exp i (\theta - \phi)/2 )</td>
</tr>
</tbody>
</table>

Table I.

We identify \( L_3^0 \) with the operator characterizing spin directions and \( T_3^0 \) with the operator specifying the proton and neutron. Then the eigenfunctions of the ground state \((\gamma=0)\) of physical nucleons is given by
This new characterization of the proton and neutron by the operator $T^3$ necessitates a rederivation of the expression for charge density. We perform it in the next section.

§ 4. Charge density

The charge density operator of nucleons is defined by the following expression, contrary to the previous result (Eq. (6.1) of reference 3):

$$\rho(x) = \rho_{\text{mean}}(x) + \rho_{\text{source}}(x)$$

$$= T^0_{12}(x) + \frac{1}{2} U(r).$$

Here $T^0_{12}(x)$ is defined by

$$T^0_{12}(x) = \phi^a(x) \rho^a(x) - \phi^b(x) \rho^b(x).$$

We insert the expressions (2.6) and (2.7) for $\phi^a(x)$ and $\rho^a(x)$ and the previous result to this equation. We obtain

$$T^0_{12}(x) = \sum_{k, l, m} (B_{rl}B_{st} - B_{rs}B_{tl}) A_{rk}A_{ul}Q_{r} \left[ \frac{1}{2} L_{r}^{st} + T_{r}^{st} + \frac{1}{2} L_{r}^{st} - T_{r}^{st} \right]$$

$$+ \frac{1}{2} \frac{L_{r}^{st} + T_{r}^{st}}{Q_{r} - Q_{s}} \frac{1}{2} \frac{L_{r}^{st} - T_{r}^{st}}{Q_{r} + Q_{s}} \frac{\partial F}{\partial x_{k}} \frac{\partial F}{\partial x_{l}}. \quad (4.4)$$

In evaluating the expectation value of this operator for the ground state of nucleons, we note the following results of elementary computations. First we can easily prove that the expectation value of the term containing a factor $\delta_{0} (P_{r} + iQ_{r} \sum_{m \neq 0} \frac{1}{Q_{r}^{2} - Q_{m}^{2}})$ vanishes, because the function $(B_{rl}B_{st} - B_{rs}B_{tl}) \times (A_{rk}A_{ul} + A_{ul}A_{rk})$ gives a vanishing expectation value for all combinations of
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indices $r$, $s$, $k$ and $l$ and for all states of nucleons concerning the directions of spin and isotopic spin. The term containing a factor $(L_0^{is} + T_0^{is})$ gives a vanishing result for states $(\gamma = 0)$ under consideration. The last term containing a factor $(L_0^{is} - T_0^{is})$ alone does give a nonvanishing result.

Inserting for $Q_r$ its strong coupling theoretic approximation value $If/\kappa$ we obtain

$$\langle \text{proton} | \sum_{r,k} (B_{rl}B_{kl} - B_{rl}B_{kl}) A_{rk} A_{tk} (L_0^{is} - T_0^{is}) \rangle \text{neutron} \frac{\partial \bar{\gamma}(r)}{\partial x_k} \frac{\partial U(r)}{\partial x_l} \rangle = \pm \frac{1}{9} \sum_{k} \frac{\partial \bar{\gamma}(r)}{\partial x_k} \frac{\partial U(r)}{\partial x_k}. \quad (4.5)$$

Therefore the charge density of the nucleon due to meson fields is given by

$$\rho_{\text{meson}}(x) = \pm \frac{1}{6} \sum_{k} \frac{\partial \bar{\gamma}(r)}{\partial x_k} \frac{\partial U(r)}{\partial x_k}. \quad (4.6)$$

Despite of the different definition (4·1) of $\rho_{\text{meson}}(x)$ we have obtained the same result.

At this point we discuss the relation between the Fourier transform of the expectation value $\rho(x)_{\text{classical}}$ of the charge density operator, given by Eqs. (4·1) and (4·6), and the charge form factor $G_E(q^2)$. The transition probability $\omega$ of the nucleon from the initial state with momentum $-q/2$ to the final state with momentum $+q/2$ due to an interaction with external electromagnetic field $A_{\mu}^{\text{ext}}(x)$ is given by

$$\omega = \langle q/2 | \langle \rho(x)A_{\mu}^{\text{ext}}(x) - j(x) \cdot A^{\text{ext}}(x) \rangle dx | -q/2 \rangle. \quad (4.7)$$

If we assume that

$$A_{\mu}^{\text{ext}}(x) = \varphi \exp(\frac{iq \cdot x}{\kappa})$$

and

$$A^{\text{ext}}(x) = a \exp(\frac{iq \cdot x}{\kappa}), \quad (4.8)$$

then we have$^{11)$

$$\omega = G_E(q^2) \varphi - \frac{ie}{2\kappa} (\sigma \times q) G_M(q^2) \cdot a. \quad (4.9)$$

The two functions $G_E(q^2)$ and $G_M(q^2)$ are the form factors in the static theory.

These functions are related to the relativistic form factors $F_1(q^2)$ and $F_2(q^2)$ in the following way$^{15)$

$$G_E(q^2) = F_1(q^2) - (q^2/4m_p^2) \kappa F_2(q^2) \quad (4.10)$$

and

$$G_M(q^2) = F_1(q^2) + \kappa F_2(q^2).$$
Here the functions $F_i(q^2)$ and $F_j(q^2)$ are defined as follows:

$$
\langle p' | f_\mu(0) | p \rangle = \frac{i e}{(2\pi)^3} \frac{m_p}{(p_\nu p'_\nu)^{1/2}} \bar{u}(p') \left[ \tau_\mu F_i(q^2) - \frac{\sigma_\mu q_\nu F_j(q^2)}{2m_p} \right] u(p), \quad (4.11)
$$

where $p = (p, i\nu_0), q_\nu = p_\nu' - p_\nu$ and $\kappa$ is the anomalous magnetic moment in nuclear magnetons.

From Eq. (4.7) we should have, for small $q^2$, the following relations:

$$
\int \rho(x)^{\text{classical}} e^{i q \cdot x} dx = e G_E(q^2)
$$

and

$$
\int j(x)^{\text{classical}} e^{i q \cdot x} dx = \frac{i e}{2m_p} (\sigma \times q) G_M(q^2).
$$

Then we add a final remark that the expressions (4·1) and (4·6) explain well the following experimental facts\(^{13}\) concerning charge distributions, (1) $G_{E}^n(q^2) \approx 0$ and (2) $G_{M}^p(q^2) = (1 + q^2/0.71)^{-1}$ for the region $q^2 \leq 3 \text{(GeV/c)}^2$, when we assume an exponential form for the source function $U(r)$ with a cutoff parameter $A = 1.12 \times 0.210 \text{ F}$. Because for $A = 0.210 \text{ F}$ we have already shown\(^\text{b}\) that $G_{E}^n(q^2) \approx 0$ and that $G_{E}^p(q^2)$ and $G_{M}^p(q^2)$ are separately nearly equal to $\frac{1}{2} (1 + q^2/0.71)^{-2}$ though this $A$ is rather small in comparison with $A = 1.77 \times 0.210 \text{ F}$ determined from the Roper resonance.

§ 5. Isovector magnetic moments

In this section we calculate the expectation values of $M^{(\text{proton})}_{1s}$ and $M^{(\text{neutron})}_{1s}$ in Eqs. (2·14) and (2·15) for the ground states of physical nucleons given by Eqs. (3·9) and (3·10). First of all we substitute for $Q_\nu$ its strong coupling theoretic approximation value $1f/\kappa$ as in the calculation of charge density. Then for convenience we put

$$
K_{rs} = A_{r1}A_{s2} - A_{r2}A_{s1} \quad (5·1)
$$

and

$$
H_{rs} = B_{r1}B_{s2} - B_{r2}B_{s1}. \quad (5·2)
$$

We express the expectation values $\langle \text{proton} | K_{rs} H_{rs} | \text{proton} \rangle$ by matrices with row and column indices $r$ and $s$. They are given as follows:

$$
\langle \text{proton} | K_{rs} H_{rs} | \text{proton} \rangle = \frac{1}{9} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}. \quad (5·3)
$$

We insert this expression into Eq. (2·14) and obtain the same result as
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those given by Pauli and Dancoff with a complete omission of the kinetic energy terms in characterizing the state vectors:

\[ M_{\text{meson}} = \pm \frac{e}{2m_p} \frac{r^3 m_p}{\kappa^2} \left( \int X(r) U(r) \, dx - \frac{\kappa^2}{4\pi} \int X^2(r) \, dx \right). \]  

(5.4)

Similarly for \( M_{\text{source}} \) we have, from Eq. (2.15),

\[ M_{\text{source}} = \pm \frac{1}{6} \frac{e}{2m_p}, \]  

(5.5)

where we have used the numerical values given, in a notation such that \( r \) stands for the \( r \)th row, by

\[ \frac{\langle \text{proton} \mid A_{r\alpha} B_{r\beta} \mid \text{proton} \rangle}{\langle \text{neutron} \mid A_{r\alpha} B_{r\beta} \mid \text{neutron} \rangle} = \pm \frac{1}{9} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \]  

(5.6)

From Eqs. (5.4) and (5.5) we see that the magnetic moment is, in this approximation, just the isovector part alone, so that \( \beta = \mu(n)/\mu(p) = -1 \). This approximation which takes states with \( \sigma_z = \tau_z = 1 \) and the conditions of \( y = 0 \) and of no vibrational excited state, correspond to a band of nucleon isobars, \((1/2, 1/2), (3/2, 3/2), (5/2, 5/2), \ldots \) in terms of \((J, T)\), namely, the unitary irreducible representation \((\infty, 0, 0)\) of \( SU(4) \) and also of the strong coupling group \([SU(2)]_r \times \), considered by Cook, Goebel and Sakita. The generators of this group are, in the Pauli and Dancoff representation, total angular momentum of the system \( L^\theta \), total isotopic spin \( T^\phi \) and tensor operator \( A_{\alpha\beta} \) which connects different isobar states. This operator \( A_{\alpha\beta} \) is given by\(^5\)

\[ A_{\alpha\beta} = \sum_{r,s} A_{r\alpha} B_{r\beta} \left[ \left( P_r + iQ_r \sum_{s}(Q_s^2 - Q_s^2) \right) \delta_{rs} + \frac{1}{2} \frac{L_{rs}^\alpha + T_{rs}^\phi}{Q_r - Q_s} - \frac{1}{2} \frac{L_{rs}^\beta - T_{rs}^\phi}{Q_r + Q_s} \right]. \]  

(5.7)

The structure of the Lie algebra satisfied by these generators is as follows:

\[ [L_{\alpha}^\theta, L_{\beta}^\theta] = i\epsilon_{\alpha\beta\gamma} L_{\gamma}^\theta, \quad [T_{\alpha}^\phi, T_{\beta}^\phi] = i\epsilon_{\alpha\beta\gamma} T_{\gamma}^\phi, \]

\[ [L_{\alpha}^\theta, A_{\beta\gamma}] = i\epsilon_{\alpha\beta\gamma} A_{\alpha\gamma}, \quad [T_{\alpha}^\phi, A_{\beta\gamma}] = i\epsilon_{\alpha\beta\gamma} A_{\gamma\alpha}, \]  

(5.8)

\[ [A_{\alpha\gamma}, A_{\beta\delta}] = 0. \]

The tensor operators \( A_{\alpha\gamma} \) become the ideal of the Lie algebra and further they commute with each other. Therefore we see that in the Pauli and Dancoff scheme the underlying group is already non-compact\(^4\) for finite values of coupling constant, in contrast to the Cook, Goebel and Sakita scheme where the non-compact group is obtained in the limit of coupling constant \( = \infty \).

This particular property arises from the Pauli and Dancoff prescription given by
\[ \varphi_a(x) = \varphi'_a(x) + \frac{\partial}{\partial x} \varphi_a(x), \quad \int \varphi'_a(x) \nabla U(r) \, dx = 0 \]

and

\[ \pi_a(x) = \pi'_a(x) + \frac{\partial}{\partial x} \pi_a(x), \quad \int \pi'_a(x) \nabla \xi(r) \, dx = 0, \]

supplemented by Eqs. (2.6) and (2.7), and the following commutation relations:

\[ [\varphi_{ab}, \pi_{\beta\ell}] = i\partial_{a\beta} \delta_{b\ell}, \]

\[ [\varphi_{ab}, \varphi'_{\beta\ell}] = [\varphi'_{ab}, \pi_{\beta\ell}] = 0, \]

\[ [\pi_{ab}, \varphi'_{\beta\ell}] = [\pi_{ab}, \pi'_{\beta\ell}] = 0, \quad (5.10) \]

\[ [\varphi'_a(x), \pi'_{\ell\beta}(y)] = i\partial_{a\ell} \left[ \delta(x - y) - \frac{1}{4\pi} \sum_k \frac{\partial \xi}{\partial x_k} \frac{\partial U}{\partial y_k} \right]. \]

So far the discussion has been limited to the zeroth approximation with respect to nondiagonal operators \( \delta_{\ell\in(\pm)} \) and \( \tau_{\in(\pm)} \). To obtain a better value we must go over to higher approximations. A brief discussion of this point and the evaluation of numerical values of magnetic moments will be taken up in the subsequent section for studying the implications of the \((3/2, 3/2)^+\)-resonance (1240 MeV) and \((1/2, 1/2)^+\)-resonance (1470 MeV) for absolute values of magnetic moments.

§ 6. Higher approximations and numerical values of isovector magnetic moments

In accordance with the fact that the calculations with new state vectors turn out to give the same result for magnetic moments in the zeroth approximation as those given by Pauli and Dancoff, we obtain the same result as those given by Hourieh \(^{10} \) for higher approximations also. Therefore we skip the details of the derivation of relevant expressions.

We write down the perturbation energy \( \Delta E \) in the second order with respect to the transformed nondiagonal electromagnetic interaction Hamiltonian produced by an external field \( A(x) = (\hbar/2)(-x_3, x_1, 0) \) and so \( h_t(x) = \delta_{\ell\in} \hbar \) as follows:

\[ \Delta E = \sum_n \frac{\langle \text{proton} N_{\text{neutron}} | \hat{H}_{\text{non}} | n \rangle}{E_{\text{proton}} - E_n} \langle n \rangle_N \hat{H}_{\text{non}} \langle \text{proton} N_{\text{neutron}} | \rangle \]

\[ \approx \frac{1}{(E_{\text{proton}} - E')} \left( \frac{\langle \text{proton} | \hat{H}_{\text{non}} | \text{neutron} \rangle^2}{E_{\text{proton}} - E'} \right). \quad (6.1) \]

Here the intermediate states \( |n\rangle \) are characterized by the condition that at least one of two operators \( \delta_3 \) and \( \tau_3 \) assumes \((-1)\) while the ground states are characterized by \( \delta_3 = \tau_3 = 1 \). The Hamiltonian \( \hat{H}_{\text{non}} \) is the nondiagonal part of an operator,
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\[ S_2S_1[H^0(\text{Eq. (3.1)}) + H'](S_2S_1)^i, \]  

where

\[ H' = -\frac{e}{2m_p} \left( \frac{1 + \tau_3}{2} \right) \sigma \cdot \int h(x) U(r) dx - e \int S(x) \cdot A(x) dx. \]  

Further we have made an approximation that \( E_n \) is replaced by its lowest value \( E' \) and that states \( |\mu\rangle \) span a complete set.

Then the contribution to magnetic moment by this perturbation energy is

\[ \Delta M_{12} = \left[ -\frac{\partial}{\partial h^i} (\Delta E) \right]_{h=0} \]

\[ = \sum_{i=1}^{3} \Delta M_{12}^{(i)}. \]  

\( \Delta M_{12}^{(i)} \) are given by elementary calculations as follows:

\[ \Delta M_{12}^{(1)} = -\frac{e}{2m_p} \frac{N \kappa^2}{8I^2f^3} \frac{1}{\text{proton}} \sum_{k} A_{R^b}(L^b - T^b_0) \left| \begin{array}{c} \text{proton} \\ \text{neutron} \end{array} \right\rangle \langle E_0 - E' \rangle \]

\[ = \frac{1}{(E' - E_0)} \frac{N \kappa^2}{6I^2f^3} \frac{e}{2m_p}, \]  

\[ \Delta M_{12}^{(2)} = \frac{e}{2m_p} \frac{N \kappa^2}{32I^2f^3} \frac{1}{\text{proton}} \sum_{r, \gamma, i, a} \left[ (L^r_0 - L^r_0'), A_{R^b}B_{R^b} \right] (\sigma_i + \tau_3) \sigma_3 \sigma_r \]

\[ \times \left| \begin{array}{c} \text{proton} \\ \text{neutron} \end{array} \right\rangle \langle E_0 - E' \rangle \]

\[ = \pm \frac{1}{(E' - E_0)} \frac{N \kappa^2}{12I^2f^3} \frac{e}{2m_p}, \]  

and

\[ \Delta M_{12}^{(3)} = \frac{e}{2m_p} \frac{N}{48I^2} \left( \int \left[ U(r) (x \cdot \nabla X(r)) dx \right] \langle \text{proton} \rangle \sum_{r, \gamma, i, a} \left[ (L^i_0 - T^i_0), \right. \]

\[ (A_{R^b}A_{R^b} - A_{R^b}A_{R^b}) (B_{R^b}B_{R^b} - B_{R^b}B_{R^b}) \left( -\sigma_i + \tau_3 \right) \sigma_3 \sigma_r \left| \begin{array}{c} \text{proton} \\ \text{neutron} \end{array} \right\rangle \langle E_0 - E' \rangle \]

\[ = \pm \frac{1}{(E' - E_0)} \frac{N m_p}{18I^2} \left( -A \int U(r) (x \cdot \nabla X(r)) dx \right) \frac{e}{2m_p}, \]  

where \( E_0 \) is the energy of physical nucleons.

Then we use a previous result and equate \( 3N \kappa^2/(4I^2f^3) \) to 300 MeV (mass of \( (3/2, 3/2)^+ \)-resonance). We obtain, independently of distribution functions \( U(r) \), the following results:

\[ \Delta M_{12}^{(i)} = \frac{2}{3} \frac{100}{(E' - E_0)} \frac{e}{2m_p}. \]
In order to give a quantitative value to the expression (6.7) we introduce a simple exponential form for the source function \( U(r) \) as follows:

\[
U(r) = \frac{1}{8\pi A^3} e^{-r/A}.
\] (6.10)

Then the function \( X(r) \) for which \( dX(r)/dr \) and therefore \( \psi_\sigma(x) \) is normalizable, is given by

\[
X(r) = \frac{1}{(1 - \kappa^2 A^2)^3/2} \frac{e^{-r}}{r - \frac{1}{2(1 - \kappa^2 A^2)} \left( \frac{r + 2}{A} \right) e^{-r/A}}.
\] (6.11)

We insert these expressions into Eq. (6.7) and obtain

\[
\Delta M_{12}^{(3)} = \frac{5}{12} \frac{m_p}{E' - E_0} \frac{e}{2m_p},
\] (6.12)

where we have used the integration given by

\[
\int \left( U(r) \cdot \nabla X(r) \right) dx = \frac{1}{2A} \frac{1}{(1 - \kappa^2 A^2)^2 (1 + \kappa A)^2} - \frac{3}{16 (1 - \kappa^2 A^2)}
\]

\[
- \frac{1}{2(1 - \kappa^2 A^2)} + \frac{2\kappa A}{(1 - \kappa^2 A^2)^2} \left[ \frac{1}{16} + \frac{1}{4(1 - \kappa^2 A^2)^2} + \frac{1}{2(1 - \kappa^2 A^2)^3} \right] \sim - \frac{5}{32A}.
\] (6.13)

It is very conceivable that \( E' \) lies at high values beyond the low-lying resonances, though we could not easily estimate it from the Hamiltonian (3.1). Therefore we can expect that \( \Delta M_{12}^{(3)} \) is small in magnitude. We limit ourselves to the discussion of the isovector magnetic moment in the zeroth approximation. This is given by

\[
M_{12}^{(3\text{magnetic})} \sim \frac{4}{3} \left( \frac{5}{16} - 3\kappa A \right) \frac{m_p}{e} \frac{5}{100 2m_p}.
\] (6.14)

Here we have used the following integration:

\[
\int \left( X(r) U(r) - \frac{\kappa^2}{4\pi} X^3(r) \right) dx = \frac{1}{2A(1 - \kappa^2 A^2)} \left[ \frac{1}{(1 - \kappa^2 A^2) (1 + \kappa A)^2} - \frac{1}{4(1 - \kappa^2 A^2)} - \frac{1}{8} \right]
\]

\[
- \frac{1}{A} \left[ \frac{\kappa A}{2(1 - \kappa^2 A^2)^2} + \frac{1}{(1 - \kappa^2 A^2)^3} \right] \left[ \frac{1}{16} \frac{1}{4(1 - \kappa^2 A^2)^2} + \frac{1}{2(1 - \kappa^2 A^2)^3} \right]
\]

\[
- \frac{2}{(1 - \kappa^2 A^2) (1 + \kappa A)^3} \frac{1}{(1 - \kappa^2 A^2)^2 (1 + \kappa A)} \right] \sim \frac{1}{A} \left( \frac{5}{16} - 3\kappa A \right).
\] (6.15)

Now we fix the cutoff parameter \( A \) by the previous result\(^3\) which says that the first excited vibrational state corresponds to the Roper resonance. This con-
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The equation is equivalent to demanding \((N/I)^{1/2} = 530\) MeV (mass of \((1/2, 1/2)^+\)-resonance). Since \((N/I)^{1/2}\) is given by

\[
\left( \frac{N}{I} \right)^{1/2} = A \left[ \frac{16 (\kappa A)^4}{(1 - \kappa^2 A^2)^4} - \frac{1}{(1 - \kappa^2 A^2)^2} - \frac{6}{(1 - \kappa^2 A^2)} + \frac{24}{(1 - \kappa^2 A^2)^3} - \frac{16}{(1 - \kappa^2 A^2)^2} \right]^{1/2} = \frac{1}{A} \quad (6.16)
\]

we have \(A = 1.77 \times 0.210\) F.

Then we have the following relation:

\[
M_{12}^{(\text{source})} (\text{Eq. (5-5)}) + M_{12}^{(\text{neon})} (\text{Eq. (6-14)}) = 2.36 \frac{e}{2m_p}. \quad (6.17)
\]

This equation provides us with \(\kappa = 0.174 \times 140\) MeV. And from the previous result \(3N\kappa^2/4I_3f^2 = 300\) MeV we have \(f = 2.11\). This small effective mass\(^9\) of bound pions clearly expresses the fact that a strong attractive pion-pion interaction is important for isovector magnetic moments. The addition of the pion-pion interaction Hamiltonian, \(- (\lambda/4\kappa^2) (\sum a \varphi_a^2)^2\) to the original Hamiltonian \(H\) (Eq. (2-1)) changes the equation of motion for the pion field to the following form:

\[
\left[ \frac{\partial^2}{\partial t^2} - \nabla^2 + \left( \kappa^2 - \frac{1}{\kappa^2} \sum a \varphi_a^2 \right) \right] \varphi_a (x) = - \sqrt{\frac{4\pi}{\kappa^2}} \frac{f}{\kappa} \tau_a \sigma \cdot \nabla U(r). \quad (6.18)
\]

Therefore\(^*)\) the effective mass of bound pions is reduced to \(\kappa (1 - 3\lambda f^2/\kappa^6)^{1/2}\).

\[\text{§ 7. Dependence of } \beta = \mu(n)/\mu(p) \text{ on dimensions of } SU(4)\]

Bég, Lee and Pais have raised a question concerning the relation between the \(SU(6)\)-predictions and the point of view of the local Lagrangian field theory. They say that \(\beta = -2/3\) is a pure number independently of coupling constants of baryon fields with meson fields. The field theory should predict, however, \(\mu(n) = 0\) and \(\mu(p) = e/2m_p\) when no strong interaction is applied. The observed value \(\beta = -0.684\) is obtained for existing magnitudes of coupling constants.

To answer partially to this question we consider, in this section, the dependence of \(\beta\) on the dimensions of the irreducible representations of a group \(SU(4)\). The generators of this \(SU(4)\) are \(1/2 \sigma_i\), \(1/2 \tau a\) and \(1/2 \sigma_i \tau a\).

We limit discussions to the unitary irreducible representations with the

\(^{9)}\) When we insert the effective mass of bound pions \((\kappa_{\text{eff}})\) into \(\kappa\), a caution must be taken, because we have the \(\kappa\) on both sides of Eq. (6-18). Confusion can be easily avoided when we write the original equation of motion as follows:

\[
\left( \frac{\partial^2}{\partial t^2} - \mathbf{p}^2 + \kappa^2 \right) \varphi_a (x) = - \sqrt{\frac{4\pi}{\kappa^2}} \frac{f}{\kappa} \tau_a \sigma \cdot \mathbf{p} U(r).
\]

Then in the final expressions we insert \(\kappa_{\text{eff}} = \kappa \left(1 - 3\lambda f^2/\kappa^6\right)^{1/2}\) into \(\kappa\) and 140 MeV into \(\kappa\). In this notation, e.g. we have \(3N\kappa^2/4I_3f^2 = 300\) MeV. The value \(f = 2.11\) is obtained from this equation.
Young tableaux \((N, 0, 0)\), because the band of nucleon isobars obtained in §5 corresponds to the Young tableaux \((\infty, 0, 0)\) of the non-compact and non-invariance group \([(SU(2))_{\nu} \otimes (SU(2))_{\tau}] \times T_{\nu} \).

Differently from the strong coupling group and in accordance to the conventional quark model of \(SU(6)\) theory we suppose that nucleon isobars are made of fundamental objects (baryonettes). In this paper, however, we do not discuss the meson as representations of \(SU(4)\). Therefore the question of whether we can really construct the invariant interaction Hamiltonian corresponding to all dimensions of representations by nucleon isobars, is left open.

As in the quark model of \(SU(6)\)-theory we assume that the total magnetic moments of nucleon isobars are ascribed to the intrinsic magnetic moments of individual baryonett which are (charge of baryonett)/2 (mass of baryonett). Further we assume that the magnetic moment operator has the same form \(M=Q \otimes \sigma\) as does the individual baryonett, where \(Q=T_3+\frac{1}{2} Y+\frac{1}{2} n_B\), because baryonettes have zero strangeness in the present discussions.

From the point of view of \(SU(4)\), magnetic moment operators of the form \(M=(\text{isoscalar})+(\text{isovector})=a_1 \otimes \sigma + b_2 \otimes \sigma\) are all equally acceptable. We demand, however, that \(a_1 + b_2 = \frac{1}{2} n_B + \frac{3}{2} n_B = Q\), because this expression is valid both in 4 (Wigner's \(SU(4)\) where nucleons belong to a multiplet 4 and \(n_B=1\)) and in 20 (submultiplet of 56 of \(SU(6)\) where nucleons belong to a multiplet 20 and \(n_B=1/3\)).

For \(N=1\) we have a multiplet \((J, T)=(1/2, 1/2)\). The charge operator is given by \(Q=\frac{1}{2}(1+\tau_a)\). We have \(\beta=0\).

For \(N=3\) we have a multiplet \((J, T)=(1/2, 1/2), (3/2, 3/2)\). The isobars in this multiplet are constructed by three baryonettes, which can be seen from the equation, \(4 \times 4 \times 4 = (3) + 2 \times (2, 1) + (1^3)\). The charge operator is given by \(Q=\frac{3}{2}(\frac{1}{2} + \tau_a)\). We have the following normalized wave function symmetric in three indices:

\[
N_{ABC} = \left( \frac{\delta J C}{\sqrt{2}} \right)_{ab} \xi_{ce}^{(m)} A_a B_b C_c + \frac{1}{3} \left[ \left( \frac{C}{\sqrt{2}} \right)_{ab} \xi_a^{(e)} \epsilon_{\alpha \beta} B_\gamma (\sigma) \right. \\
- \left. \left( \frac{C}{\sqrt{2}} \right)_{ab} \xi_a^{(e)} \epsilon_{\alpha \beta} B_\gamma (\sigma) + \left( \frac{C}{\sqrt{2}} \right)_{ab} \xi_b^{(e)} \epsilon_{\alpha \beta} B_\gamma (\sigma) \right], \tag{7.1}
\]

Here \(\xi_{ce}^{(m)}\) is the non-relativistic Rarita-Schwinger wave function\(^{(16,17)}\) of \(J=3/2\), and satisfies the symmetry condition that \(\psi_{abc} = (\sigma C/\sqrt{2})_{abc} \xi_{ce}^{(m)}\) are completely symmetric in \(a, b\) and \(c\). The operator \(C\) is given by \(C=i\sigma_2\). The isotopic wave functions of \(T=3/2\) are normalized in the following conventional way:

\[
A_{111} = N_{3/2}^+, \quad A_{112} = (1/\sqrt{3}) N_{3/2}^+, \quad N_{112} = (1/\sqrt{3}) N_{3/2}^0, \quad A_{122} = N_{3/2}^-.
\tag{7.2}
\]

The magnetic moments of nucleon isobars are obtained by
By elementary calculations we have $\beta = -2/3$.

For $N=5$, where nucleons belong to 56 of $SU(4)$, and $n_B=1/5$ we have a multiplet $(J, T) = (1/2, 1/2), (3/2, 3/2), (5/2, 5/2)$. The fact that the isotopic wave functions of $(5, 0, 0)$ are constructed from five baryonettes, can be easily seen in the following way. Namely we have

$$4 \times 4 \times 4 \times 4 \times 4 = (5) + 4 \times (4, 1) + 5 \times (3, 2) + 6 \times (3, 1^+)$$

$$+ 5 \times (2^2, 1) + 4 \times (2, 1^+).$$

(7.4)

If we have one pair of baryonett and antibaryonett, then we have

$$4 \times 4 \times 4 \times 4 \times 4 = (5, 1^+) + 3 \times (4, 2, 1) + 4 \times (3^2, 1)$$

$$+ 3 \times (3, 2^2) + 8 \times (2, 1) + 4 \times (1^3).$$

(7.5)

The charge operator is then given by

$$Q = \frac{1}{2}((1/5) + r_\pi).$$

We have the following normalized wave function symmetric in five indices:

$$N_{ABCD} = (30)^{-1/2}N_{ABCD}^{(3, 3/2, 3/2)} + (2^4 \cdot 5^3)^{-1/2}N_{ABCD}^{(3, 3/2, 2)} + (2^4 \cdot 3^3 \cdot 5)^{-1/2}N_{ABCD}^{(3, 3/2, 1/2)}.$$  

(7.6)

The wave function $N_{ABCD}^{(3, 3/2, 3/2)}$ of $(J, T) = (5/2, 5/2)$ is given by

$$N_{ABCD}^{(3, 3/2, 3/2)} = \left\{ \left[ (\sigma C)_{ab} (\sigma C)_{cd} + (\sigma C)_{ac} (\sigma C)_{bd} + (\sigma C)_{bc} (\sigma C)_{ad} \right] \xi^{(m)}_{ab} 
+ \left[ (\sigma C)_{bd} (\sigma C)_{ca} + (\sigma C)_{bc} (\sigma C)_{de} + (\sigma C)_{de} (\sigma C)_{ca} \right] \xi^{(m)}_{bd} 
+ \left[ (\sigma C)_{cd} (\sigma C)_{ea} + (\sigma C)_{ce} (\sigma C)_{da} + (\sigma C)_{de} (\sigma C)_{ea} \right] \xi^{(m)}_{cd} \right\} A_{i \beta j \rho} (m).$$

Here $\xi^{(m)}$ is the non-relativistic Rarita-Schwinger wave function of $J=5/2$ and we have used this fact alone in constructing the above wave function. The isotopic wave functions of $T=5/2$ are normalized in the following conventional way:

$$A_{1111} = N_{5/2}^{5/2}, \quad A_{1112} = (1/\sqrt{5}) N_{5/2}^{5/2}, \quad A_{1113} = (1/\sqrt{10}) N_{5/2}^{5/2}, \quad A_{1114} = (1/\sqrt{10}) N_{5/2}^{5/2}, \quad A_{1115} = (1/\sqrt{10}) N_{5/2}^{5/2}.$$  

(7.7)

The wave function $N_{ABCD}^{(3, 3/2, 3/2)}$ of $(J, T) = (3/2, 3/2)$ is given by

$$N_{ABCD}^{(3, 3/2, 3/2)} = \epsilon_{ab} (\sigma C)_{cd} \xi^{(m)}_{ab} \Delta_{i \beta} (m) + \epsilon_{ac} (\sigma C)_{bd} \xi^{(m)}_{ac} \Delta_{i \beta} (m) + \epsilon_{ad} (\sigma C)_{bc} \xi^{(m)}_{ad} \Delta_{i \beta} (m) + \epsilon_{ae} (\sigma C)_{bd} \xi^{(m)}_{ae} \Delta_{i \beta} (m).$$

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* An indication of the existence of resonance $(5/2, 5/2)^+$ has been reported in the $\pi^-p$ system with a mass of 1.71 BeV in the reaction $\pi^- + p \rightarrow \pi^- + \pi^- + \pi^+ + n$, p. See T. G. Schumann, Phys. Rev. Letters 15 (1965), 531.
The functions $\epsilon_{ab}$ and $\epsilon_{a\beta}$ are the Levi-Civita symbols in two dimensions. Since we cannot construct the Levi-Civita symbols in higher dimensions, the above expression for $N_{ABCD\bar{E}}$ might be a unique one. The magnetic moments of nucleon isobars are given by $5N_{ABCD\bar{E}}(M_{J})_{\beta}^{\gamma}N_{ABCD\bar{E}}$. By elementary calculations we have $\beta = -16/19$.

As the number $N$ and therefore the dimension and so the magnitude of coupling constants of strong interaction increases the isoscalar parts of magnetic moments become small and tend to zero. Thus for an infinite band of nucleon isobars given by $(J, T) = (1/2, 1/2), (3/2, 3/2), (5/2, 5/2), \ldots$, we have the same value as that obtained in the Pauli and Dancoff scheme of strong coupling theory of pion and nucleon system. In fact we see from Eq. (2·15) that the isoscalar magnetic moment of nucleon sources vanishes when we calculate the expectation value of this operator and that we are left with the isovector magnetic moment alone. Then it is well expected to have $\beta = -1$ from the Chew and Low static theory.

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**References**

16) W. Rarita and J. Schwinger, Phys. Rev. 60 (1941), 61.