Closure to "Discussion on 'Prediction of Time-Varying Vibroacoustic Energy Using a New Energy Approach' " (2005, ASME. J. Vib. Acoust., 127, p. 100)

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This short letter offers a reply to the discussion on our previous paper, "Prediction of Time Varying Vibroacoustic Energy Using a New Energy Approach" [1] made by Savin [2]. Generally speaking, the discussion is interesting, and the provided comments (particularly points 1 and 3) help to clarify several points that have not been fully covered in the paper. Having said that, the discussion should be corrected to include the following remarks.

As a preliminary, it should be noted that the aim of our paper [1] was to propose a novel, alternative method to the usual diffusion equation (or, transient SEA), hoping that some members of the research community may find it useful. The main remarks corresponding to the Savin's comments [2] are summarized as follows:

1. The first point of the comments focuses on the application of transport equation. It should be stated that the equation (Eq. 10 in [1]) is proposed to describe the time-varying behavior of high-frequency vibrational energy in a damped and bounded system. In a simple propagation domain, the space-time energy density W and power flow density $\mathbf{I}$ are the integration of phase-dependent $W(\mathbf{k})$ and $\mathbf{I}(\mathbf{k})$ with respect to $\mathbf{k}$. In the case of multipropagation in a multidimensional domain (for example, the model in [3]), an important assumption is that the correlation of each propagation mode is neglected. Therefore, it is considered in the paper that the vibrational energy from each propagation mode can be superposited. However, some unexpected errata have been found in Eqs. (25), (36), and (37) where $\hat{\mathbf{k}}$ should be taken away; consequently, Eq. (8) is the same as Eq. (25).

2. The authors attempt to derive the energy equation in a more general way, then the start point is set by the symmetric hyperbolic system (Eq. (13) in [1]). Equations (11) and (12) in [1] are just proposed to introduce the symmetric hyperbolic system with a damping term. It is unfortunate that the notation of μ is repeated once again in Eqs. (20) and (21). They should be represented by different notations, because they are not strictly equal.

3. The third point is related to the first one. In [3], the transport equation was applied to the waves in *unbounded* media; the energy components $W^{+}(\mathbf{x},\mathbf{k},t)$ and $W^{-}(\mathbf{x},-\mathbf{k},t)$ correspond to two simple eigenvalues $\pm\omega$, so that $W^{+}(\mathbf{x},\mathbf{k},t)=W^{-}(\mathbf{x},-\mathbf{k},t)$. In fact, they are the same quantities but in inverse direction to infinity ($\pm\infty$), and they cannot be superposed. Thus,

$$W(\mathbf{x},t)=\int_{\mathbb{R}^3} W^{+}(\mathbf{x},\mathbf{k},t)\mathrm{d}k, \quad I(\mathbf{x},t)=c(\mathbf{x})\int_{\mathbb{R}^3} W^{+}(\mathbf{x},\mathbf{k},t)\hat{\mathbf{k}}\mathrm{d}\mathbf{k} \tag{1}$$

In the paper, W^{+} and W^{-} represent the progressive and regressive wave energies propagating in *bounded* media. Especially in damped cases, $W^{+}\neq W^{-}$. By the principle of superposition under some assumptions, the energy density and power flow density could be written as

$$W(\mathbf{x},t)=\int_{\mathbb{R}^3}[W^{+}(\mathbf{x},\mathbf{k},t)+W^{-}(\mathbf{x},-\mathbf{k},t)]\mathrm{d}\mathbf{k} \tag{2}$$

$$I(\mathbf{x},t)=c(\mathbf{x})\int_{\mathbb{R}^3}[W^{+}(\mathbf{x},\mathbf{k},t)-W^{-}(\mathbf{x},-\mathbf{k},t)]\hat{\mathbf{k}}\mathrm{d}\mathbf{k} \tag{3}$$

4. As explained in item 2, Eqs. (1) and (2) is just *artificially* proposed to introduce the general symmetric hyperbolic system, and we agree with Savin's comment that *there is no need to invent a new damping model for Euler's equation*. However, it is believed that damping model does not affect the format of the proposed energy equation. In addition, the damping model at high-frequency domain in [1] expressed by Eq. (7), where ω should be thought of as the center frequency of a frequency band. The time history of the vibrational energy is then corresponding to the high-frequency band centered by a specific frequency value.

5. It is agreed that the bending vibrations' equation of the Euler-Bernouilli cannot simply be regarded as a hyperbolic one. However, it can be converted to the hyperbolic format via a change of variables. The applications on hyperbolic waves has been published (see [4]). In fact, accomplishment of the derivation of the time-varying energy equation for dispersive waves in beams or plates has been done, and it can demonstrated that the time-varying energy of high-frequency dispersive waves can also be modeled by the proposed energy equation (10) in [1].

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