boiling in forced convection) has been shown to be valid as postulated by Snyder in two sets of experimental efforts. First, rapid temperature decreases have been observed in the solid material under a bubble during bubble growth. This result shows that the hypothesized thin liquid layer below the bubble was rapidly vaporizing as postulated. Second, it has been shown that large amounts of vapor can condense on the bubble's top surface during its lifetime with heat being carried away by turbulent eddies at the bubble surface. This second observation shows that simultaneous evaporation and condensation can occur, and latent heat transport rates through the bubble can, therefore, be large. These results indicate that the mass transfer mechanism is the predominant heat transfer mechanism in subcooled nucleate boiling.

**Observations in Artificial Bubble Investigations**

**Experimental.** 1 The bubble heat transfer coefficient varied from approximately \(4 \times 10^{-4}\) to \(4 \times 10^{-6}\) Btu/hr sq ft as the cooling stream velocity was varied from 0.2 to 38 fps (Reynolds number varied from approximately 900 to 2 \(\times\) 10^5) and as the cooling stream temperature was varied from 140 to 80 F, the pressure was near atmospheric.

2 The ratio of the actual amount of heat removed from the bubble to the amount of latent heat necessary to form one volume of steam equal to the maximum observed volume of the bubble varied from approximately 10 to 100.

3 The presence (or absence) of a thin thermal boundary layer on the heated plate does not appreciably affect the turbulent diffusion of heat from the bubble surface and, thus, does not appreciably affect the amount of condensation occurring thereon.

**Theoretical.** The value for the thermal diffusivity for heat transfer effective over the life of one stationary bubble was approximately the same as the eddy diffusivity \(\varepsilon_a\) in fully developed pipe flow approximately one bubble radius from the wall. It ranged from 0.06 to 3.0 sq ft/hr for the conditions in this experiment.

**Theoretical Observations on Bubble in Real Boiling**

Using the mass transfer model of Snyder along with the dynamics of bubbles and transient heat conduction in the heating plate one can predict experimental results with a proper choice of the effective turbulent diffusivity for heat transfer at the bubble top. Value used which resulted in agreement between this theory and the experiments of Gunther [20] were approximately 0.1 of the value of the eddy diffusivity for momentum transfer calculated using the velocity of the cooling stream. Calculations based on this model also showed that the heat removed from the plate due to one bubble was approximately 30 times more than that calculated on the basis of the latent heat associated with the mass required to form one bubble volume.

**Acknowledgment**

This work was partially supported by NASA Grant No. NsG-657.

**References**

1 Snyder, N. W., Lecture Notes on Diffusion and Mass Transfer, University of California, 1952.


**DISCUSSION**

S. G. Bankoff

In the work of Bankoff and Mason, steam was injected through a small aperture in a horizontal plate in contact with a pool of water, and a jet of cold water was directed downward onto the aperture. It is very interesting, therefore, that the heat transfer coefficients measured in this work agree so well with the previous measurements. One would suspect, as pointed out earlier that boundary-layer effects are in agreement with these rapidly growing bubbles and that turbulence extends all the way to the bubble wall. With these enormous heat transfer coefficients, it is quite certain that mass transfer accounts for a major portion of the heat transfer in highly subcooled nucleate boiling.

One can also be quite sure that inertial effects are quite important for highly subcooled boiling. Gunther and Kreith took photographs indicating that the growth and collapse times were roughly equal in water at atmospheric pressure with sufficiently

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5 Northwestern University, Evanston, Ill.
large steam velocities and subcoolings. This would imply that the bubble grows and collapses as a Rayleigh bubble, under constant pressure difference.

**M. G. Cooper**

Recently, I have been supervising and carrying out studies of nucleate pool boiling, reaching some conclusions which are relevant to the use of the microlayer in the theory of Drs. Snyder and Robin. As shown in references [27, 28], Dr. Lloyd and I developed a theory to predict the formation and thickness of such layers under hemispherical bubbles. We conclude that its initial thickness is $C\sqrt{R}$, where $R$ is the kinematic viscosity of the liquid, $t_0$ is the time taken for the bubble to grow to the point under consideration, and $C$ is a constant of order 0.8. In our experiments, $C$ lay in the range 0.6 to 1.0. Taking $C = 0.8$ and $t_0 = 100$ microsec (typical for Dr. Snyder’s experiments) this predicts a thickness of some 200 microin., which is close to the thickness required for their theory. Toward the center of the bubble, the microlayer would be thinner, and might therefore dry out, but if this were confined to a small region their theory would not be greatly affected by it.

Recently, I have developed a theory for bubble growth resulting solely from evaporation of the microlayer, the bulk liquid being nearly at saturation temperature. The theory has been submitted for publication (reference [29]). It uses certain extreme assumptions in order to simplify the mathematics, and a rate of growth can then be predicted. A point which emerges relevant to the present paper is that early in bubble growth the inertia stress, $\rho(R^2 + \frac{2}{3}R^2)$, is much larger than surface tension stress, $2\sigma/R$, so the bubble tends to be hemispherical. However, when the bubble grows and collapses as a Rayleigh bubble, under constant pressure difference. Experimental data on bubble shapes under various conditions of pressure, heat flux, subcooling, and velocity would be valuable in evaluating the relevance of the mass transfer mechanism. For the case of highly subcooled boiling in forced convection at atmospheric pressure, Gunther [20] observed hemispherical bubbles (see p 117 of reference [20]). For higher pressures the surface tension term, $2\sigma/R$, in the bubble dynamics equation decreases for the same size bubble. For example, the surface tension term decreases by about 50 percent from atmospheric to 1000 psi, for the same size bubble. Jihi and Clark [2] have reported bubbles larger than Gunther’s for pressures as high as 500 psi. One might expect their bubbles to show less of a tendency to round and become spherical than Gunther’s atmospheric bubbles. Bubble shapes were not easily observed from the photographs reported by Jihi and Clark.

Hospeti and Mesler [4] have recently reported an interesting study of bubble shapes for saturated pool boiling at atmospheric pressure. They observed that the bubble shape depended on the bubble growth rate. The growth rate was high for high surface temperature. Relating this to the present model, a higher liquid film temperature would result in a higher rate of mass input to the bubble. The additional mass would result in a higher bubble vapor pressure. $P - P_a$, the “driving force” in the bubble dynamics equation, would be larger and would cause a faster growth rate. Growth rates are also affected by the rate of condensation because of its affect on $P - P_a$. For example, for curves one and two of Fig. 12 of this paper, the following were recorded for time equal 50 sec:

<table>
<thead>
<tr>
<th>Curve Number</th>
<th>$\epsilon'$ (ft$^3$/hr)</th>
<th>$P - P_a$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.087</td>
<td>1.47</td>
</tr>
<tr>
<td>2</td>
<td>0.105</td>
<td>0.37</td>
</tr>
</tbody>
</table>

Bubble shape seems to be a function of several variables. Additional experimental work to establish the relation between bubble shape and such variables as pressure, surface tension, subcooling, heat rate, and velocity appears warranted.

It should be tacitly observed that the mass transfer model used in this paper was able to predict the performance of the experimentally observed bubbles of Gunther (Fig. 12 versus Fig. 11). Three assumptions were made: (1) the bubble was hemispherical (Gunther’s bubbles were essentially this); (2) a thin liquid film existed at all times at the base of the bubble; and (3) an eddy thermal diffusivity was determined to make the growth and collapse of the bubbles fit the Gunther data.

A bubble history of the bubbles shown in Fig. 12 was very sensitive to the value of eddy thermal diffusivity as well as the degree of subcooling.

The mechanism of bubble dynamics when the bulk fluid temperature is near saturation becomes quite different. The bubble continues to grow in size with little or no condensation until the forces of inertia of the liquid literally drags the bubble off the surface. This is an entirely different regime from the highly subcooled case. Thus, Dr. Cooper’s calculations in reference [29] deviates somewhat from the cases to which we have paid attention.

**Authors’ Closure**

The authors would like to thank Dr. Bankoff and Dr. Cooper for their observations and interesting comments.

We would agree that the mass transfer effect would be smaller for a spherical bubble with a small contact area than for a hemispherical bubble with a large contact area. The rate of heat removal from the thin liquid film would certainly be a function of the available area for evaporation. Experimental data on bubble shapes under various conditions of pressure, heat flux, subcooling, and velocity would be valuable in evaluating the relevance of the mass transfer mechanism. For the case of highly subcooled boiling in forced convection at atmospheric pressure, Gunther [20] observed hemispherical bubbles (see p 117 of reference [20]). For higher pressures the surface tension term, $2\sigma/R$, in the bubble dynamics equation decreases for the same size bubble. For example, the surface tension term decreases by about 50 percent from atmospheric to 1000 psi, for the same size bubble. Jihi and Clark [2] have reported bubbles larger than Gunther’s for pressures as high as 500 psi. One might expect their bubbles to show less of a tendency to round and become spherical than Gunther’s atmospheric bubbles. Bubble shapes were not easily observed from the photographs reported by Jihi and Clark.

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Additional References
