rises gradually from $T_d\,(=T_{in})$, and then steeply near the pad surface.

As discussed so far, this 3-D analysis can investigate the bearing performance more extensively and more precisely than is possible with the 2-D analysis, and the results obtained with the former often differ from those obtained with the latter both qualitatively and quantitatively. From the view point of safe operation of bearings, the 2-D analysis should not be adopted because it tends to underestimate maximum temperature rise and minimum film thickness decrease, even though it has the advantage of greatly shortening the calculation time.

Conclusion

The authors have analyzed the thermohydrodynamic performance of sector-shaped thrust pad bearing which can tilt both radially and circumferentially, taking into account the three-dimensional variation of lubricant viscosity and density. The temperature distribution in the pad and film were also investigated. The effect of pivot position, disk speed, minimum film thickness, inlet temperature of lubricant, ambient temperature and heat transfer coefficient were elucidated. The results were considerably different from those obtained when the isoviscous or the two-dimensional THD analysis is used. It was shown that the two-dimensional analysis can not substitute for the three-dimensioal analysis. The present theory predicts the bearing performance more accurately and is considered to be useful for design purpose.

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DISCUSSION

I. Etsion[1]

The authors presented an interesting 3D THD analysis of sector-shaped tilting pad thrust bearings. They showed optimum pivot location (Fig. 3(*a*)) for maximum load capacity and compared it with 2D THD and isoviscous analyses showing considerable differences between the results of the three analyses.

In reference [7] an isoviscous analysis of arbitrarily pivoted sector pad thrust bearings was performed taking into account both pitch and roll of the pad. The analysis in [7] did not require any iteration for moment balance convergence and was very efficient for optimization of pivot location. It was found that maximum load capacity is obtained whenever the pad is tilted so that a uniform minimum film thickness is maintained along its trailing edge ($\alpha_v = 0$ in the present analysis). For a bearing of inner to outer radius ratio of 0.7 and an angular extent $\theta_0 = 30$ deg the optimum pivot location in [7] is $\bar{r}_p = 0.51$ and $\bar{\theta}_p = 0.39$. The dimensionless load capacity (using the definition of $\bar{w}$ in the present paper) corresponding to this case is $\bar{w} = 4.16 \times 10^{-2}$. For a bearing of radius ratio 0.675 and $\theta_0 = 22.5$ deg which corresponds to Fig. 3(*a*) the isoviscous analysis of [7] will result in $\bar{w}$ even smaller than 4.16×10^{-2}. This is much closer to the 3D THD result than the isoviscous analysis shown in Fig. 3(*a*). Hence it seems that the present 3D THD analysis produces maximum load capacity results that do not differ much from those obtained by the much simpler isoviscous analysis of [A1]. The accuracy of the results in [A1] was checked against other analyses (see discussion of [A1]) and the agreement was found excellent. Thus the authors' conclusion regarding the differences between their 3D THD analysis and the isoviscous analysis (based on Fig. 3(*a*)) seems questionable.

Additional References

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C. M. McC. Ettles[2]

The authors make an interesting comparison of two and three dimensional solutions of thrust bearing films. It is clear from their work that a three dimensional solution will yield much more complete and reliable information, particularly for heavily loaded high speed bearings. In addition to the previous work by Rhode, Tieu and others quoted in the paper, the authors may not be aware of three dimensional solutions by Huebner [8] and, very recently El Saie [9]. In [9] heat conduction into the rotor is also considered, together with thermo-elastic deflection of the shoes for any arbitrary support system. A further three dimensional analysis is mentioned in [10].

One of the most interesting results given by the authors is the sensitivity of the radial pivot position $\bar{r}_p$ on shoe performance. Can the authors suggest a simple method for calculating an optimum value of $\bar{r}_p$?

The development of a three dimensional analysis requires great attention to detail, for example convection effects at any given node in a three dimensional grid should apparently be considered in three coordinate directions. This seems to be particularly necessary in the inlet region where, if the film convergence is greater than about 2.5:1, local reverse flow can occur. This results in (imaginary) stream lines being predominantly vertical at some nodes, where the vertical

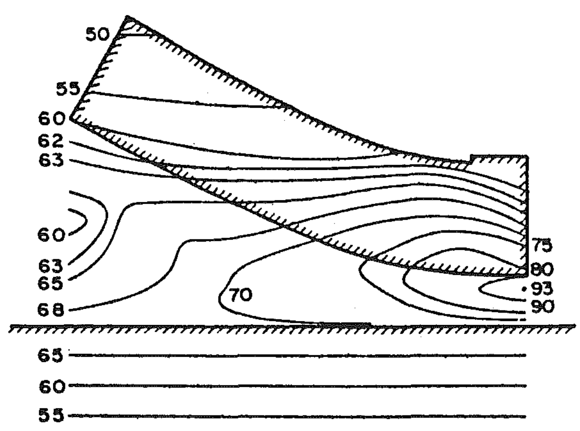

Fig. 10 Example velocity distribution showing reverse flow

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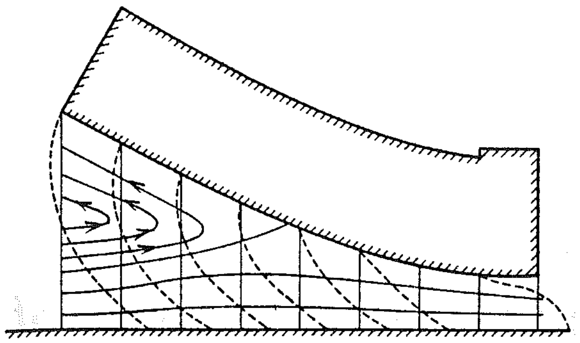

Fig. 11 Temperature distribution through film and components

velocity $\bar{v}_z$ is dominant. Could the authors give some details as to how this component velocity is eliminated from the energy equation?

The authors mention that central differences are used except for a backward difference for $\partial T/\partial \Theta$. Is the temperature solution treated as an initial value (propagation) problem, with the temperature field being obtained in a single pass? Did the authors find that this was affected by local reverse flow? Figures 10 and 11 (discussion) taken from [11] show a two dimensional solution viewing the film in elevation. The particular case shown is for a convergence of about 3:1 where reverse flow does occur.

The question of how to model the inlet temperature (hot oil carry over) is most difficult. In their studies the authors have assumed the inlet profile to be uniform and equal to the rotor surface temperature and to the leading edge temperature of the shoe. Vohr [12] has recently made a substantial contribution to hot oil carry over theory which may be useful in further work.

In Fig. 8 of the paper a wide range of surface heat transfer coefficients is used to allow for heat transfer from the free surfaces of the pad. Kaiwaicke [13] has shown experimentally that the thermal distortion of shoes is crucial to the performance of large bearings. Have the authors obtained any insight as to appropriate values of surface heat transfer coefficient?

Additional References

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Authors' Closure

The authors would like to thank Dr. Etsion for his comments and interest in this paper.

The authors agree that $\bar{W}$ (using the notation in this paper) corresponding to the maximum W/KA for $\theta_p/\beta = 1$ (the authors' case), $R_i/R_o = 0.7$ and $\beta = 30$ deg in Fig. 5(*b*) [7] is 4.16×10^{-2} as the discusser stated. He also stated that $\bar{W}$ would become smaller for the smaller values of R_i/R_o and β, that is, 0.675 and 22.5 deg respectively (the authors' case). However, the authors would like to point out the contrary by using the discusser's results [7].

Table 1 [7] shows the value 0.312×10^{-2} of W/KA at $h_1/h_2 = 3$ for the smaller $R_i/R_o = 2/3$ with the same θ_p/β and β. The corresponding $\bar{W}$ is 5.23×10^{-2}, which means that $\bar{W}$ increases with the decrease of R_i/R_o, contrary to the discusser's comment. Incidentally, $\bar{W} = 5.23 \times 10^{-2}$ is nearly equal to the result of the authors' isoviscous analysis in Fig. 3(*a*).

The broken lines in Fig. 2[7] show W/KA for various β and constant $R_i/R_o = 0.7$ which is nearly equal to the authors' ratio 0.675. One can see that W/KA increases with the decrease of β from 90 to 30 deg. Therefore, it would be reasonable to assume that $\bar{W}$ increases too with the further decrease of β from 30 to 22.5 deg (the authors' case).

It seems to the authors that the discusser's comment on the isoviscous analysis is contrary to his own results.

The authors appreciate Professor Ettles' valuable discussion in which he asked four questions.

The first one is on the possibility of suggesting a simple method for calculating an optimum value of $\bar{r}_p$. It would be always difficult to propose a simple method for the three dimensional THD problem, but as long as the pad configuration is ordinary, the optimum value of $\bar{r}_p$ would not go too far from 0.51.

The process of eliminating $\bar{v}_z$ in the energy equation is as follows. Integrating the dimensionless equation of continuity with respect to $\bar{z}$ between the limits 0 and $\bar{z}$, $\bar{v}_z$ can be obtained in the following expression

$$\frac{R}{h_0\bar{h}}\bar{v}_z = \frac{\bar{z}}{\bar{h}}\left(\frac{R}{\Delta R}\bar{v}_r\frac{\partial \bar{h}}{\partial \bar{r}} + \frac{\bar{v}_\theta}{\theta_0}\frac{\partial \bar{h}}{\partial \bar{\theta}}\right) - \frac{R^2}{\bar{\rho}\bar{h}(R+\Delta R\bar{r})\Delta R}\frac{\partial}{\partial \bar{r}}\left\{\bar{h}\frac{(R+\Delta R\bar{r})}{R}\int_0^{\bar{z}}\bar{\rho}\bar{v}_r d\bar{z}\right\} - \frac{1}{\bar{\rho}\theta_0\bar{h}}\frac{\partial}{\partial \bar{\theta}}\left(\bar{h}\int_0^{\bar{z}}\bar{\rho}\bar{v}_\theta d\bar{z}\right)$$

which is substituted for every $\bar{v}_z$ term in the energy equation.

The authors can not give a clear answer to the third question about the effect of local reverse flow on the calculation of temperature distribution in the film. No peculiar phenomena were experienced in the calculation. The calculated cases by the authors may not have resulted in such great attitude angles, or in the three dimensional case the radial flow of lubricant would suppress the local reverse flow to appear until the attitude angle becomes much greater than for the two dimensional case Professor Ettles described.

With regard to the last question about the heat transfer coefficient on the free surfaces of the pad, the authors at present can not give a definite answer. The coefficient would vary greatly, depending on the type and configuration of the bearing, but the order of its magnitude usually ranges from 1 to 10 kilowatt over square meter degree.