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Interpolation of Daily Temperature in Finland

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The present approach for daily temperature interpolation of the Watershed Simulation and Forecasting System of the Finnish Environment Institute is based on the inverse distance weighted interpolation. In order to improve the calculation, three alternative methods were tested: 1) modified inverse distance weighted model, 2) regression with dummy variables for taking into account time and 3) regression equation calibrated for each day. The regression model calibrated for each day proved to be the most promising model. By average, the difference between the accuracy of it and the inverse distance weighted methods wasn't big but some indication was found that in single cases it can make a difference. The estimated parameters were found to have realistic physical meanings.

Introduction

The Watershed Simulation and Forecasting System (WSFS) (Huttunen and Vehviläinen 2001) of the Finnish Environmental Institute covers the whole area of Finland and the border watersheds. The model system is driven by the spatially distributed values of hydro-meteorological observations. The initiative of this work comes from the need to improve the process descriptions and to search for more accurate methods to interpolate and to calculate areal values, particularly for temperature and precipitation, and to assess the possible increased forecast precision.

This study represents the tested temperature interpolation methods and results with comparisons to the ones of the present method. The original idea was to test the spatial interpolation algorithms in two stages. First, a trend surface was to be fitted to the empirical daily temperature field using explanatory parametric and semi-parametric methods, constructed of several explanatory variables (latitude, longitude, elevation, distance from the sea and relative area of lakes). Secondly, a spatial error component was to be fitted by linear interpolation methods, for instance by different kriging methods (*e.g.* Deutsch and Journel 1998). Also the effect of space-time interaction was to be considered. However, when the trend surface was fitted by a regression model it was found out that the resulting errors were distributed with no spatial correlation between different observation stations obviating the second stage. This is probably due to the relatively sparse station network since Henttonen (1991) was able to derive a spatial correlation structure for the errors with a significantly denser network using the ordinary kriging method. It is also possible that the daily calibration abolishes the correlation and it could be found using a longer time step, *e.g.* one month. Due to the above-mentioned reason, in this study two different model types, regression models and inverse distance weighted (IDW) models, were tested without any spatial model for the residuals.

Data

The data used in this study was provided by the Finnish Meteorological Institute. The temperature data set of 1961-2002 used in the interpolation consists of 81 stations in different parts of Finland (see Fig. 1). The geographical information, *i.e.* the distance of the stations from the sea and the relative amount of lake area around the station, was derived from the database of the Finnish Environment Institute.

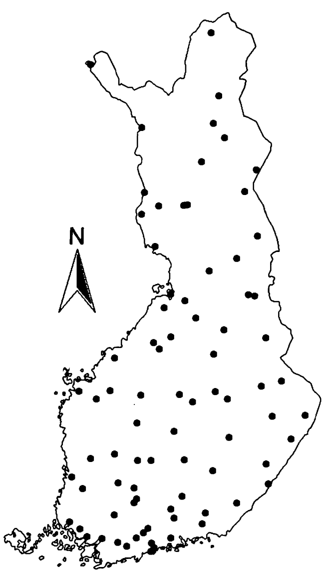


Fig. 1. Temperature observation stations in 1961-2002.

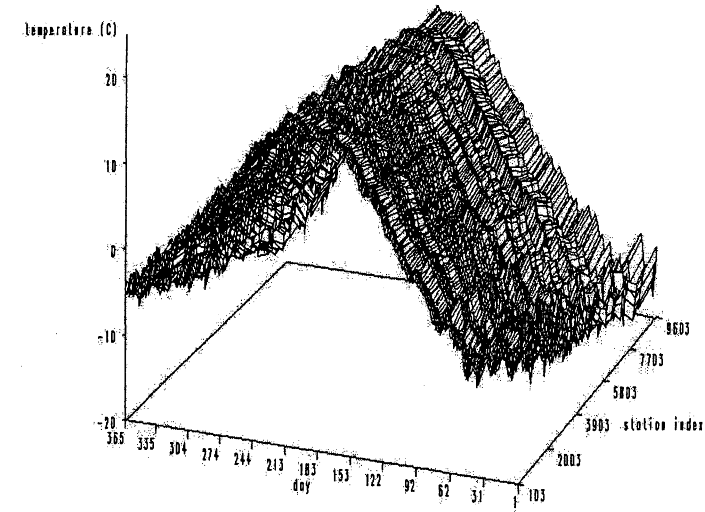


Fig. 2. Mean daily temperatures in 1961-2002.

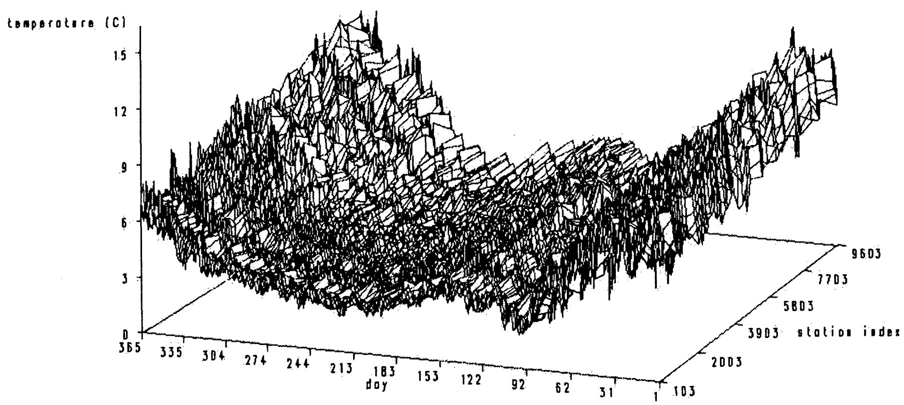


Fig. 3. Standard deviations of daily temperatures in 1961-2002.

Fig. 2 shows the mean daily temperature at each observation station. The mean temperature is calculated for each day (axis 1) over the years and the station index (axis 2) increases from south to north and from west to east. It is easy to see periodicity within a year at each station. Moreover, during the summer the temperature is quite constant through the whole country whereas at the beginning of the year the southern Finland is warmer than the northern part of the country by average. This is due both to the warming effect of the sea in southern and western Finland and to the northern Finland belonging to the influence of the polar air mass.

The standard deviation of the daily temperature at each station is represented in Fig. 3. Also this figure shows that during summer the variation in temperature is

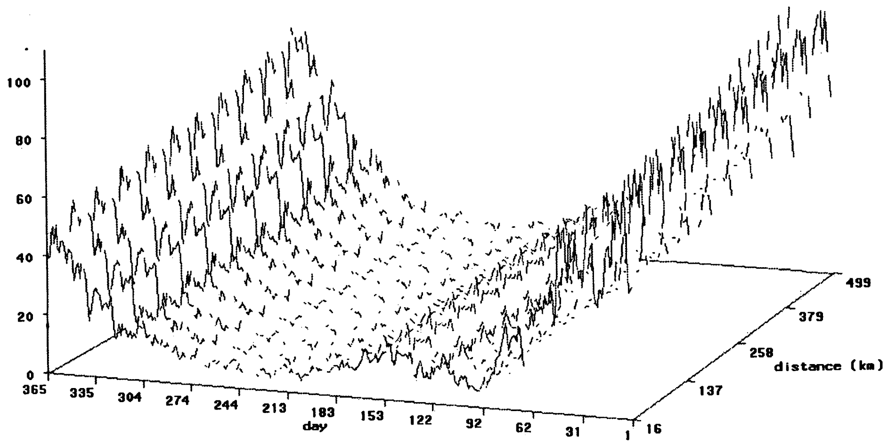


Fig. 4. Semivariogram of daily temperatures (vertical axis in (°C)2) in 1961-2002.

very small by average in different parts of Finland. During winter the temperature varies more in northern Finland obviously due to the balancing effect of the sea in southern Finland.

Spatial correlation of the temperature observations T can be characterised by semivariogram. Empirical semivariogram $\gamma_2(h)$ is calculated as

$$2\gamma_2(h) = \frac{1}{|N(h)|}\sum_{N(h)}(T(r_i) - T(r_j))^2 \tag{1}$$

where $N(h)$ is given for a range of pairwise distances by

$$N(h+\delta h) = \{i,j : |r_i - r_j| \in (h-\delta h, h+\delta h)\} \tag{2}$$

and $|N(h)|$ is the number of such pairs (i,j).

Fig. 4 represents the semivariogram of the daily temperature measurements during 1961-2002. Again it is obvious that Finland is homogeneous during spring and summer (days 100-280) in terms of the daily temperature. At the beginning and end of the year, there is noticeable variation in temperature in different parts of the country.

Also preliminary time series analysis was done by fitting different kinds of time series models to the data. Because no satisfactory solution was found in reducing the periodicity of the data, probably due to high heteroscedasticity in space and time, the effect of time was taken into account by other means in the tested models.

Methods

The present model in WSFS for the temperature interpolation is the IDW interpolation. The following alternative models were tested and compared to the present one:

1) modified IDW model that includes the distance from the sea and the relative amount of lake area as explanatory variables (Uusitalo 2002), 2) regression with dummy variables for taking into account time and 3) regression equation calibrated for each day.

The explanatory variables in the models were selected based on their potential to explain physical phenomena or properties which the temperature depends on, availability for each station and interpolation point and practicality for operational watershed modelling. Table 1 shows a summary of these factors.

Table 1 – Basis for selected explanatory variables in interpolation.

variable	physical phenomenon or property
x coordinate	continental/coastal air mass
y coordinate	temperate/polar zone
z coordinate (elevation)	adiabatic lapse rate
relative amount of lake area	temperature difference between land and lakes when no ice cover
distance from sea	temperature difference between land and sea when no ice cover

Inverse Distance Weighted Models

IDW interpolation is a well-known method in hydrology (*e.g.*. Tabios and Salas 1985; Dirks *et al.* 1998; Johansson 2000a). The basic formula of IDW can be expressed as

$$T_j = \sum_{i=1}^{n} w_{ij} T_i / \sum_{i=1}^{n} w_{ij}, \qquad (3)$$

$$w_{ij} = f(d_{ij}) = \frac{1}{d_{ij}^{b}} \qquad (4)$$

where: $T_{i,j}$ = temperature, i, j = station indexes, n = number of stations, w_{ij} = weight, d_{ij} = distance between stations i and j, b = exponent

WSFS uses $b = 2$, *i.e.* inverse distance squared weighting, and takes into account the three nearest stations. It also accounts for the elevation of the stations by reducing temperature about 0.7°C for each 100 m increase in elevation (Johansson 2000b). As a matter of fact, the reduction is an estimated parameter in WSFS but in this study it was fixed to 0.7°C.

The model 1 differs from the present model in *d*. In this case it is written as

$$d_{ij} = \sqrt{(x_i-x_j)^2+(y_i-y_j)^2+(z_i-z_j)^2+q_s(s_i-s_j)^2+q_l(l_i-l_j)^2} \qquad (5)$$

where: x, y = geographical location, z = elevation above sea-level, s = distance from sea, l = relative amount of lake area around x, y within 50 km, q_s, q_l = empirical parameters.

Regression Models

A regression model is commonly used in spatial interpolation in both hydrology (*e.g.* Hosang and Dettwiler 1991; Henttonen 1991; Wotling *et al.* 2000) and meteorology (Venäläinen and Heikinheimo 1996; Ninyerola *et al.* 2000).

The regression with dummy variables (model 2) can be written as

$$T(v,x,y,z,s,l,m) = cv + \sum_{m=1}^{365} b_m t_m + a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 xy + a_6 z + a_7 s + a_8 l + \varepsilon \qquad (6)$$

where: $c, a_0 - a_8$ = empirical parameters, b_m = empirical dummy variable parameter, v = year (= 1961-1989 for calibration and 1990-2002 for validation), t_m = dummy variable for day, m = index of day (= 1-365), ε = identically and independently distributed random variable

The dummy variables take values of either 0 or 1 depending on m. The explanatory variables v, x, y, z and s were (0, 1) standardised and l was transformed by taking the square root and arcsin, *i.e.* l_{tr} = arcsin $(\sqrt{l})$.

The purpose of testing the model 2 was to find out if only one equation can be used for each day through the years. The variation in time is accounted for so that the first term describes a long-term linear trend with an increase or decrease of magnitude c per year. The seasonal variation within the years is taken into account by the dummy variables in the second term.

The basic regression model, model 3, accounts for time so that it is calibrated for each day. It can be expressed as the model 2 but without the first two factors on the right hand side of Eq.(6). The variables were transformed in the same way.

Results

The interpolation models were calibrated using the least squares method and the data set of 1961-1989. Validation was carried out with the data set of 1990-2002. Because of different model types, the idea of these experiments is first explained in detail. In the case of the present model, there was no difference between calibration and validation. The parameters q_s and q_l of the model 1 were estimated for each day over the years 1961-1989 so that they were the same for each station, thus resulting 365 values for both of them. Then the modified distance was calculated and interpolation carried out through 1961-2002. The parameters of the model 2 were also estimated using the 1961-1989 data and thereafter the 1990-2002 data was used running the model with these estimates. The daily regression (model 3) was calibrated using the measurements of each station in 1961-1989 and then cross-validated by excluding each station in turn in 1990-2002.

Overall Results with Accuracy Measure Comparisons

The measures of accuracy used in comparison of the models are represented in Table 2. Accuracy measures MAD, RMSE and MAPE all reflect more or less the same thing, *i.e.* the typical error of the model for given data. RMSE is more sensitive to large errors than MAD due to squaring. MAPE is a useful measure because it relates the error to the observed value. The median measures MEAD and MEAPE should be compared to the corresponding mean measures MAD and MAPE, respectively. If

Table 2 – Measures of accuracy used in assessment of models (Armstrong 1985 and Nash and Sutcliffe 1970). Variables obs and pred stand for observed and predicted value, respectively. mean(), median() and sum() are mathematical operators indicating taking mean, median (50% fractile) and summing variables, respectively.

measure of accuracy	abbreviation	formula
mean absolute deviation	MAD	$mean(obs - pred)$
median absolute deviation	MEAD	$median(obs - pred)$
root mean square error	RMSE	$\sqrt{mean((obs - pred)^2)}$
mean absolute percentage error	MAPE	$\left(mean\left(\frac{\lvert obs - pred\rvert}{obs}\right)\right)100$
median absolute percentage error	MEAPE	$\left(median\left(\frac{\lvert obs - pred\rvert}{obs}\right)\right)100$
coefficient of fitness	FIT	$\frac{sum((obs - mean(obs))^2) - sum((obs - pred)^2)}{sum((obs - mean(obs))^2)}$

Table 3 – Model accuracy statistics in the calibration period (1961-1989).

model	MAD	MEAD	RMSE	MAPE	MEAPE	FIT
present	0.73	0.47	1.17	–0.20	1.60	0.99
model 1	0.80	0.54	1.23	0.11	1.80	0.99
model 2	3.80	2.97	5.05	10.62	9.08	0.78
model 3	0.65	0.45	0.96	–0.11	1.54	0.99

the predicted values are unbiasedly distributed around the observed value, MEAD is close to MAD and MEAPE is close to MAPE. FIT is probably the most common measure for models accuracy in hydrology. It describes the proportion of the variation of the data that a model is able to take into account. The highest value for FIT is 1.

Table 3 shows that the present model and the models 1 and 3 are almost equally accurate by any measure and the model 2 is clearly the most inaccurate. The main difference between the models 2 and 3 is the way they handle the time dimension; the model 2 has dummy variables for days and the year is taken into account as a continuous explanatory variable as the model 3 is calibrated for each day without any time related explanatory variables. It is obvious that the way the model 2 accounts for time is not effective enough even though the estimate of c (positive) and nearly all the estimates of b_m were statistically significant in the 1% risk level.

All the models show quite similar relative sensitivity to large errors explained by the comparison between RMSE and MAD. MAPE values are very low and FIT values close to 1 for all the models except model 2. MAD and MEAD of all the models differ less than 1 degree reflecting reasonably symmetric error distribution. Also the percentage values of MAPE and MEAPE confirm this conclusion. However, when MEAD of each model is smaller than MAD, MEAPE is bigger than MAPE except for model 2.

A parameter identification problem was noticed in the calibration of model 1. Parameters q_s and q_l were estimated for each day over 29 years and the SIMPLEX method (O'Neill 1971) was sometimes unable to get out of local minima. The procedure was also found to be quite sensitive to the initial values. Moreover, the last three terms under the square root on the right hand side of Eq.(5) were not significant but almost equal results were obtained by neglecting them.

Table 4 – Model accuracy statistics in the validation period (1990-2002).

model	MAD	MEAD	RMSE	MAPE	MEAPE	FIT
present	0.70	0.45	1.11	0.14	1.93	0.99
model 1	0.79	0.55	1.21	0.32	2.24	0.98
model 2	3.86	3.10	4.98	13.42	11.02	0.74
model 3	0.76	0.51	1.18	–0.10	2.12	0.99

The same measures are represented for validation in Table 4. The main interest between calibration and validation is how the performance of the models 1 and 2 changes. By comparing the accuracy measures, it can be said that the calibration was successful and/or dependency structure between the independent and dependent variables is similar in these two data sets. The present model and model 3 can be compared in pair due to the corresponding experiments.

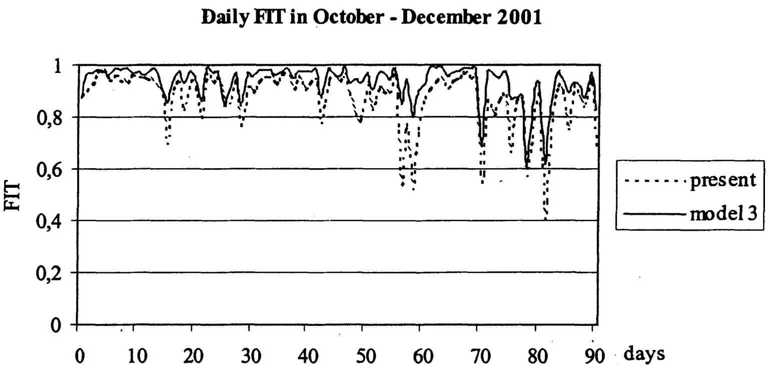


Fig. 5. Daily FIT at the beginning of the snow cover period (October – December) for the present model and model 3 in 2001.

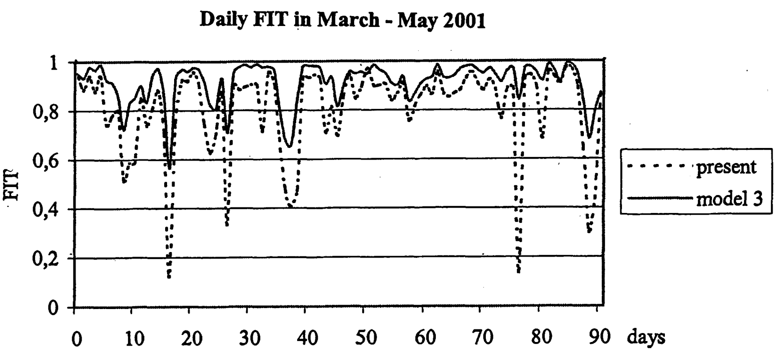


Fig. 6. Daily FIT in a typical snow melting period (March – May) for the present model and model 3 in 2001.

Table 5 – Model accuracy statistics for the temperature interval –5- +5°C in 1961-2002.

model	MAD	MEAD	RMSE	MAPE	MEAPE	FIT
present	0.53	0.36	0.80	0.04	1.88	0.91
model 3	0.50	0.37	0.71	–0.06	1.86	0.93

Comparing Present Model and Model 3

Because the accuracy measures shown in Tables 3 and 4 are averages of a long period, it is difficult to see major differences between different models, particularly within a short period of time. In the data description it was shown that during summer Finland is homogeneous in terms of temperature and the models can be expected to produce similar results. However, during the winter the spatial variation can be significant hence providing a harder test for the models. Especially of great concern to WSFS are periods when the temperature is close to zero and defining the form of precipitation based on the temperature becomes more crucial. Also the periods when the snow cover starts to form in autumn (October-December) and when it melts in spring (March – May) are highly important.

In order to study these effects concentrating on comparing the present model and the best alternative model, model 3, the accuracy measure FIT was calculated for temperatures –5- +5 °C only during 1961-2002. From the results in Table 5 it can be concluded again that there are no major differences between the models. Model 3 is slightly better in terms of MAD, RMSE and FIT.

Next, FIT of the present model and model 3 were drawn for October-December 2001 reflecting the snow cover starting period (Fig. 5) and for March-May 2001 representing the snow melting period (Fig. 6). It is easy to see the model 3 being more accurate in these cases.

Further Assessment of Model 3

Because the variables of the model 3 were standardised, their parameter estimates are a measure of the significance of the variable. Fig. 7 a-e shows the mean daily values of the parameter estimates. In Fig. 7a, the estimate of the intercept term a_0 is basically the mean daily temperature of Finland. A realistic yearly cycle of the mean temperature can be seen. Fig. 7b shows that the quadratic x and y don't explain virtually anything about the spatial variation of the temperature at any time of the year

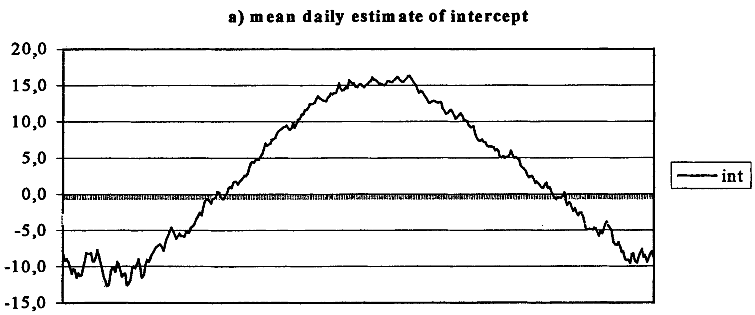
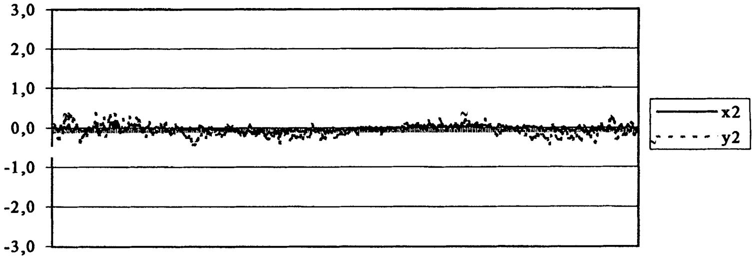
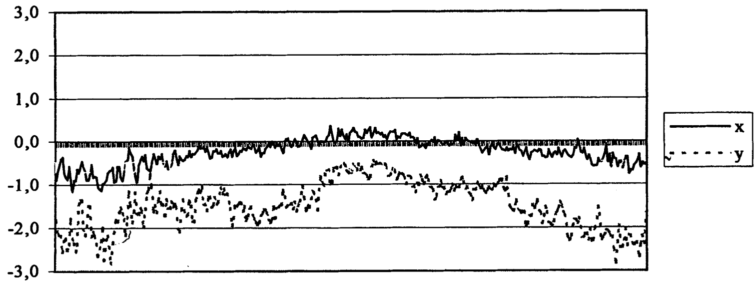


Fig. 7 a-e. Mean parameter estimates of model 3.

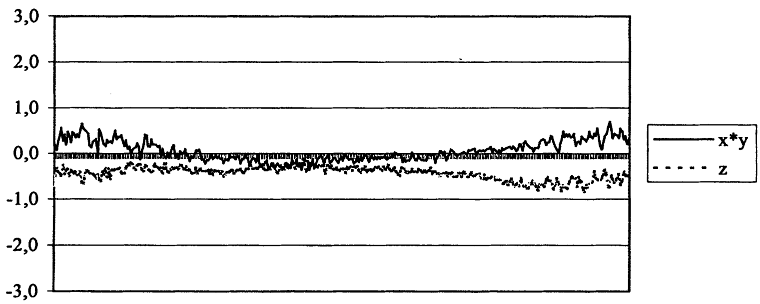
b) mean daily parameter estimates of x^2 and y^2



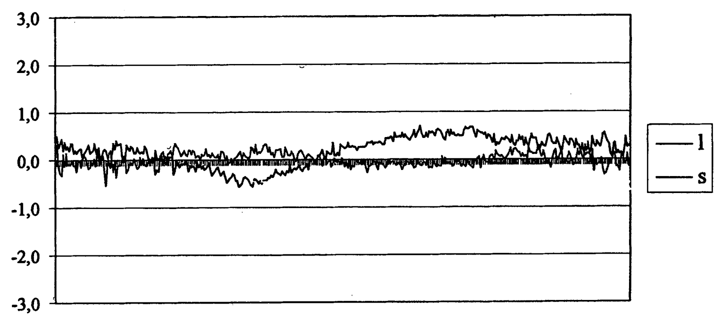
c) mean daily parameter estimates of x and y



d) mean daily parameter estimates of x*y and z



e) mean daily parameter estimates of l and s



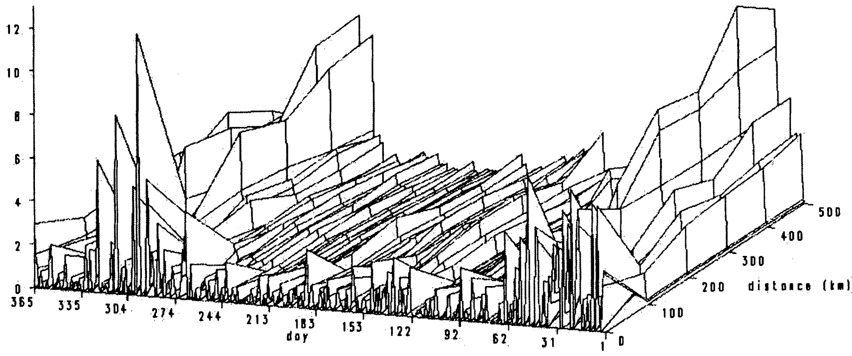


Fig. 8. Semivariogram of model 3 residuals (vertical axis in (°C)2) in 1992.

but their first-order forms reveal an obvious linear relationship between these variables and temperature (Fig. 7c). Especially, the longitude co-ordinate y is of remarkable significance telling that Finland gets colder towards the north at the beginning and end of the year. The distance from the sea s is not significant but the other variables in Fig. 7 d and e have some importance, in particular, the relative amount of lake area l; the more there are surrounding lakes, the colder it is in spring and warmer in autumn.

Finally, Fig. 8 represents the daily semivariograms of the model 3 validation residuals for 1992 as a typical example. It can be seen that there is no obvious spatial correlation structure to be modelled by theoretical variograms. Therefore, there was no basis for using kriging methods in interpolation.

Conclusions

In the comparison of three different alternative daily temperature interpolation methods, the regression model calibrated for each day (model 3) proved to be the most promising candidate to replace the present IDW based model. By average, the difference between the accuracy of the present method and the models 1 and 3 wasn't big but some indication was found out that in single cases of short time periods the model 3 is the most accurate. Moreover, the regression model analysis is useful in describing the data and the information related to the variables can be utilised. In this case, the estimated parameters had realistic physical meanings. It is a matter of future research to implement the model 3 into WSFS and to test if any improvement in the predictions can be gained.

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