Copula-based depth-duration-frequency analysis of typhoons in Taiwan

Jenq-Tzong Shiau, Hsin-Yi Wang and Chang-Tai Tsai

ABSTRACT

Typhoons are an inevitable and frequently occurring natural hazard in Taiwan which cause severe economic damage and loss of life. The common practice for flood-mitigation planning and design uses univariate frequency analysis. However, separate univariate analysis cannot reveal the significant relationship among correlated variables. This study therefore employs copulas to construct the joint distribution of rainfall depth and duration for typhoon data. Using copulas to construct a multivariate distribution means that the effects of marginal variables can be separated from that of dependent variables. We derive the depth-duration-frequency (DDF) formula based on using copulas to represent the joint distribution of rainfall depth and duration. Typhoon data recorded at the Kaohsiung Weather Station located in southern Taiwan are used as an example to illustrate the proposed methodology. The marginal distributions for rainfall depth and duration are fitted as the three-parameter gamma and Gumbel distributions, respectively. The Plackett copula is selected to construct the DDF curves. The DDF allows rainfall depth for a specific rainfall duration and return period to be estimated. This DDF formula improves the understanding of complex hydrologic processes and enhances the design safety criterion of hydraulic structures.

Key words | bivariate distribution, copula, depth-duration-frequency, frequency analysis, joint probability, return period

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INTRODUCTION

Typhoons are an inevitable and frequently-occurring natural hazard in Taiwan. Each year, flooding and mudslides induced by high-intensity rainfalls cause disastrous economic damage and loss of lives. The increasing extreme hydrological events due to global climate change have recently received global attention. According to statistics, the average number of typhoons hitting Taiwan per year, as well as the total rainfall during typhoon periods, have increased. For example, the mean number of typhoons over a 25-year period has increased from 4 (1958–1983) to 6 (1984–2009) according to the Central Weather Bureau (CWB) typhoon database (Central Weather Bureau 2009).

Typhoon Herb in 1996 and Typhoon Morakot in 2009 had record-breaking rainfalls of 1,994 mm in two days and 2,884 mm in three days, recorded at Alishan Weather Station by the Central Weather Bureau. Moreover, the maximum 48-hour rainfall of 2,361 mm for Typhoon Morakot is almost equivalent to the current world record of 2,467 mm, measured at Aurère of the east coast of Madagascar (WMO World Weather and Climate Archive, http://wmo.asu.edu). Since Taiwan suffers from severe typhoons, accurate analysis of rainfall data induced by typhoons is an important issue in the design and planning of flood-mitigation engineering.

Rainfall depth, intensity, duration and associated frequency are the essential variables in hydrological design. Intensity-duration-frequency (IDF) curves or depth-duration-frequency (DDF) curves of rainfall data are often

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used for urban drainage problems. Bernard (1932) established rainfall intensity formulae for long-duration rainfalls. Chow *et al.* (1988) used probability distribution functions to calculate return periods of storms to build DDF curves. Burlando & Rosso (1996) derived general distribution-free DDF curves based on either scaling or multi-scaling conjectures with log-normal models.

The traditional way to construct IDF or DDF is the graph-type approach. That is, recorded rainfall data for various intensity (or depth), duration and frequency (in terms of return period) are plotted in a graph. Some empirical equations are developed to represent IDF or DDF to avoid reading the required rainfall information from a graph. For example, Di Baldassarre *et al.* (2006) used DDF curves to estimate rainfall durations from 5 to 45 minutes and Muller *et al.* (2008) used the bivariate extreme logistic distribution to build DDF curves. A theoretical derivation of the IDF or DDF must build on the joint distribution functions of rainfall intensity (or depth), duration and frequency. Since these variables are correlated and may have different distributions, limited multivariate models are available to simultaneously construct such relationships. This is the reason that few studies are devoted to establish IDF or DDF equations theoretically.

A copula is a function that links univariate marginals to construct the multivariate distribution. Since copulas separate the effects of dependence among correlated variables from the effects of their marginals, the difficulties in forming multivariate distributions with different types of marginals are alleviated. Developed by Sklar (1959), copulas appear a flexible tool to construct multivariate distribution and have been widely used to investigate insurance claims and financial losses (Frees & Valdez 1998; Hürlimann 2004).

In an effort to model relationships among correlated variables in complex hydrological processes, copulas have increasingly received attention for hydrological studies such as frequency analysis of rainfalls, floods (Shiau *et al.* 2006), droughts (Shiau 2006; Shiau *et al.* 2007; Shiau & Modarres 2009) and sea storms (De Michele *et al.* 2007). For example, De Michele & Salvadori (2003) used a bivariate copula to describe the dependence between rainfall intensity and duration. Kao & Govindaraju (2007) determined the dependence between average rainfall intensity and duration to obtain the probability of surface runoff volumes. Serinaldi (2009) used copula-based mixed models for bivariate rainfall data. Grimaldi & Serinaldi (2006) constructed a trivariate stochastic rainfall model from Archimedean copulas, assuming the same dependence level for each pair of selected variables. Salvadori & De Michele (2006) performed a trivariate rainfall frequency analysis using the Frank copula with generalized Pareto distributions using storm intensity and duration as well as the dry periods. Kao & Govindaraju (2008) modelled hourly data from extreme rainfall events by a trivariate Plackett family of copulas. Favre *et al.* (2004) determined a conditional joint return period using copulas and the joint probability distribution of peak flow and volume. Zhang & Singh (2007) applied the Gumbel-Hougaard family of Archimedean copulas to calculate the conditional return period. Singh & Zhang (2007) used the conditional Frank copula to derive an IDF in Louisiana and compared this to the univariate frequency analysis by the US National Weather Service. Shiau (2006) and Shiau *et al.* (2007) used copulas to analyze the drought frequency of various locations.

This study aims to derive the depth-duration-frequency (DDF) curves of typhoon rainfalls in Taiwan using copulas. The joint distribution of rainfall depth and duration is first constructed by copulas. A conditional copula is then employed to derive the DDF. The typhoon data from Kaohsiung Weather Station located in southern Taiwan from the period of 1960–2006 were used as an example to illustrate the proposed methodology. Associated properties from the derived DDF are also explored in this study.

STUDY REGION AND DATA

Study region

Taiwan lies approximately between 120–122°E and 21.8–25.3°N. With an area of 36,000 km^2 it extends nearly 400 km from north to south and around 145 km from east to west. There are more than 200 peaks higher than 3,000 m in the island. Since steep mountains over 1,000 m constitute 31% of Taiwan, rivers are generally short and steep. Mean annual rainfall in Taiwan is approximately 2,500 mm.

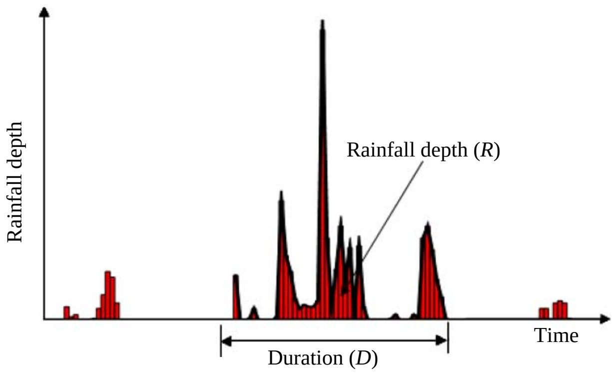

Figure 1 | Definition sketch of a typhoon event.

However, uneven spatial and temporal distributions are observed. Nearly 90% of annual rain falls within the period of May–October in southern Taiwan.

Typhoon data

The typhoon events used in this study were selected from the Central Weather Bureau (CWB) typhoon database. The data selection criteria is 'heavy rain', which is defined as a cumulative rainfall depth exceeding 50 mm during a 24-hr period and 1-hr rainfall intensity exceeding 15 mm/hr. For the period of 1960–2006, 77 typhoon events were labelled as 'heavy rain' and abstracted from records of the Kaohsiung Weather Station, located in southern Taiwan. A graph of a typical typhoon event is illustrated in Figure 1. Two important characteristics, rainfall depth and rainfall duration, are abstracted from each typhoon event and employed to construct DDF. Rainfall duration is the time period of continuous rainfall, where the rainfall depth represents the cumulative rainfall depth within the rainfall duration. The annual maximum rainfall depth and duration for the typhoon events show similar trends from 1960 to 2009, as shown in Figure 2, and it can be seen that the maximum rainfall depths over 500 mm occur more often from 2001 to 2009.

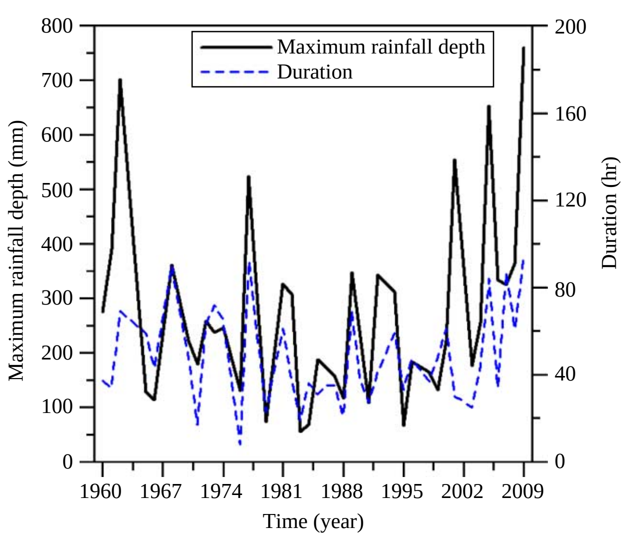

Figure 2 | The trend of maximum rainfall depth and duration for typhoon events.

METHODOLOGY

DDF relationship of typhoon events

In hydrological frequency analysis, the return period of the partial duration series can be defined as the average inter-arrival or inter-hydrological event time (McCuen 2005). The return period is related to the cumulative probability as

$$T = \frac{1}{\lambda P(X > x)} = \frac{1}{\lambda(1 - P(X < x))} = \frac{1}{\lambda(1 - F(x))} \tag{1}$$

where T is the return period in years; λ is the arrival rate of events per year, which is estimated as the ratio of total typhoon events to the total years and is determined by the peak over threshold method (see Figure 3); and $F(x)$ is the cumulative distribution function.

The above relation can be extended to include two variables using the conditional distribution. Thus, the relationships among the depth, duration and frequency (in terms of return period) for typhoon events can be represented by the conditional return period, which is given by

$$T_{R|D}(r|d) = \frac{1}{\lambda[1 - F_{R|D}(r|d)]} \tag{2}$$

where R and D denote rainfall depth and duration, respectively; $F_{R|D}(r|d)$ is the conditional cumulative distribution function (CCDF) of R given $D = d$, i.e. $F_{R|D}(r|d) = \Pr[R \le r|D = d]$; and $T_{R|D}$ is the return period of R given D.

The conditional distribution function $F_{R|D}(r|d)$ can be expressed as a function of the joint cumulative distribution function (JCDF) $F_{R,D}(r,d)$ and the cumulative distribution

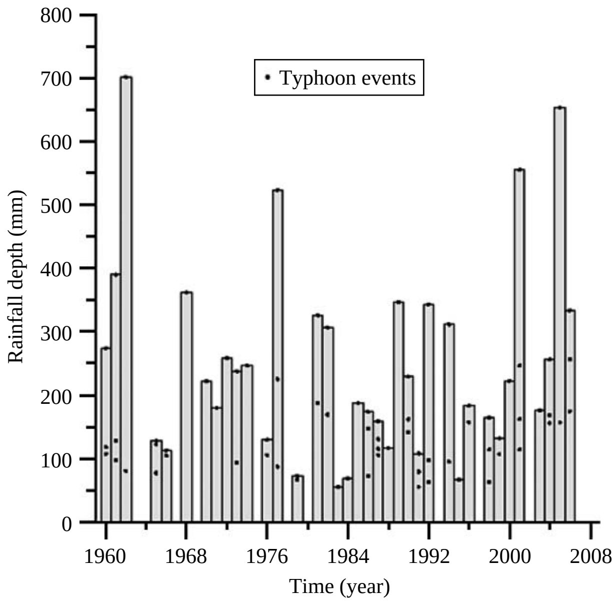

Figure 3 | Typhoon events of Kaohsiung Weather Station in 1960–2006 selected using peak over threshold method.

function (CDF) of rainfall duration, $F_D(d)$, i.e.

$$F_{R|D}(r|d) = \frac{1}{f_D(d)}\frac{\partial F_{R,D}(r,d)}{\partial d} = \frac{\partial F_{R,D}(r,d)}{\partial F_D(d)} \tag{3}$$

where $f_D(d)$ and $F_D(d)$ denote the probability density function (PDF) and cumulative distribution function (CDF) of rainfall duration, respectively; $F_{R,D}(r,d)$ is the JCDF of R and D.

Combining Equations (2) and (3) implies that the derivation of DDF requires either the CDF $F_{R|D}(r|d)$ or the JCDF $F_{R,D}(r,d)$ and the CDF $F_D(d)$. Since $F_{R,D}(r,d)$ and $F_{R|D}(r|d)$ are not easily fitted from observed data, copulas are used in this study to construct these functions.

Copula-based DDF

According to the Sklar's theorem (1959) of correlated rainfall depth R and duration D, the joint distribution of R and D can be constructed by the univariate distributions and a copula function used to describe dependency between R and D. The relationship is given by

$$F_{R,D}(r,d) = C(F_R(r), F_D(d)) \tag{4}$$

where $F_R(r)$ and $F_D(d)$ are cumulative distribution functions for R and D, respectively; C is a copula function. Furthermore, the conditional distribution of R given D is calculated as:

$$F_{R|D}(r|d) = \frac{\partial F_{R,D}(r,d)}{\partial F_D(d)} = \frac{\partial C(F_R(r), F_D(d))}{\partial F_D(d)} = C_{F_R|F_D}(F_R(r)|F_D(d)) \tag{5}$$

where $C_{F_R|F_D}(F_R(r)|F_D(d))$ is a conditional copula (Joe 1997) of R given $D = d$. The conditional return period $T_{R|D}$ defined in Equation (2) thus becomes a function of conditional copula, that is

$$T_{R|D}(r|d) = \frac{1}{\lambda[1 - C_{F_R|F_D}(F_R(r)|F_D(d))]} \tag{6}$$

Therefore, $T_{R|D}$ depends only on the univariate distributions of $F_R(r)$ and $F_D(d)$ and the relationship between R and D, which is described by a copula function.

Goodness-of-fit test

A goodness-of-fit test is required to check how well the distributions fit the data. Yue *et al.* (1999) and Zhang & Singh (2007) suggest the Gringorten plotting-position formula (Gringorten 1963; Cunnane 1978) to represent the empirical probability, which is

$$F(r,d) = P(R \le r_i, D \le d_j) = \frac{\sum_{m=1}^{i}\sum_{l=1}^{i} n_{ml} - 0.44}{N + 0.12} \tag{7}$$

where N is the total number of observations; j is the sorted ranks in ascending order of d_j, $j = l, \ldots, N$; m is the sorted ranks in ascending order of r_i, $i = 1, \ldots, j$; and n_{ml} is the number of occurrences of the combinations of r_{i} and d_{j}.

Akaike's Information Criteria (AIC) (Akaike 1974), as suggested by Smith (2003) and Zhang & Singh (2007), are then used to determine whether the observed typhoon events are well characterized by the proposed copulas. The Akaike's Information Criterion (AIC) is defined as

$$\mathrm{AIC} = -2\log(\text{maximized likelihood for model}) + 2(\text{no. of fitted parameters}) \tag{8}$$

RESULTS

The rainfall depth and duration for the selected 77 typhoon events are shown in Figure 4. Some basic statistics including

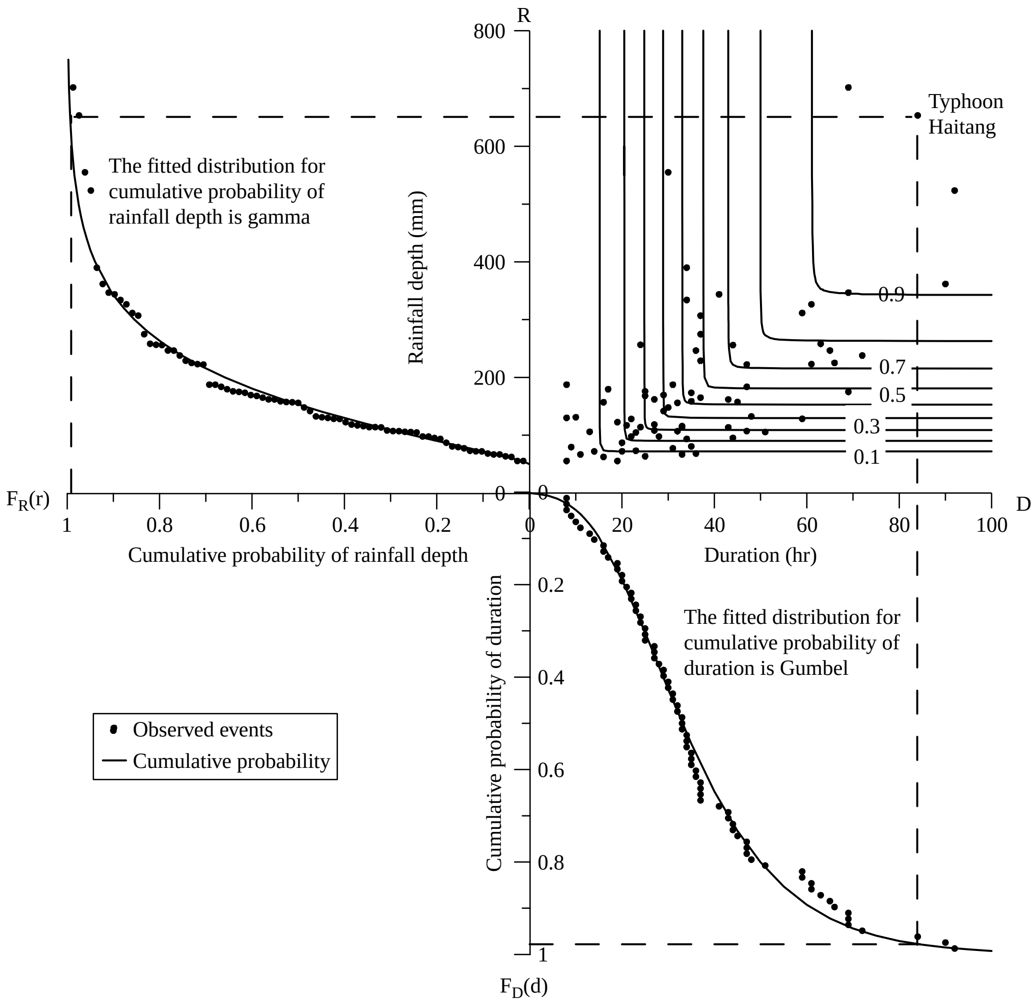

Figure 4 | Isopleths of joint cumulative probabilities of the Plackett copula and the associated marginal distributions of rainfall depth and duration.

mean value, standard deviation, coefficient of variations, coefficient of skewnesses, coefficient of kurtosises, maximum, minimum and correlation coefficient are reported in Table 1. The selected typhoon events demonstrate a positive correlation between rainfall depth and duration, with a coefficient of correlation being 0.62.

Univariate distribution of rainfall depth and duration for selected typhoon events

To model the typhoon event probability, the three-parameter gamma distribution (Equation (9)) was used to describe the rainfall depth and the Gumbel distribution (Equation (10)) was used for rainfall duration. The parameters for the gamma and Gumbel distributions were estimated by the maximum likelihood method. The parameters for gamma distribution are $\alpha = 1.301$, $\beta = 104.389$ and $\xi = 50.0$, and parameters for the Gumbel distribution are $\gamma = 14.891$ and $\mu = 27.611$. The observed data and

Table 1 | Basic statistics of observed rainfall depth and duration

	Rainfall depth *R* (mm)	Duration *D* (hr)
Mean	185.81	36.40
Standard deviation	130.17	19.40
Coefficient of variation	0.70	0.533
Coefficient of skewness	2.03	0.92
Coefficient of kurtosis	4.77	0.54
Maximum	701.90	92.00
Minimum	55.40	8.00
Correlation coefficient	0.62	

fitted distributions are shown in Figure 4. Both distributions fit well to the observed data and are not rejected by the Kolmogorov-Smirnov (K-S) and chi-square goodness-of fit tests at a 5% significance level:

$$F_R(r) = \int_0^r \frac{(t-\xi)^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} \exp\left(-\frac{t-\xi}{\beta}\right) dt = \int_0^r \frac{(t-50.0)^{0.301}}{379.504} \exp\left(-\frac{(t-50.0)}{104.389}\right) dt \quad (9)$$

$$F_D(d) = \exp\left[-\exp\left(-\frac{d-\mu}{\gamma}\right)\right] = \exp\left[-\exp\left(-\frac{d-27.611}{14.891}\right)\right] \quad (10)$$

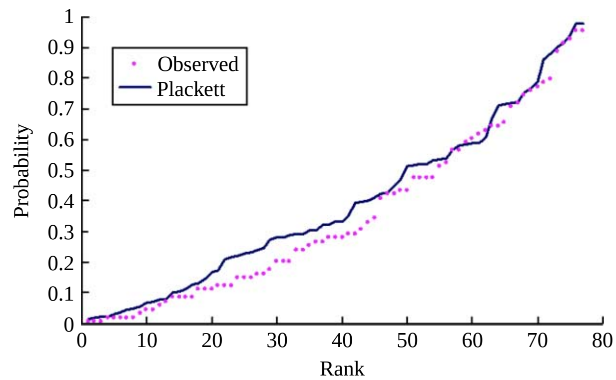

Figure 5 | Comparison of the empirical joint distribution of rainfall depth and duration for observed events and the corresponding values derived by the Plackett copula.

Parameter estimation and goodness-of-fit tests of copulas

In this study, one-parameter copulas such as Clayton, Galambos, Gumbel-Hougaard and Plackett are considered to construct the relationship between rainfall depth and duration for typhoon events. These copulas, as well as other copulas, can be found in Nelsen (2006) and Genest & Favre (2007). The method of inference function of margins (IFM) is used here to estimate the copula parameter (Joe 1997). IFM is a two-step method to construct multivariate models by estimating parameters of the univariate marginals distributions first and then the parameters of the copula.

Table 2 reports the parameters estimated by the IFM method for these copulas and associated AIC values. The p-values of the K-S test are also summarized in Table 2. Since the p-values range between 0.910 and 0.912, it is concluded that all four copula-based joint distributions considered in this study are suitable to model the correlated rainfall depth and duration of typhoon data. The Plackett copula is selected to illustrate the derivation of DDF because of its lowest AIC values. Figure 5 shows the empirical joint distribution defined by Equation (7) and the fitted Plackett copula of the typhoon data.

Table 2 | Estimation of copula parameter, AIC value and p-value of K-S test

Copula	Parameter θ	AIC	p-value of K-S test
Galambos	8.133	1,309.99	0.912
Clayton	13.877	1,308.64	0.911
Gumbel-Hougaard	8.875	1,308.54	0.912
Plackett	707.350	1,244.58	0.910

Copula-based bivariate distribution

The joint distribution of rainfall depth and duration for typhoon data in terms of the Plackett copula thus becomes

$$F_{R,D}(r,d;\theta) = \frac{1}{2}\frac{1}{\theta-1}[1+(\theta-1)(F_R(r)+F_D(d))] - \frac{1}{2}\frac{1}{\theta-1}[(1+(\theta-1)(F_R(r)+F_D(d)))^2 - 4\theta(\theta-1)(F_R(r)F_D(d))]^{1/2} \quad (11)$$

where θ is the copula parameter (707.350 for the typhoon data); and $F_R(r)$ and $F_D(d)$ are the CDF of rainfall depth and duration, respectively, defined by Equations (9) and (10).

The joint cumulative distribution $F_{R,D}(r,d)$, in terms of isopleths of joint probability of the rainfall depth and duration, and the corresponding univariate $F_R(r)$ and $F_D(d)$ associated with the observed typhoon data are depicted in Figure 4.

Return period in terms of copula

The return period for both rainfall depth and duration exceeding certain values can be defined in terms of copulas (Shiau 2003; Salvadori 2004; Salvadori & De Michele 2006):

$$T_{RD} = \frac{1}{\lambda(1-F_R(r)-F_D(d)+C(F_R(r),F_D(d)))} \quad (12)$$

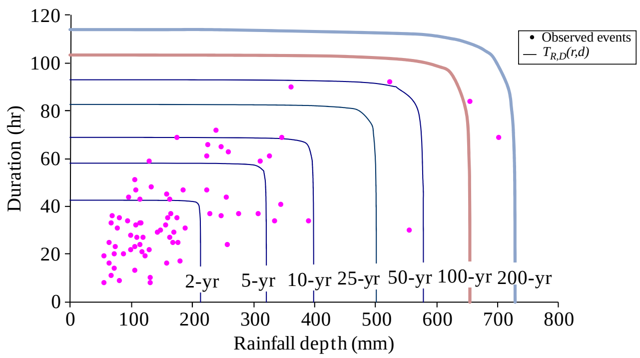

Figure 6 | Isopleths of the joint return periods $T(r,d)$.

where λ denotes the arrival rate of typhoon events (estimated as 77/47 = 1.638 events per year); $F_R(r)$ and $F_D(d)$ are univariate distributions of rainfall depth and duration, respectively; and $C(F_R(r), F_D(d))$ is the Plackett copula representing the joint probability distribution as defined in Equation (11). Isopleths for return periods of 2, 5, 10, 25, 50, 100 and 200 years defined by Equation (12) are depicted in Figure 6.

Conditional joint distribution

The conditional copula of the Plackett copula can be found in Joe (1997). The conditional distribution in terms of the conditional Plackett copula is therefore

$$F_{R|D}(r|d) = \frac{1}{2} - \frac{\frac{1}{2}[707.350F_D(d) + 1 - 707.350F_R(r)]}{[(1 + 707.350(F_D(d) + F_R(r)))^2 - 1998546.690F_D(d)F_R(r)]^{1/2}} \quad (13)$$

where $F_R(r)$ and $F_D(d)$ are defined in Equations (9) and (10), respectively.

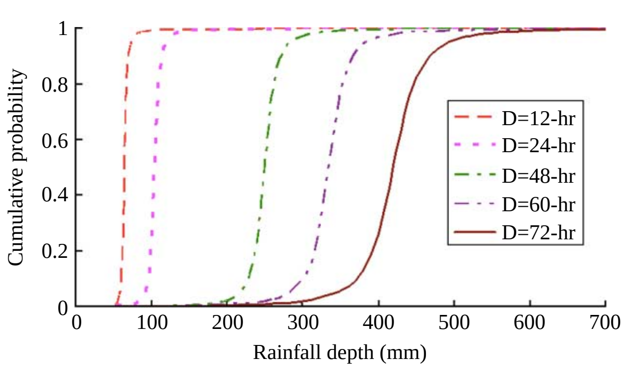

Figure 7 | Conditional cumulative probability of rainfall depth in typhoon events with given duration as D.

Figure 7 illustrates the cumulative probability of rainfall depth conditional on specific rainfall durations of 12, 24, 48, 60 and 72 hours. The range of rainfall depth rapidly increases with increasing duration. To illustrate this trend, the range of rainfall depth with cumulative probability of 0.01–0.99, 0.05–0.95, 0.1–0.9 and 0.25–0.75 for various rainfall durations are summarized in Table 3. For example, the rainfall depth of a design storm with a 48 hr duration ranges between 217.1 and 284.0 mm with a probability of 0.9.

Copula-based DDF of typhoon events

The rainfall depth-duration-frequency (DDF) relationship in terms of the Plackett copula is therefore

$$T_{R|D}(r|d) = \frac{1}{1.638[1 - F_{R|D}(r|d)]} \quad (14)$$

where $F_{R|D}(r|d)$ is the conditional distribution, defined in Equation (13) for the typhoon data considered in this study.

Depth-duration-frequency curves for return periods of 2, 5, 10, 25, 50, 100 and 200 years are shown in Figure 8. Rainfall depth of a design storm with a certain return period obviously increases with increasing duration. Some typical values of rainfall depth with rainfall duration of 12, 24, 48, 60 and 72 hours and return periods of 2, 5, 10, 25, 50, 100 and 200 years are summarized in Table 4.

Table 3 | Rainfall depth (mm) ranges of specific duration for various quantile values

	Cumulative probability			
Duration (hr)	0.01–0.99	0.05–0.95	0.10–0.90	0.25–0.75
12	54.6–91.9	58.6–73.2	60.2–69.5	62.3–66.3
24	80.8–144.4	93.2–120.0	97.1–114.4	101.6–109.1
48	178.6–332.5	217.1–284.0	228.2–271.6	240.0–258.9
60	222.6–452.3	284.8–384.8	301.7–366.7	319.7–348.0
72	264.4–584.7	347.1–493.9	371.9–468.0	398.7–440.6

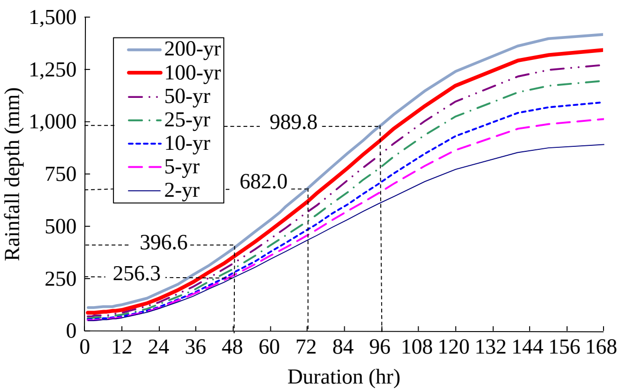

Figure 8 | Depth-duration-frequency (DDF) curves of typhoons for the Kaohsiung Weather Station, Taiwan.

DISCUSSION

Separate univariate frequency analysis of correlated variables in a complex process such as typhoons may lead to conflicting results. For example, Typhoon Haitang in 2005 had produced a rainfall depth of 653.5 mm within a period of 84 hours. Separate univariate frequency analysis of the Typhoon Haitang indicates that the return periods of rainfall depth and duration are 100 and 27 years, respectively. Similarly, for typhoon Morakot in 2009, the rainfall depth was 759.5 mm within a period of 93 hour (recorded at the Kaohsiung Weather Station). Univaraite analysis gives return periods of 264 and 50 years for depth and duration analysis, respectively.

However, bivariate frequency analysis results in a return period of Typhoon Haitang of 108 years if rainfall depth and duration are considered simultaneously. Table 5 summarizes the results of univariate frequency analysis, which include the rainfall depth and duration of 2, 5, 10, 25, 100 and 200 year return periods. The joint return period defined in Equation (12) for the values of rainfall depth and duration reported in Table 5 and univariate return period defined in Equations (9) and (10) for the values of rainfall depth and duration are also listed in Table 5. Generally, the univariate return periods and joint return period can be related by (Shiau *et al.* 2006):

$$T_{RD} = \frac{1}{\frac{1}{T_R} + \frac{1}{T_D} + C\left(1 - \frac{1}{T_R}, 1 - \frac{1}{T_D}\right) - 1} \tag{15}$$

where T_R and T_D are the return period for rainfall depth and duration, respectively.

The limitation of the current study is that the maximum one-hour rainfall depth is not considered in the bivariate frequency model, which is important on spillway design and flood peak attenuation in reservoir operation. Extension of the current model to include total rainfall depth, maximum one-hour rainfall depth and rainfall duration requires trivariate copulas to construct the rainfall peak-depth-duration-frequency (PDDF) relationships. Another topic for future studies is to incorporate the hyetograph (rainfall pattern) in the DDF analysis.

Table 4 | Rainfall depth (mm) of typhoon events for various return periods with specific duration

	Return period (year)						
Duration (hr)	**2**	**5**	**10**	**25**	**50**	**100**	**200**
12	65.7	68.7	72.0	79.0	88.3	103.3	127.0
24	108.0	113.1	118.2	128.2	140.0	157.7	183.8
48	256.3	268.5	280.0	301.3	324.4	355.7	396.6
60	344.2	362.3	379.1	409.5	441.4	482.6	533.5
72	435.0	461.5	485.8	528.3	570.7	622.7	682.0

Table 5 | Rainfall depth and duration for specific return periods determined by gamma and Gumbel distributions, respectively, and comparison of joint return period T_{RD}

Return period *T* (years)	Rainfall depth (mm)	Duration (hr)	T_{RD} (years)
2	213.5	42.7	2.1
5	320.3	58.0	5.5
10	398.8	68.8	11.6
25	501.0	82.7	31.7
50	577.5	93.1	70.0
100	653.5	103.5	160.8
200	729.1	113.8	388.9

CONCLUSIONS

Rainfall depth-duration-frequency curves were constructed using copulas. There are 77 typhoon events meeting the ‘heavy rain’ criteria recorded by the Kaohsiung Weather Station in southern Taiwan to construct the DDF relationship. The observed rainfall depth and duration for these typhoon events are fitted by the three-parameter gamma distribution and two-parameter Gumbel distribution. Four copulas are employed to construct the joint distribution of rainfall depth and duration. The results show that four copulas are suitable to model the relationship between rainfall depth and duration. However, only the Plackett copula is used to construct the DDF of the typhoon data.

The equation for the DDF can be expressed in terms of conditional distribution of rainfall depth given rainfall duration. In this study, this conditional distribution can be represented by the conditional copula which is a function of univariate distributions of rainfall depth and duration. Thus, the DDF of typhoons is expressed in terms of univariate distributions of rainfall depth and duration and a copula to link these two univariate distributions.

Rainfall depth for various return periods and rainfall durations are demonstrated in graph and table forms, which offer more information than the results of univariate frequency analysis. For the design of stormwater management facilities and delineating flooding, the information presented can enable engineers to obtain the design discharge and make appropriate choices regarding hydrological design.

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