According to tests which were available to the writer, this exponent is reasonably valid up to a 1-in. ball. For bearings having larger balls, the use of this exponent overrates the bearing capacity. Therefore the question arises whether the range of bearing sizes as given in Fig. 7 of the paper, is sufficient, or whether the functional relationship given in Equation [6] is the correct assumption.

The other point is the relation between load and life as shown in Fig. 6. The bearing industry has been using this cube relationship for many years. We may remark, however, that there is some doubt that this relationship holds good for lighter and heavier loads than are used in accelerated tests. For practical purposes, we may accept the cube law until further refinements are in sight.

Because of its potential benefits, the authors' present contribution deserves our careful study. It may be possible that with more tests, the empirical constants as given by the authors may be modified to correspond to actual bearing performance. If this task can be accomplished, it would have the influence of bringing about a more uniform system of bearing rating. The co-operative effort of the bearing industry can achieve such beneficial results.

AUTHORS' CLOSURE

It has been a pleasure to note the favorable reception given the theory on the dynamic capacity of rolling bearings, and the support which the theory has gained through comprehensive experiences of bearing engineers. The present discussion deals only with details from which can be concluded that the principal features of the theory have been accepted.

The statistical theory is based on a dispersion of the strength qualities of the material. It is assumed that the material is made up of small parts of varying strengths, distributed in a way that is statistically uniform. In no way is the theory confined to the assumption that the weak parts consist of slag inclusions. Slag is mentioned only to give a definite example. The appearance of fatigue cracks both in the rolling direction as well as diagonally at 45 deg would indicate that the difference of the fatigue stresses is not very great in either plane. Unfortunately, Fig. 1 of the paper has been turned 90 deg clockwise, and in this connection we refer to the original paper in *Acta Polytechnica*.

Mr. Haager objects: "if the product law is introduced in determining the bearing rating, then the actual bearing rating might show a greater value than that obtained by using the weakest members." This opinion is incorrect. Each bearing member contributes to the fatigue of the bearing and reduces the capacity of the bearing. Actually, and also according to the product law, the bearing as a whole is weaker than its weakest member. Only if other members are infinitely strong, the weakest will be the only deciding one. Load ratings which are based only on the weakest part result in too high values.

The bearing capacity is shown by the load the bearing can endure for a certain number of revolutions, with a probability of 90 per cent. The number of revolutions (or time × speed), on which the catalog ratings are based is of course a matter of taste. In order to give the consumers the greatest simplicity in formulas and calculations, the "basic capacity" should be based on just 1,000,000 revolutions.

The reason for the low capacity of the rigid thrust bearings is not yet definitely determined. Unfortunately, only a few laboratory tests with these bearing types are available.

As all other tests reported, these were made with SKF standard bearings, which are made with at least the accuracy now mentioned by Mr. Haager. Probably the reason for the necessary reduction may be found in the rigidity of the bearing design.

Mr. Lee asks if "the process of manufacture produces a variation in material dependent upon the size," for material for rolling bearings. We do not want to exclude the possibility of this being so, but we do not have sufficient data for judging the question. Although the functional relation $z^{h}{}_{0}$ has proved satisfactory within the range of tests, it is possible that a more complicated relation is prevailing, and that modifications will prove necessary in extreme cases.

We fully agree with Mr. Spicacci that further tests must be made before it is possible to establish for certain all the necessary coefficients and exponents and decide if any of them are of a more complicated form.

Supersonic Diffusers for Wind Tunnels[1]

J. H. KEENAN.[2] This paper is of obvious and immediate utility, partly because of its empirical nature and partly because of the clarity of presentation.

It illustrates and emphasizes the departure from simple transverse-shock conditions which invariably occurs in the presence of a boundary layer. Failure to allow for the "length" of the shock is a common defect in supersonic diffusers which have been designed for use in wind tunnels, compressors, and other supersonic passages.

The definition of diffuser efficiency employed here is simple and logical, although it is not generally accepted. Some further discussion of the various possible ways of defining diffuser efficiency doubtless would help to reduce the great variation in the meaning of this term.

ARTHUR KANTROWITZ.[3] The writer will discuss the diffusers, type 4 and type 5, which are shown in Fig. 7 of the paper. It is clear from this figure that these diffusers are the most efficient of those investigated. The writer will give an alternative explanation of their operation, which was that originally given when these long-throat supersonic diffusers were first proposed.[4]

It is shown in the reference, on the basis of one-dimensional unsteady frictionless-flow theory, that the shock position is stable in passages which diverge downstream and unstable in passages which converge downstream. Therefore the equilibrium-shock position on the basis of frictionless-flow theory should be shown somewhat downstream from the long throat instead of at the beginning of the long throat, as is shown in Fig. 7 of the paper. Furthermore, the furthest upstream stable position of a diffuser normal shock is determined by the level of disturbances coming from the region downstream. Finally, the inclusion of a long throat will make it possible for a normal shock to move further upstream toward the throat and so to smaller channel areas and to a position of higher efficiency if a long throat is provided in which the shock may oscillate. This explanation of the operation of these long throats is illustrated in Fig. 1 of this discussion.

In this figure the stable equilibrium-velocity distribution in the channel is shown as a solid line, and the unstable equilibrium-velocity distribution is shown as a dashed line. The cross-hatched area represents a disturbance approaching the throat

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[1] By E. P. Neumann and F. Lustwerk, published in the June, 1949, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 71, pp. 195–202.

[3] Professor of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Mass. Mem. ASME.

[3] Associate Professor, Graduate School of Aeronautical Engineering, Cornell University, Ithaca, N. Y.

[4] "The Formation and Stability of Normal Shock Waves in Channel Flows," by Arthur Kantrowitz, NACA TN 1225, March, 1947.

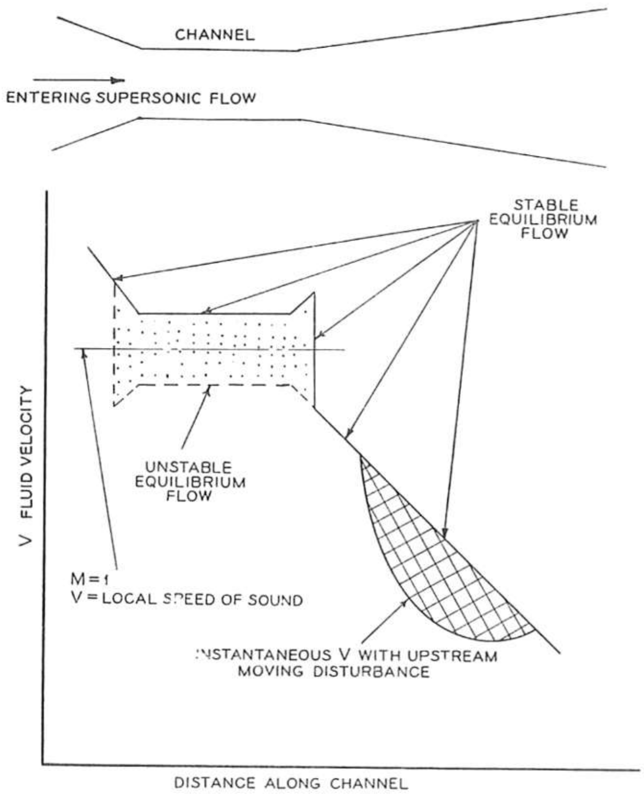

Fig. 1 Diagrammatic Representation of Long-Throat Supersonic Diffuser

from the rear of the channel. If the effect of this disturbance is to move the shock to a position upstream from the unstable-equilibrium shock position, then the shock will move out ahead of the diffuser and the starting procedure will have to be repeated. If, on the other hand, the disturbance is not sufficiently strong to move the shock as far as the upstream (unstable) equilibrium position, then it is shown in the reference that the shock will return to its original stable-equilibrium position. It is also shown in this report that the minimum disturbance necessary to move the shock just to the unstable-equilibrium position is obtained roughly when the "area of the disturbance" on a velocity-length plot (the cross-hatched area in Fig. 1), is equal to the dotted area on the same figure.

It will be seen that the use of the long throat increases the dotted area, and thus increases the maximum disturbance level, which may be tolerated in the diffuser without forcing the shock to jump out upstream. Alternatively, it is possible to increase the back pressure or the efficiency of the diffuser, thus reducing the strength of the equilibrium shock, without reducing the dotted area, when a long throat is used, and the disturbance level is maintained constant. This latter situation is apparently the case in the experiments reported in the subject paper and explains the increase in efficiency observed with the use of long throats.

It is clear that this one-dimensional frictionless theory cannot give the whole story of the shock motions in the diffuser, and this apparently is illustrated in the paper by the authors' observations that the shock position is stable in constant-area channels, whereas this theory predicts only neutral stability for this case. Thus it may be that, for a considerable part of the time, the shock would be found in the long throat of these diffusers and that the mixing mechanism postulated by the authors might be responsible for a part of the observed improvements in efficiency.

However, the authors observed the occurrence of "unstable points" of higher efficiency only with the long-throat diffuser. We take this to mean that they observed more efficient shock positions, which could be obtained only for limited periods of time, after which a sufficiently large disturbance would come along and force the shock past the throat and the unstable-equilibruim position, and so necessitate another starting procedure. Such unstable points have been observed previously in other investigations of supersonic diffusers but usually the difference between maximum unstable and maximum stable efficiencies was quite small. However, in the long-throat diffusers, this effect is emphasized because of the ability of these diffusers to resist relatively large disturbances and so to be disturbed only by comparatively rare occurrences. It is thus seen that this phenomenon finds explanation only from the disturbance point of view and not from the mixing point of view presented by the authors.

Authors' Closure

The idealized analysis presented by Professor Kantrowitz probably helps to explain some of the phenomena encountered in diffuser instability. However, there is some question whether or not any explanation of the phenomena Professor Kantrowitz refers to can be defended completely on the basis of either boundary-layer effects or an analysis basis on one-dimensional unsteady frictionless-flow theory.

The "mixing point of view" that Professor Kantrowitz attributes to the authors appears to be of his own making. It probably comes from the statement by the authors that "once separation occurred, the stream did not again fill the passage for a distance equal to 8 to 12 diameters of the tube." This statement was intended to describe the observations that normal shocks were usually not present in the straight section but that a separation of the stream from the tube walls usually occurred in their stead. This observation was made through a series of high-speed schlieren photographs similar to those contained in the subject paper. It was occasionally observed that the stream would sometimes detach itself from only one of the walls of the rectangular passageway under observation.

Stability of Linear Oscillating Systems With Constant Time Lag[1]

H. Poritsky.[2] Fashions come and go even in technical sciences. Nowadays it is customary to treat every problem by means of Laplace's transforms, and no author feels that he has done his subject justice without them. It is worth pointing out, therefore, that the stability problem in question can be handled directly without Laplace transforms.

Assuming that the linear difference-differential equation of the free system has exponential solutions $e^{\lambda t}$, it is found on substituion that λ must satisfy a certain transcendental equation. The similarity between ordinary linear systems, resulting in ordinary differential equations with constant coefficients, and the linear systems with a constant time lag, considered by the author, lies in the fact that in each case the solution of the homogeneous system, corresponding to free oscillations, can be obtained by means of exponentials. The difference lies in the fact that in ordinary systems one is led to an algebraic equation for the solution of these exponentials, while in a system with a constant time lag, one is led to a proper transcendental equation. The stability of the free system thus leads to the location of roots λ in the negative-real half of the complex plane.

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[1] By H. I. Ansoff, published in the June, 1949, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 71, pp. 158–164.

[2] Engineering General Division, General Electric Company, Schenectady, N. Y. Mem. ASME.