Equating equation (8) to equation (1) gives for $x > 0.5$

$$k = A \frac{0.401}{(1 - x)^{1/2}}$$  \hspace{1cm} (9)

For the cases when the crack depth is an intermediate fraction of the width of the beam ($0.15 < x < 0.5$) in which case equation (7) and equation (9) cannot be used, the course of the stress-intensity factor can be traced approximately. A method of calculating the strain-energy release rate for notched bars in bending with notches of arbitrary depth was given by Bueckner [7] but, in the present discussion, the least-squares method, based on values of $k$ for which functions were known, was used. In order to simplify the problem mathematically $k = f(x)$ was expressed in the form

$$k = Bx^a$$  \hspace{1cm} (10)

where $a$ and $B$ are constants (in the case analyzed $k = 2.204A^{x^{0.75}}$). It is seen from the curve defined by the open circles in Fig. 5 that the approximate function given by equation (10) produces good results when $x$ has a value between 0 and 0.7.

One of the most common forms of describing fatigue crack propagation by the stress-intensity factor is the expression given next:

$$\frac{dl}{dN} = Mk^a$$  \hspace{1cm} (11)

where $n$ and $M$ are constants and $N$ is the number of cycles. When written in logarithmic form, equation (11) is a straight line

$$\log \frac{dl}{dN} = M' + n \log k$$  \hspace{1cm} (12)

or after substituting equation (10)

$$\log \frac{dl}{dN} = M'' + na \log x$$  \hspace{1cm} (13)

If, after a simple mathematical treatment, the experimental results, too, can be expressed in logarithmic form to give a straight line, the exponent $n$, which is a controlling factor in propagation and the constant $M$, which includes among others the influence of mean stress on the propagation, can be determined easily.

Experiments have shown [3] that the exponent $n$, which is generally a constant value for various materials, is particularly sensitive to localized (heterogeneous) microscopic structural changes such as heterogenous selective precipitation, temper brittleness, etc. The constant, $M$, is basically sensitive to macroscopic changes such as those produced by mechanical properties, sample geometry, loading conditions, etc.

The rate of fatigue crack propagation in hardened and tempered 6145 steel as a function of fatigue crack length is shown in Fig. 6. The experimental results exhibit by the three curves, each with different combinations of $\sigma_{min}$ init and $\sigma_{max}$ init, support the theoretical evaluation given in the frequency and suggest that a model based on one value of the stress-intensity factor can be used for studying the effect of mean stress or mean load on fatigue crack propagation.

Conclusions

1. In cyclic loading the plastic zone is especially sensitive to structural factors (probably to a much greater degree than in unidirectional loading) and, for this reason, it is unreliable to use the plastic zone size as defined by the authors of the paper in the analysis of crack propagation.

2. Despite the different concepts held by the authors and the discussers, the final results may be quite similar because of the second basic assumption made by the authors; namely, that the changes in plastic zone size are proportional to $(\Delta b)^2$.

Authors' Closure

These authors would like to thank the discussers for their interest in the paper. The discussers indicate they will show that

1. The rate of fatigue crack propagation cannot be adequately described as a sole function of the plastic zone size; and

2. The rate of fatigue crack propagation can be described as a function of the stress-intensity factor alone, without prior dependence of relationship to plastic zone size.

Unfortunately this author is at a disadvantage in commenting on the discussers' observations since the ASME did not deem it necessary to include copies of the figures with the discussers' text. However, a few things can be said with a certain degree of confidence.

1. Item 2 above is not new. This has been reported previously in numerous places.

2. Item 1 above is not proven by the discussers' results.

3. Equations (1) through (10) of the discussion are not necessary and in fact are subject to question. The correct stress-intensity factor for the specimen configuration used by the discussers can be found in ASTM STP 410.

4. The discussers' statement in the paragraph before the conclusion section that their data suggests a model based on one value of the stress-intensity factor can be used for studying the effect of mean stress is not correct for all materials. The materials discussed in the present paper, 2024-T3 and 7075-T6 aluminum, do exhibit a mean stress effect.

An Extension of the Woods' Theory for Unsteady Cavity Flows

T. Y. WU

Dr. Kelly has extended an earlier theory of L. C. Woods to include two important effects of (1) nonzero angle of attack in the mean flow and (2) finite cavity size. A work of such an extended scope is certainly welcome since it can offer useful information urgently needed.

With respect to the first effect, it is of basic interest to note that, unlike the linearized theory of oscillating thin airfoils, the unsteady parts of the force coefficients and moment coefficients now depend quite significantly on the incidence angle $\alpha_0$ of the primary (steady) flow, as shown by the final results, Figs. 2-5. Since the original Woods' theory is used as a starting point, presumably both $\alpha_0$ of the basic flow and the amplitude of oscillation are as-
assumed to be small. If so, should we expect that the principle of linear superposition must hold valid? I would appreciate hearing from Dr. Kelly any clarification or comment regarding the possibility if the dependence of the unsteady solution on \( \alpha_0 \) of the basic flow may be due to some sort of quasi-linear approximations made in the analysis such as the small displacement of the stagnation point expressed in terms of \( \delta \) (equation (27) of the text), or as the expression \( ds/\gamma \) (equation (14) being not linearized. If such is the case, is this type of approximation consistent?

In the second aspect of this paper, the semiempirical correction suggested by Woods is adopted to represent cavities of finite size. May I ask the question whether the cavity volume can be allowed to change with time. Experimental evidences are overwhelming that fluctuations of cavity size, generally beyond the experimenter's control, have strong effect on the force coefficients. If the cavity volume varies with time, in an otherwise unbounded flow, then a physically realistic theory requires to have a time-dependent sink at infinity, as discussed by Wang and Wu. In fact, L. C. Woods later adopted this model to remove the defect of his earlier work, finding that the results with or without this sink can be drastically different.

I know Dr. Kelly has also done some experimental studies on this problem, which I believe is cited as reference [2]. I think anyone having experience on oscillating cavity flow test would agree that this is a very challenging task—my congratulations to Dr. Kelly for his accomplishment! I hope that in time, he would make available the comparison between this theory and the experimental results.

Author's Closure

The comments of Dr. Wu are very well taken, and the author agrees with them for the most part, particularly since Dr. Wu has spent much time and effort investigating the fundamental background of this problem.

In regard to the first comment of Dr. Wu, the original theory of Woods does not seem to explicitly assume the incidence angle \( \alpha_0 \) to be small. It is left as a free parameter until such time as he takes the drastic step of setting \( \alpha_0 = 0 \), “to simplify our account of the theory.” Woods makes the statement that the unsteady perturbation will be independent of mean incidence to first order, which does not prove to be true in the logical extension of his results. Perhaps further investigation would reveal some implicit dependence that would invalidate these results.

The author agrees with Dr. Wu that the fluctuations of cavity size should be taken into account. This points to the need for a better theory, perhaps based on the works of Wang and Wu.

The Behavior of a Spherical Bubble in the Vicinity of a Solid Wall

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