Asymptotic Behaviour of Total Cross Sections and Relations between Reduced Regge Residues

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Recently it has been pointed out by Lipkin\(^1\) that meson-baryon and baryon-baryon total cross sections\(^2\) in the 6–20 GeV energy range exhibit linear dependence on \(s^{-1/2}\), where \(s\) is the square of the total energy in the barycentric system. In this letter, it is reported that this property is derived from simple assumptions on the Regge trajectories and the total cross section data are analysed in this scheme.

The asymptotic energy dependence of the total cross sections arises from \(\rho\), \(\omega\), \(\varphi\) and \(R\), \(f\), \(f'\) meson trajectory exchanges in the crossed channel and is given by the sum \(\sum \gamma \epsilon_{\rho i}^{\epsilon_{\omega i}}^{\epsilon_{\varphi i}}\) (the suffix \(i\) runs over \(\rho\), \(\omega\), \(\varphi\) and \(R\), \(f\), \(f'\)), when we neglect other low-lying trajectories. If linear approximation and exchange degeneracy\(^3\) are valid for meson trajectories, we obtain \(\alpha_{\rho}(0) = \alpha_{\omega}(0) \approx 1/2\) and \(\alpha_{R}(0) = \alpha_{f}(0) \approx 1/2\) and smaller values for \(\alpha_{\varphi}(0)\) and \(\alpha_{f'}(0)\), without detailed calculations. These values are compatible with the results of previous analyses.\(^4\) Thus the nearly equal value 1/2 for each trajectory intercept yields \(\alpha_{\varphi}(0) = -1/2\approx -1/2\) and the linear dependence of the total cross sections on \(s^{-1/2}\) is obtained.

We give a more elaborate formulation to analyse the data in this scheme. To separate the contributions of the vector meson and tensor meson trajectory exchange, we consider the sums and differences of particle and antiparticle total cross sections: \(\Sigma(AB) = \sigma_{i}(\bar{A}B) + \sigma_{i}(AB)\) and \(\Delta(AB) = \sigma_{i}(\bar{A}B) - \sigma_{i}(AB)\). The quantity \(\Delta(AB)\) is represented by the contributions of only the vector meson trajectories, because of charge conjugation invariance, as follows,

\[
\Delta(AB) = \sum \gamma \epsilon_{\varphi i}^{\epsilon_{R i}}^{\epsilon_{f i}} \frac{\sqrt{\pi}}{\prod(\epsilon_{\varphi i}^{\epsilon_{R i}}^{\epsilon_{f i}})} \times \frac{\Gamma(\alpha_{\varphi}(0) + 3/2)}{\Gamma(\alpha_{\varphi}(0) + 1)} \left( s - M_{A}^{2} - M_{B}^{2} \right)^{\alpha_{\varphi}(0)}
\]

The suffix \(i\) runs over \(\rho\) and \(\omega\), as we neglect the contribution of \(\varphi\) trajectory noting that \(\alpha_{\varphi}(0) < \alpha_{\rho}(0) \approx \alpha_{\omega}(0)\). Since \(q_{AB}(s) \approx s^{1/2}/2\) and \((s - M_{A}^{2} - M_{B}^{2})/s \approx 1\) in the 6–20 GeV range as far as we are concerned with pseudoscalar mesons, proton and deuteron, we get the expression \(\Delta(AB) = \sum \gamma_{\varphi i}^{\epsilon_{R i}}^{\epsilon_{f i}} \times 1^{s^{1/2}}\), where we take \(\alpha_{\varphi}(0) = \alpha_{\rho}(0) = 1/2\) as the first approximation, and kinematical factors are absorbed into reduced residues \(\gamma\).

The quantity \(\Sigma(AB)\) is represented by the contributions of only the tensor meson trajectories as follows, \(\Sigma(AB) = \sum \gamma_{\varphi i}^{\epsilon_{R i}}^{\epsilon_{f i}} s^{1/2}\) + Energy independent (Pomeranchuk) term, where the suffix \(i\) runs over \(R\) and \(f\) and we take \(\alpha_{R}(0) = \alpha_{f}(0) = 1/2\). We also neglect the contribution of \(f'\) trajectory and use the same unit as that in the case of \(\Delta(AB)\). From the slopes of the plots of \(\Delta(\pi\rho)\), \(\Delta(Kp)\), \(\Delta(Kn)\), \(\Delta(pp)\), \(\Delta(pn)\), \(\Delta(Kd)\) and \(\Delta(pd)\) versus \(s^{-1/2}\), we obtain relations for the reduced residues of the vector meson trajectories. We have \(\gamma_{Kd}/\gamma_{Kn} = 3.0 \pm 0.3\), from the relations for \(\Delta(Kd)\) and \(\Delta(pd)\), and \(2\gamma_{Kd}/\gamma_{Kn} = 1.0 \pm 0.2\), from the relations for \(\Delta(\pi\rho)\), \(\Delta(Kp)\) and \(\Delta(Kn)\).

The contents of these relations are the same as the relations (1a) and (4a) in re-
ference 5), respectively, and these follow from the universality assumption proposed by Levinson, Wall and Lipkin, which states that $\omega$ trajectory couples to non-strange quark number and that $\rho$ trajectory couples to isospin. The relations for the reduced residues of the tensor meson trajectories are obtained by the same procedure as that in the vector meson trajectory case.

\[
\begin{align*}
\sum(\pi p): \gamma_{\pi p} &= 45 \pm 5 \text{ (mb} \cdot \text{GeV)} \\
\sum(Kp): \gamma_{Kp} + \gamma_{Kn} &= 25 \pm 5 \\
\sum(Kn): \gamma_{Kn} - \gamma_{Kp} &= 20 \pm 3 \\
\sum(pp): \gamma_{pp} + \gamma_{pn} &= 112 \pm 11 \\
\sum(pn): \gamma_{pn} - \gamma_{pp} &= 112 \pm 10 \\
\sum(\pi d): \gamma_{\pi d} &= 160 \pm 8 \\
\sum(Kd): \gamma_{Kd} &= 53 \pm 4 \\
\sum(pd): \gamma_{pd} &= 255 \pm 5.
\end{align*}
\]

From Eqs. (6), (7) and (8), we obtain

\[
\gamma_{Kp}: \gamma_{\pi p}: \gamma_{pp} = 1:3:4.8.
\]

This relation suggests some kind of universality as for $f$ meson. From the viewpoint of the quark model, the vector and the tensor mesons are considered to be the $^3S_1$-state and $^3P_2$-state of quark pairs, respectively. Therefore, the tensor meson trajectory may have some properties analogous with the vector meson trajectory if the internal orbital motion is connected very little with those properties. But it follows from the assumption that the $f$ meson trajectory couples to non-strange quark number, analogous to $\omega$ meson, that $\gamma_{Kp}: \gamma_{\pi p}: \gamma_{pp} = 1:2:3$. This discrepancy from the value obtained in Eq. (9) means that the vector and tensor meson trajectories do not have completely analogous properties. From Eqs. (4) and (5), we see that $\gamma_{p\bar{n}}$ is much smaller than $\gamma_{p\pi}$, though large experimental errors are included. This is compared with the fact that $\gamma_{p\bar{p}}$ is much smaller than $\gamma_{p\pi}$. The discussions of the additivity of the deuteron target and the scaling factor of the Regge asymptotic expansion, which are extracted from the relations obtained above, will be given elsewhere. Finally, we would like to mention the approximation which we used. If we have quark model in our mind, i.e. the $\varphi$ and $f'$ mesons consist of the strange quark pairs $\lambda$ and $\bar{\lambda}$ and have small coupling with proton or deuteron target which consists of non-strange quarks, the approximation of neglecting the contribution of $\varphi$ and $f'$ meson trajectories would be justified and it would be a natural assumption that $\omega$ trajectory couples to non-strange quark number.

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