Forward $\pi$-$p$ Scattering and a Modified Interference Model

Shigeo MINAMI and Koushi SASAKI

Department of Physics, Osaka City University, Sumiyoshi-ku, Osaka

(Received February 15, 1969)

Forward $\pi$-$p$ elastic scattering at 1.5±5 GeV/c is described in terms of the amplitude due to a modified interference model. The results are in good agreement with data. The existence of new parity doublets of nucleon resonances is discussed on the basis of our results.

§ 1. Introduction

In a previous paper\(^1\) we have shown that the observed structure of backward $\pi$-$p$ elastic scattering can be described well by a modified interference model\(^1\) in which the scattering amplitude $T$ is given by $T = T_{\text{Regge}} + T_{\text{res}} - \langle T_{\text{res}} \rangle$, where $T_{\text{Regge}}$, $T_{\text{res}}$ and $\langle T_{\text{res}} \rangle$ are the Regge exchange amplitude, the direct channel resonant amplitude and the locally averaged value of the resonant amplitude, respectively.

Because the correct value of $\text{Im} \ T(0^\circ)$ can be obtained by using the optical theorem from experimental total cross sections, we think it important to examine the modified interference model in the description of forward $\pi$-$p$ scattering. In addition to this, we are interested in the following points:

(i) Parity doublets of nucleon resonances have no effect on backward scattering, while they contribute with constructive interference to forward scattering. Therefore detailed study of forward scattering is one of the best means to discover new parity doublet of nucleon resonances.

(ii) We may suppose that in an intermediate energy region such as 1.5±3.5 GeV/c, the ratio of real to imaginary parts of forward scattering amplitude depends strongly on the incident energy. We wish to predict the ratio on the basis of the modified interference model.

(iii) It has been proposed\(^5\) that all the Regge trajectories except for Pomeranchon are mainly generated by the low energy resonances. Although Harari et al.\(^6\) have shown that this assumption is consistent with their analysis of forward $\pi$-$p$ scattering up to about 1.8 GeV/c, it is necessary to examine their assumption by analysis of $\pi$-$p$ scattering at $\geq 1.8$ GeV/c.

It is the purpose of this paper to examine these problems along the same line with our previous approach\(^1\) to backward $\pi$-$p$ elastic scattering. The forward scattering amplitude $T(0^\circ) = T_{\text{Regge}}(0^\circ) + T_{\text{res}}(0^\circ) - \langle T_{\text{res}}(0^\circ) \rangle$ is estimated by assuming that the $\langle T_{\text{res}} \rangle$ has a functional form similar to that of the $T_{\text{Regge}}$. Comparing our estimated values of $(4\pi/k) \ \text{Im} \ T(0^\circ)$ with those of experimental total
cross section, we wish to emphasize the following points: (1) The modified interference model gives a correct description of \( \pi-p \) elastic scattering not only in the backward direction\(^{13} \) but also in the forward direction. (2) There would exist, at least, a new parity doublet of nucleon resonances with \( I=\frac{1}{2} \) and mass about 2.3 GeV. Finally we discuss whether or not the Regge exchange amplitude due to the \( P' \) trajectory can almost be generated by the direct channel resonances.

\section{Estimation of the forward scattering amplitude}

As is well known, the Regge exchange amplitude for \( \pi-p \) elastic scattering at 0° is expressed by\(^{9,14} \)

\[ T_{\text{Regge}}(0^\circ) = -C\alpha(\alpha + 1)\left(\frac{E_L}{E_0}\right)\frac{\sin \pi\alpha}{\sin \pi\alpha} \] for \( P \) and \( P' \),

\[ = -C(\alpha + 1)\left(\frac{E_L}{E_0}\right)\frac{\sin \pi\alpha}{\sin \pi\alpha} \] for \( \rho \).

\( E_L \) is the total pion lab-system energy, \( \alpha(\tau) \) is the trajectory, and \( C \) is a parameter in which the residue is included. We choose, for simplicity, a scale factor \( E_0 \) to be 1 GeV.

Although there seem to be various sets** of the Regge parameters with which elastic scattering at high energy (\( \geq 5 \) GeV/c) can be well described, we consider here the following sets of Regge parameters \( \alpha \) and \( C \) suggested by Rarita et al.\(^{4} \)

Solution (1):

\[ \begin{align*} \alpha(0) &= 0.73, \\ C &= 16.35 \text{ (mb GeV)} \quad \text{for } P, \\ \end{align*} \]

\[ \begin{align*} \alpha(0) &= 0.58, \\ C &= 1.47 \text{ (mb GeV)} \quad \text{for } P', \end{align*} \]

Solution (2):***

\[ \begin{align*} \alpha(0) &= 0.57, \\ C &= 16.58 \text{ (mb GeV)} \quad \text{for } P, \end{align*} \]

\[ \begin{align*} \alpha(0) &= 0.57, \\ C &= 1.57 \text{ (mb GeV)} \quad \text{for } P', \end{align*} \]

Hereafter we refer to as cases (1) and (2), respectively, when the solutions (1) and (2) are employed. Note that the values of \( C \) in Eqs. (3) and (4) correspond to those in \( \pi^-p \) elastic scattering. Then for \( \pi^-p \) elastic scattering, the \( P \) and \( P' \) terms stay the same while \( \rho \) changes sign, for charge exchange

---

\(^{9}\) \( T_{\text{Regge}} \) in this paper corresponds to \( A' \) in reference 4).

\(^{10}\) We shall discuss the difference between them in another place.

\(^{11}\) This solution (2) corresponds to the solution (3) in reference 4).
scattering, the $P$ and $P'$ terms vanish and $\rho$ is multiplied by $-\sqrt{2}$.

The resonant amplitude $T_{\text{res}}$ is estimated by the Breit-Wigner formula

$$T_{\text{res}}(0^\circ) = A_{\text{res}} + iB_{\text{res}} = \sum \frac{\xi}{2k} \frac{\Gamma_r}{(\omega_r - \omega)^2 + (\Gamma/2)^2} (J + \frac{1}{2}),$$

where $\xi$ is the isospin factor. The resonances and their parameters $(J^P, \omega_r, \Gamma)$ for our calculations are the same as in the previous paper (see Table I in reference 1). When $|\omega_r - \omega| \gg \Gamma$, the Breit-Wigner formula with constant width could not give correct estimation for the resonant amplitude. However, this defect may almost be removed by the prescription $T_{\text{res}} - \langle T_{\text{res}} \rangle$.

Next we consider how to estimate the $\langle T_{\text{res}} \rangle$. As is well known, the Regge exchange amplitude $T_{\text{Regge}}$ has some connection with the direct channel resonances. Moreover, it was recently proposed that the $T_{\text{Regge}}$ may correspond to the averaged value of the $T_{\text{res}}$. In view of these facts, it would be reasonable to assume that the $\langle T_{\text{res}} \rangle^{*}$ has a functional form similar to that of the $T_{\text{Regge}}$ with respect to the energy dependence. That is, the $\langle A_{\text{res}} \rangle$ is expressed as

$$\langle A_{\text{res}} \rangle = b \left( \frac{E_f}{E_0} \right)^{-n}.$$  

(6)

The parameters $b$ and $n$ are determined so that (i) the $\langle A_{\text{res}} \rangle$ may have the same asymptotic behavior as the $A_{\text{res}}$ and (ii) a relation

$$\int_{1.5}^{3.2} A_{\text{res}} dp \simeq \int_{1.5}^{3.2} \langle A_{\text{res}} \rangle dp$$

may be satisfied, since the dip-bump structure of $d\sigma/d\Omega$ or $A_{\text{res}} (B_{\text{res}})$ is remarkable in the $1.5 \sim 3.2 \text{ GeV}/c$. The estimated values of $b$ and $n$ are shown in Table I.

Thus, we can obtain the forward scattering amplitude. In Fig. 1 are shown the estimated values of $T_{\text{Regge}}(0^\circ)$, $T_{\text{res}}(0^\circ)$ and $\langle T_{\text{res}}(0^\circ) \rangle$ in a region from 1.5 to 5 GeV/c. The values of $\text{Im} T(0^\circ)$ in case (1) are nearly equal to those in case (2).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$b$ (1/GeV)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle A_{\text{res}}^+ \rangle$</td>
<td>-2.26</td>
<td>0.85</td>
</tr>
<tr>
<td>$\langle B_{\text{res}}^+ \rangle$</td>
<td>7.73</td>
<td>2.49</td>
</tr>
<tr>
<td>$\langle A_{\text{res}}^- \rangle$</td>
<td>-1.69</td>
<td>0.46</td>
</tr>
<tr>
<td>$\langle B_{\text{res}}^- \rangle$</td>
<td>6.37</td>
<td>1.77</td>
</tr>
</tbody>
</table>

$^*$ As an expression of $\langle T_{\text{res}} \rangle$ for backward $\pi p$ elastic scattering, a form similar to (6) has been adopted in a previous paper.

$^{**}$ The values of parameters $b$ and $n$ are different from those in the description of backward scattering. That is, these parameters are functions of $\cos \theta$. 
Fig. 1. The forward scattering amplitudes for (a) $\pi^+p$ and (b) $\pi^-p$ elastic scattering in an energy region from 1.5 to 5 GeV/c. The solid, dash-dotted and dashed curves show the $T_{\text{res}}(0^\circ)$, $\langle T_{\text{res}}(0^\circ) \rangle$ and $T_{\text{Regge}}(0^\circ)$, respectively, where $T_{\text{res}}(0^\circ) = A_{\text{res}} + i B_{\text{res}}$ and $T_{\text{Regge}}(0^\circ) = A_{\text{Regge}} + i B_{\text{Regge}}$.

§ 3. Total cross section and a possible parity doublet of nucleon resonances

Since we have the well-known relation $(4\pi/k) \text{Im } T(0^\circ) = \sigma_{\text{tot}}$, it is possible to examine whether or not the imaginary part of calculated $T(0^\circ) = T_{\text{Regge}}(0^\circ) +$
Forward $\pi$-$p$ Scattering and a Modified Interference Model

Fig. 2. Energy dependence of $(4\pi/k) \text{Im } T(0^\circ)$ and total cross section. The solid and dashed curves show our theoretical values of $(4\pi/k) \text{Im } T(0^\circ)$ for $\pi^+\bar{p}$ and $\pi^-\bar{p}$ scattering, respectively. Experimental data for the total cross section are from references 8)~13).

$T_{\text{res}}(0^\circ) - \langle T_{\text{res}}(0^\circ) \rangle$ gives the correct description of $\pi^\pm p$ elastic scattering in the intermediate energy region 1.5~5 GeV/$c$. The results shown in Fig. 2 indicate that agreement between the estimated values of $(4\pi/k) \text{Im } T(0^\circ)$ and experimental total cross sections is good except for $\pi^-p$ scattering in the vicinity of 2.3 GeV/$c$. How should we interpret this discrepancy? This would probably suggest the existence of a new parity doublet of nucleon resonances with $I=\frac{1}{2}$ and mass about 2.3 GeV, because they have no large effects on backward scattering and contribute considerably to forward scattering.

Let us try to make assignment of $J^P$ for the new resonances on the basis of consideration about the Regge trajectory. Recently Kohsaka et al.\(^5\) have assumed the existence of an additional resonance $N^*$ with $I=\frac{1}{2}$, $J^P=9/2^-$, $M=2200$ MeV, $\Gamma=250$ MeV and $x(=\Gamma_*/\Gamma)=0.13$.\(^*\) The $N_{1/2}^*(2200)$ probably does

\(^*\) Kohsaka et al.\(^5\) have studied forward $\pi^-p$ elastic scattering by using a modified interference model. However, their model is quite different from ours.
not correspond to the resonance predicted in this paper, because the experimental
total cross section for $\pi^-p$ scattering at about 2.3 GeV/c cannot be reproduced
by taking into account the effects of the $N_{7/2}^*(2200)$. As a possible case we
can consider the existence of a new parity doublet of resonances with $J=5/2$
and $M\simeq 2.3$ GeV. They belong to the Regge trajectories whose members are
the $N_{7/2}^*(1750)$ with $J^P=\frac{3}{2}^+$ and $N_{3/2}^*(1710)$ with $J^P=\frac{3}{2}^-$, respectively. Now we
tentatively estimate their width and elasticity so that experimental cross section
in $\pi^-p$ scattering at about 2.3 GeV/c may be reproduced. The results are as
follows: $\Gamma=0.25$~0.30 GeV and $x (=\Gamma_*/\Gamma)=0.3$~0.4.

Harari et al. have studied the sum rule for the imaginary part of the crossing-even, forward $\pi-N$ scattering amplitude and have emphasized that the Regge exchange amplitude due to the $P'$ trajectory corresponds to the averaged value of the direct channel resonant amplitude. In order to examine whether their consideration is correct or not, we compare the values of $M_i(0^o)=\frac{1}{2}[\langle T_{reg}^+(0^o)\rangle + \langle T_{reg}^-(0^o)\rangle]$ with those of the $T_{regge}(0^o)_{P'}$, where $T_{regge}(0^o)_{P}$ and $T_{regge}(0^o)_{P'}$ are the Regge exchange amplitudes due to the $P$ and $P'$, respectively (cf. Fig. 3).

On the basis of the results shown in Fig. 3, we can say the following: It is impossible to verify their assumption unless a lot of additional resonances (probably parity doublet) with large masses are discovered. In other words, their assumption means the existence of a large number of new parity doublets of nucleon resonances whose masses are larger than 2.1 GeV.

![Fig. 3. Energy dependence of Im $T_{regge}(0^o)_{P'}$ and Im $M_i(0^o)=Im [\frac{1}{2}\langle T_{reg}^+(0^o)\rangle + \langle T_{reg}^-(0^o)\rangle]$. The dashed and solid curves show the values of the former and latter, respectively.](https://academic.oup.com/ptp/article-abstract/42/2/275/1841153)
Finally we examine the ratio \( R \) of real to imaginary parts of the forward scattering amplitude. As is shown in Fig. 4, there is a slight difference between the results in cases (1) and (2). So far as the \( R \) at 1.5\( \sim \)5 GeV/c is concerned, our results seem to be consistent with those obtained by Barashenkov. The value reported by Dubna group\(^7\) is

\[
R = -0.18 \pm 0.06 \text{ for } \pi^- p \text{ scattering at } 3.5 \text{ GeV/c,}
\]

and our theoretical values of \( R \) for \( \pi^- p \) at 3.5 GeV/c are equal to \(-0.180\) and \(-0.188\) for cases (1) and (2), respectively.

References