The Internal Structures of the Planets

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(Received 1974 August 2)

Summary

Prompted by a paper of Sir Edward Bullard (1948), some problems of the internal structures of the planets are discussed. The structures must be inferred from the sparse dynamical data (mass, density, \( J_2 \)) which are all that are available, and must be culled from analogies with the Earth for the Moon and the terrestrial planets or from theoretical calculations of behaviour of simple materials at high pressures for the outer planets. The importance of the value of the dimensionless moment of inertia, \( C/Ma^2 \), is emphasized and the validity of applying Bullen's compressibility-pressure hypothesis to other planets than the Earth is considered.

1. Introduction

In 1948 Sir Edward Bullard published a paper on 'The Figure of the Earth' (Bullard 1948) in which he calculated the polar flattening of a model Earth in hydrostatic equilibrium having the density distribution of an early model of Bullen's (Bullen 1940, 1942). It is well known that to first order, the polar flattening of a rotating planet is determined by the non-dimensional moment of inertia, \( C/Ma^2 \) (\( C \) is the polar moment of inertia, \( M \) the mass and \( a \) the equatorial radius) and because internal density distributions are forced to fit the value of \( C/Ma^2 \) derived from the coefficient \( J_2 \) of the second zonal harmonic in gravity, and the precessional parameter, \( H \), it might be thought that Bullard's calculation did no more than verify the consistency of Bullen's work. In fact, Bullard went further and determined the small additional term in the flattening which depends on the details of the distribution of density. He found that the reciprocal flattening was 297.338, whereas the value then derived from physical geodesy was 296.4.

When Bullen and Bullard made their calculations the best available value of \( J_2 \) was determined somewhat indirectly; we now have of course a very accurate value from orbits of artificial satellites, and our estimate of \( C/Ma^2 \) is consequently rather different from that of 30 years ago. The polar flattening corresponding to the observed value of \( J_2 \) is 1/298.25, while the value corresponding to the actual moment of inertia on the hydrostatic hypothesis is about 1/299; thus both Sir Edward's calculations and modern data show that the Earth is slightly more flattened than it would be if it were in hydrostatic equilibrium.

I remind you of this calculation of Sir Edward's because our knowledge of the Moon and planets is still at an even more primitive stage than knowledge of the Earth was in the 1940's. Since that time, one development in particular has greatly improved our knowledge of the density distribution inside the Earth—the observation...
of the free modes of vibration of the Earth and their interpretation in terms of the
distribution of density.

The mechanical data we have for the planets are, on the other hand, confined to
values of the masses, densities and, in most cases, of $J_2$ (many of them derived from
the behaviour of natural satellites, some from that of natural satellites or space probes);
in addition we know something of the seismicity and outer structure of the Moon, of
the heat flow through the lunar surface, and of the lunar magnetic field in past times;
of the magnetic field and heat flow of Jupiter; and of the surface topography of the
Moon, Mars and Venus. Indeed we do now more about the Moon and planets than
Gilbert, Galileo and Newton knew about the Earth, but not much more. The great
advantage we have today is our understanding of the interior of the Earth which we
hope may serve us as a guide in making models of the interiors of the plants. In
so doing we should recall that while the surface structure and behaviour of the Earth
are determined by the internal structure, and in particular by the non-hydrostatic
forces that drive dynamical processes, it is not possible to infer the variation of density
with radius from the surface properties. Indeed the problem of the dynamical
evolution of the outer parts is probably almost independent of that of the internal
composition and density, for the former depends on departures from hydrostatic
equilibrium, whereas the latter is little affected by such departures which are small in
relation to the hydrostatic pressure. In the absence of seismic data for the planets,
the best we can do is to interpret these values of the dynamical properties of the
planets with the help of analogies derived from the Earth, theoretical calculations on
simple solids, or other \textit{a priori} considerations. I propose to concentrate on how far
analogies with the Earth can take us for the Moon and terrestrial planets and how far
calculations on simple solids can take us for the major planets.

2. Planetary data

In the study of the structure of the planets, values of $C/Ma^2$ occupy a critical
position. For a sphere of constant density, $C/Ma^2$ is 0.4, and if the density increases
towards the centre, $C/Ma^2$ is less than 0.4 (it is about 0.33 for the Earth). Thus, the
value of $C/Ma^2$ gives us an idea of how much the density in the centre of a planet
exceeds that in the outer parts and together with the mean value of the density and
an estimate of the central pressure, it gives some idea of how much the material of
this planet is compressed under pressure, and whether the central condensation can
all be due to hydrostatic compression or whether, as in the Earth, there may be
changes of crystal structure or chemical composition within the planet. Of course,
to make such inferences, we need some idea of how the structure and density of
possible planetary constituents vary with pressure, whether it be obtained from
experiment, theory or analogy with the Earth.

Estimates of the masses, sizes and values of $J_2$ (Tables 1 and 2) are adequate for
most planets, especially those with satellites (Mars, Jupiter, Saturn, Uranus and
Neptune) for then the mass can be obtained from the semi-major axis of a satellite
orbit and the period of the satellite within it, and $J_2$ from the regression of the node
of the orbit of the satellite. Uranus is a special case, because the spin axis of the planet
is close to the ecliptic plane (instead of being, as is usual, nearly perpendicular to it)
while the orbits of the satellite are, as with the natural satellites of the other planets,
close to the ecliptic; the satellites thus execute nearly polar orbits, for which the
regression of the node is very small; $J_2$ cannot therefore be found for Uranus. Venus
has no natural satellites, but space probes have approached it closely and the mass
can be found from the acceleration of a space probe in the neighbourhood, and $J_2$ can
also be derived from the trajectory of the probe (it is less than $10^{-5}$). Only for Pluto
has it so far been impossible to find the mass from satellites or space probes, and the
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Table 1

Mechanical properties of the Moon and the Planets

<table>
<thead>
<tr>
<th>Planet</th>
<th>Radius (km)</th>
<th>Density (kg m(^{-3}))</th>
<th>Central pressure (10(^{11}) N m(^{-2}))</th>
<th>(J_2) ((\times 10^{-3}))</th>
<th>(f) (^{-1})</th>
<th>(C/Ma^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>2443</td>
<td>5400</td>
<td>0.6</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Venus</td>
<td>6055</td>
<td>5246</td>
<td>3.2</td>
<td>&lt;0.01</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Earth</td>
<td>6378</td>
<td>5517</td>
<td>4.0</td>
<td>1.082</td>
<td>298.25</td>
<td>0.3308</td>
</tr>
<tr>
<td>Mars</td>
<td>3398</td>
<td>3937</td>
<td>0.6</td>
<td>2.0</td>
<td>191.0</td>
<td>0.376</td>
</tr>
<tr>
<td>Jupiter</td>
<td>70850</td>
<td>1360</td>
<td>30.0</td>
<td>14.7</td>
<td>15.1</td>
<td>0.264</td>
</tr>
<tr>
<td>Saturn</td>
<td>60000</td>
<td>700</td>
<td>6.0</td>
<td>16.7</td>
<td>10.2</td>
<td>0.207</td>
</tr>
<tr>
<td>Uranus*</td>
<td>25400</td>
<td>1330</td>
<td>4.0</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Neptune*</td>
<td>25200</td>
<td>1570</td>
<td>4.0</td>
<td>5.0</td>
<td>46.0</td>
<td>0.26</td>
</tr>
<tr>
<td>Pluto*</td>
<td>3200</td>
<td>4800</td>
<td>1.0</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Moon</td>
<td>1738</td>
<td>3340</td>
<td>0.1</td>
<td>0.2</td>
<td>—</td>
<td>0.395</td>
</tr>
</tbody>
</table>

* Cook (1972c); see also Duncombe, Seidelmann & Klepczynski (1973).

masses estimated from the effect of these planets on the orbits of neighbouring planets are none too reliable; no estimates of \(J_2\) can be made.

The dynamical parameters of the Moon are quite well known as a result of detailed studies of orbits of artificial satellites and of preliminary results from laser ranging to the cube corners placed on the Moon by the Apollo expeditions. Many harmonic components of the gravitational potential have been estimated. They are proportional to such ratios as \((C-A)/Ma^2\) or \((B-A)/Ma^2\) where \(A\), \(B\) and \(C\) are the principal moments of inertia of the Moon. In addition improved values have been obtained for the physical librations of the Moon, the rocking motions of the Moon about her centre of mass which are driven by the varying positions of the Earth relative to the principal axes of the Moon. These motions are proportional to ratios such as \((C-A)/B\). Thus, just as the terrestrial values of \(J_2\) (equal to \((C-A)/Ma^2\)) and \(H\) (equal to \((C-A)/C\)) may be combined to give the value of \(C/Ma^2\) for the Earth, so may \(A/Ma^2\), \(B/Ma^2\) and \(C/Ma^2\) be found for the Moon. They are very close to 0.400.

Direct estimates of \(C/Ma^2\) are available for no other planets since precessional motions are unobservable, whether because the atmospheres are opaque (Venus, Jupiter) or because the motions are very small (Venus, Mars) or because the planet is very distant (Jupiter). \(C/Ma^2\) has therefore to be calculated from \(J_2\) on the assumption that the planet is in hydrostatic equilibrium. The Earth is not in hydrostatic

Table 2

Uncertainties of the masses and densities of the Moon and the Planets

<table>
<thead>
<tr>
<th>Planet</th>
<th>Uncertainty of mass</th>
<th>Uncertainty of density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moon</td>
<td>1/10^5</td>
<td>1/1500</td>
</tr>
<tr>
<td>Mercury</td>
<td>1/40</td>
<td>1/40</td>
</tr>
<tr>
<td>Venus</td>
<td>15/10^6</td>
<td>1/10^4</td>
</tr>
<tr>
<td>Earth</td>
<td>5/10^6</td>
<td>1/10^4</td>
</tr>
<tr>
<td>Mars</td>
<td>1/10^3</td>
<td>2/10^3</td>
</tr>
<tr>
<td>Jupiter</td>
<td>2/10^4</td>
<td>1/10^4</td>
</tr>
<tr>
<td>Saturn</td>
<td>3/10^6</td>
<td>5/10^4</td>
</tr>
<tr>
<td>Uranus</td>
<td>1/700</td>
<td>1/500</td>
</tr>
<tr>
<td>Neptune</td>
<td>1/600</td>
<td>1/400</td>
</tr>
<tr>
<td>Pluto</td>
<td>1/6</td>
<td></td>
</tr>
</tbody>
</table>
equilibrium but the discrepancy is small and it may be expected that it is even smaller for the major planets where the central pressure is greater and predominates even more over possible shear stresses (whether statically or dynamically sustained).

The Moon is clearly far from hydrostatic equilibrium, and it may be suspected (there is indeed evidence from the rugged topography and from the difference between the optical and dynamical ellipticity) that Mars departs more from hydrostatic equilibrium than does the Earth. That must be remembered in using the hydrostatic value of \( C/Ma^2 \) in constructing models of Mars.

3. Terrestrial materials

The mean densities of the Moon and terrestrial planets, which lie in the range 3340 to 5240 kg m\(^{-3}\), clearly invite comparison between their compositions and that of the Earth. The issue may be approached from two directions. On the one hand, we may start from some model of the composition of the terrestrial planets as a group and on the basis of the density of silicates and oxides work out models for the individual planets, how composition and crystal structure will vary with radius, and hence derive the density as a function of radius, calculating mass and \( C/Ma^2 \), and comparing them with the observed properties. Despite advances in knowledge of the physical and chemical properties of silicates in recent years, such a programme cannot be carried out more than sketchily at present. The alternative is to suppose that the properties of materials of the Moon and terrestrial planets are similar to the constituents of the Earth. K. E. Bullen (1946) again some 40 years ago, argued that the bulk modulus depended only on pressure, which, if it were applicable to the other terrestrial planets would enable models to be constructed but would, at the same time, prevent their being interpreted in chemical terms.

Bullen's hypothesis is based on the remarkable fact that the bulk modulus, \( K \), shows at the most only a slight discontinuity at the boundary of the core and the mantle despite the great change of density. He considered that the hypothesis could be used to resolve ambiguities in the determination of Earth models from \( P \) and \( S \) wave velocity distributions and his models of type \( B \) are based on the compressibility-pressure hypothesis. They are now known to agree less satisfactorily with data from free oscillations than do his models of type \( A \) in which the compressibility-pressure hypothesis is not exactly satisfied. Nonetheless, it is important to see how far current data both about possible materials and about the Earth as a whole, support the compressibility-pressure hypothesis.

Consider first, the Earth. From distributions of density and elastic moduli (or velocities) with radius, it is possible to calculate the behaviour of bulk modulus as a function of pressure. That has been done for models derived by Derr (1969) and Haddon & Bullen (1969) and also for the models derived by Press (1968, 1969) using the Monte Carlo scheme of calculation (Cook 1972b). The results show three things. First, the range of uncertainty of \( K \) at a given \( p \) is appreciable (about 10 per cent). Secondly, within that uncertainty, a significant change of \( K \) at the core-mantle boundary cannot be established. Thirdly, again within the uncertainty of \( K \), values of \( K \) within the lower mantle and outer core can be represented by a linear relation close to

\[
K = 2.2 + 3.2p \quad \text{(units: } 10^{11} \text{ N m}^{-2} \text{)}.
\]

The relation cannot be extended to the upper mantle for there \( K \) increases more rapidly with pressure and shows an abrupt increase at the boundary of the lower mantle, nor does it apply at the boundary of the inner core, where there must be a discontinuous increase of \( K \) (Cook 1972a).

Sufficient is known of the behaviour of solids at pressures up to \( 5 \times 10^{11} \text{ N m}^{-2} \) (5 million atmospheres) to be sure that the relation satisfied by the outer core and
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lower mantle cannot be generally true. Experimental and theoretical values of $\partial K/\partial p$ lie between 2 and 6 and generally decrease with pressure; were terrestrial behaviour a general guide $\partial K/\partial p$ should lie between 3 and 4 and be independent of pressure.

Studies of the compression of silicates such as olivines and pyroxenes under conditions in shock waves have shown that they not only undergo the by now well-known transition to spinel structure but that at higher pressures they break down into oxides. Deep within the lower mantle, matter is thus probably present as oxides of iron, magnesium, aluminium and silica. Such oxides and iron show an appreciable range of values of $\partial K/\partial p$ (Ahrens, Anderson & Ringwood 1969) and it is therefore surprising that there is such a small change of $K$ at the boundary between the core and the mantle. The reason seems to be that at the pressure of the boundary the values of $K$ for a number of oxides and for iron happen to be nearly the same, whereas at higher and lower pressures, there are significant differences. In brief, Bullen's compressibility-pressure hypothesis looks convincing at first sight because of the pressure that happens to be attained at the boundary of the core and mantle; were the core either smaller or larger, the hypothesis would appear much less convincing. $\partial K/\partial p$ is slightly greater for iron than for the core (the density of iron is likewise some 10 per cent greater than the density of the core). Comparisons of shock wave data with the observed density-pressure relation in the core nonetheless lead to the conclusion that iron, with some admixture of lighter elements remains the most plausible constituent of the core.

A final caveat on comparisons between Earth models and shock wave data must be entered. Conditions within a shock wave are neither isothermal nor adiabatic, and they are maintained for just a few microseconds and Knopoff & Shapiro (1969) have shown that considerable uncertainties lie in the derivation of isothermal or adiabatic pressure-density relations from shock wave data, mainly because of inadequate values of auxiliary thermodynamic parameters.

To summarize, a linear law of the form

$$K = a + bp$$

does not apply to most materials likely to enter into terrestrial planets, nor, even less does $K$ have the same value for all such materials at a given pressure. Thus while Bullen's compressibility-pressure hypothesis seems to summarise the behaviour of $K$ in a large part of the Earth, although not in the upper mantle nor inner core, it should not be used uncritically to construct models of the other terrestrial planets.

4. Models of the Moon and the terrestrial planets

It is only in recent years, since Earth models based on periods of free oscillation, and shock wave data on the behaviour of terrestrial material at high pressures both became available, that the foregoing critique of Bullen's compressibility-pressure hypothesis has become possible. Meanwhile, a number of planetary models based on the hypothesis have been constructed, as first suggested by Bullen himself. R. A. Lyttleton (1963, 1965) has made extensive calculations, and more recent work has been done by Cole (1971).

Consider first the Moon. The pressure at the centre is of the order of $1 \times 10^{10}$ N m$^{-2}$, so that if the bulk modulus is of the order of $2 \times 10^{11}$ N m$^{-2}$, the density of material of uniform composition cannot increase by more than 5 per cent from the surface to the centre. The dynamical data show that $C/Me^2$ is very close to 0-400, implying a nearly constant density; the natural inference is that the Moon does indeed have a uniform composition. This inference is reinforced by the consideration that the pressure at which the change from the upper to the lower mantle of the Earth takes place is about twice the central pressure in the Moon, so that no change of crystal structure would be expected in the minerals of the Moon. Lastly the mean density of the Moon, 3340 kg m$^{-3}$ is very close to that of the upper mantle.
of the Earth reduced to zero pressure, namely 3350 kg m\(^{-3}\). A preliminary model of the Moon, based only on dynamical data and analogy with the Earth, is thus a very simple one, a body of uniform composition, much the same as the upper mantle of the Earth, with slight self-compression at the centre.

How far is such a simple model consistent with seismic and other investigations of the Moon? The brilliant studies of seismic signals from impacts on the surface of the Moon and from moonquakes within have unfortunately only been able until recently to give information about relatively superficial layers of the Moon, in particular, the outermost region in which seismic velocity increases very rapidly with depth. But one observation quite recently can be interpreted as evidence of a core with a low velocity in the same way as Oldham first detected the core of the Earth. The radius of any lunar core would be very small, perhaps less than a fifth of the surface radius, so that its influence on the moment of inertia would be slight. The best value for \(C/Ma^2\) is probably close to 0·396 but the uncertainty is at present too great to make any useful statement about the maximum density of a core with a radius of 2–300 km.

A small core, fluid in the early stages of the Moon's history, has been invoked by many as the source of a dynamo that would produce the field, now vanished, which is needed to account for the present residual magnetization of lunar surface samples. It might be thought that it would be difficult to sustain dynamo action in such a small core as is now suggested, but the idea of Runcorn & Urey (1973), that the early Moon had a magnetic field that arose from alignment of magnetized iron particles, indicates that evidence of a past magnetic field cannot provide unambiguous information about a possible lunar core.

The central pressure of Mars, \(5\times10^{10}\) N m\(^{-2}\), is nearly three times that of the transition from the upper to the lower mantle in the Earth. A similar change of crystal structure is therefore very probable in Mars, and the mean density, 3940 kg m\(^{-3}\), well above the density of the upper mantle of the Earth, indeed suggests that such a change occurs. The representation of the transition in a mathematical model is not straightforward because the transition from the upper to the lower mantle of the Earth is not simple. In the first place, no mention has so far been made of temperature as affecting the density within a planet. Fortunately, with models of thermal conditions being ill-controlled by observation, density is little affected by temperature, for the coefficients of thermal expansion of most solids decreases to very low values at pressures such as occur at the centres of planets. However, the pressure at which a phase transition occurs almost certainly is affected by temperature, and so it is not sufficient to suppose that the transition occurs at the same pressure as within the Earth. Lyttleton (1965) calculated a series of models of Mars in which the transition occurred at all radii from the surface to the centre; he was able to find a fairly good, though not perfect fit to the observed parameters, and in particular, the least moment of inertia of his models was greater than the actual value for Mars; the simple interpretation is that there is an additional increase of density towards the centre corresponding to a small dense core. However, that interpretation is too simple, mainly because the upper to lower mantle transition in the Earth is not just a transition from olivine to spinel. Experimental data on the olivine–spinel transition show that it accounts for about half the actual increase of density, so that there must be a change of chemical composition in addition. It is curious that the change of structure and that of composition should occur in the same region—is this a second example (the other being the behaviour of \(K\) at the core–mantle boundary) where ideas about solid state properties within the Earth are conditioned by some fortuitous circumstances? In any case it seems clear that we can infer from the behaviour in the mantle neither a transition pressure nor a change of density that could be used in models of Mars. The possibility must be acknowledged that the density of the inner zone of Mars is slightly greater than would be that of material of the lower mantle of the Earth and that no denser core is needed.
Venus and Mercury must be left as enigmas. The value of \( J_2 \) for Venus is very small and therefore because the speed of rotation is small and until far more accurate data become available, \( C/Ma^2 \) cannot be estimated (it depends on the ratio of polar flattening to spin speed). Thus at present nothing is known of the central condensation of Venus; equally, nothing is known of the central condensation of Mercury.

5. Materials and structures of the major planets

The major planets are characterized by low mean densities (somewhat more than water for Jupiter and somewhat less for Saturn) and low values of \( C/Ma^2 \) (about 0.21 for Saturn) implying great central condensation, as would be expected of hydrostatic compression under pressures of more than \( 10^{12} \) N m\(^{-2} \). Considerations of densities of the solid forms of cosmically common elements show indeed that the principal components of Jupiter and Saturn must be hydrogen and helium, and ever since Wigner and Huntington (1935) made the first (quantum mechanical) calculations of the pressure–density behaviour of the metallic form of hydrogen (the simplest metal) it has become more and more generally accepted that Jupiter and Saturn are composed mainly of hydrogen and that somewhere within them, probably closer to the surface than to the centre, the transition to the metallic form of hydrogen occurs. Those statements are probably soundly based—the density precludes any great admixture of helium and the prediction that hydrogen will become metallic at a sufficiently high pressure may be made with considerable assurance. Great difficulties arise on trying to go further. The first is to estimate the pressure of the metallic transition, and comes not from calculations of the internal energy of the metallic form, but from the estimation of that of molecular hydrogen which so far is not susceptible to calculation. Currently, experiments on hydrogen at very high pressures are under way and may remove this difficulty. The second difficulty lies in estimating the effect of the admixture of helium. Helium like hydrogen will become a metal under sufficiently high pressure but the transition pressure will very probably be different in a mixture of hydrogen and helium from the value for helium by itself.

The distribution of helium in the planet will be determined to a considerable extent by the miscibility of helium with hydrogen and that again by whether the helium is metallic or not. Calculations on models with different degrees of mixing have been performed (Smoluchowski 1967) but the only conclusion that seems reasonably well established is that a pure hydrogen model does not quite fit the mass, density and gravity field of Jupiter, but that with 10 per cent of helium it should be possible to do so.

It is important to be clear what is really being said here. The values of \( \partial K/\partial p \) are close to 2 for both molecular and metallic hydrogen and the densities probably differ only slightly throughout the range of pressures in Jupiter. Essentially therefore, the comparison is between Jupiter and a model with \( \partial K/\partial p \) equal to 2 and a suitable density at zero pressure. Such a model does not quite fit but a relatively small adjustment will produce agreement.

Saturn on the other hand, must have a core of denser material. The argument is elementary. The size, mass and mean density of Saturn are less than those of Jupiter, so that the central pressure is less. Hence, if the materials are the same (hydrogen plus helium) Saturn will be less compressed at the centre and the moment of inertia ratio, \( C/Ma^2 \) will be greater. In fact \( C/Ma^2 \) is much less (0.207 for Saturn, 0.264 for Jupiter); Saturn must therefore have a core denser than the material of Jupiter.

Observations from Pioneer 10 in its passage close to Jupiter have confirmed the earlier deduction from infrared observations made from the Earth that there is a considerable flow of heat out through the surface of Jupiter (Chase et al. 1974). The temperature within Jupiter must therefore increase considerably towards the centre;
that conclusion raises the question of the state of metallic hydrogen in Jupiter; whether it is solid or liquid and whether it may be superconducting or superfluid. Professor N. Ashcroft and his colleagues at Cornell are currently studying those questions which, it need scarcely be said are of direct significance for the study of the magnetic field of Jupiter (Hammerberg & Ashcroft 1974; Stevenson & Ashcroft 1974).

Uranus and Neptune are denser than Jupiter and Saturn although they are smaller and so they must be composed of material with greater density at zero pressure. Many years ago, W. H. Ramsey studied the properties of a planet composed of methane, water, ammonia and neon in cosmic proportions. His work now needs revision, in part because the much better values of the sizes, masses and densities of Uranus and Neptune that have recently been obtained from astronomical observations, show that the two planets have remarkably similar radii although their masses and densities differ appreciably. $C/Ma^2$ is 0.26 for Neptune; as previously explained, that of Uranus cannot be obtained directly, but the observations of the geometrical flattening are not inconsistent with a similar value (Cook 1972c).

If Uranus and Neptune are mostly composed of material such as Ramsey suggested, then they must have cores of denser material. Ramsey's calculations show that the bulk modulus for his material is given by

$$K = K_0 + bp$$

where $b$ is about 4.5. This means at a given pressure, the increase of density above that at zero pressure will be less than for hydrogen for which $b$ is about 2. Hence, for planets with the same central pressure, one of Ramsey material will have the greater value of $C/Ma^2$. But the central pressures of Uranus and Jupiter are less than that in Jupiter, and so the increase of density will be still less and the value of $C/Ma^2$ yet closer to 0.4. The actual values of $C/Ma^2$ for Uranus and Neptune, much the same as for Jupiter, disagree with this prediction; it follows that Uranus and Neptune must include cores of denser material.

6. Conclusion

Starting from a paper of Sir Edward's I have tried to indicate some current problems in the study of the internal structures of the planets. Sir Edward has of course long been interested in the internal structure of the Earth. He himself carried out integrations of the Adams–Williamson equations similar to those of Bullen, and in connection with his studies of the dynamo mechanism, he has been concerned with the density distribution and movements in the core. It is pleasing to see both that rather simple arguments, of the sort of which he has shown himself a master, are effective in the study of the structure of the planets and also, that for the Moon and Jupiter at least, we cannot get very far in discussing the internal structures without having to consider the origins of their magnetic fields.

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References

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