Research Note

On the Amplitudes of Seismic Waves

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1. Introduction

Douglas, Hudson & Blamey (1972) suggest that the relative amplitudes of P and Rayleigh waves generated by underground explosions depend on the properties of the source medium. Calculations based on an assumption of a flat spectrum for P waves radiated at source, indicate that the relation between the body wave magnitude, $m_b$ (computed at 1 Hz) and the surface wave magnitude, $M_s$ (computed at 0.05 Hz) should be

$$m_b = M_s + 1.8 \text{ for explosions in alluvium}$$

and

$$m_b = M_s + 1.0 \text{ for explosions in granite.}$$

In this note we examine why these $m_b : M_s$ relationships depend on the source medium. Douglas et al. (1972) use unit sources whose amplitudes are such as to give the same P wave amplitude in unbounded homogeneous material with the same properties as the firing medium, no matter what values are taken by the elastic constants. In the discussion that follows we use the more convenient definition of a unit source as one represented by a unit force system, e.g. a unit point force is a force of unit magnitude acting at a point. A dipole is the resultant of two equal and opposite forces brought together along their line of action, and a unit dilatational point source is taken to be three equal perpendicular dipoles of unit amplitude; i.e. a body force $F$, where

$$F(x) = \nabla \delta(x - \xi) \quad (1)$$

$\xi$ being the source point. Taking such a source with amplitude $\phi(t)$, to represent an explosion in a homogeneous material, we find the far-field displacement in the P wave to be directed radially from the source, with amplitude

$$u^P = \frac{1}{4\pi \rho \alpha^2} \left[ \frac{\phi(t - R/\alpha)}{R} \right] = -\frac{1}{4\pi \rho \alpha^3} \phi'(t - R/\alpha) \quad (2)$$

(see Love 1906), where $R = |x - \xi|$, and $\rho$ and $\alpha$ are the density and P wave-speed in the material. Thus, the same magnitude of source generates higher amplitude waves in weaker materials ($\rho \alpha^3$ smaller). This is no doubt because the force is allowed to do more work in the weaker solid, and therefore generates more energy.

The same effect is apparent for a double-couple source. However, displacements from a point force have the factor $(\rho \alpha^3)^{-1}$ rather than $(\rho \alpha^3)^{-1}$.

If the dilatational source acts at depth $h$ from the surface of a homogeneous half-
space, the amplitude of the spectrum of Rayleigh waves generated at a distance $r$ along the surface from the epicentre is (taking the vertical component)

$$
\tilde{u}_R = \left( \frac{\omega^3 \gamma}{2 \pi r \beta} \right) \left\{ \frac{2 - \gamma^2/\beta^2}{2 - \gamma^2/\beta^2 - \eta^2 - \frac{1}{2}(\eta^2 + \eta^2)} \right\} \frac{\tilde{\phi}(\omega)}{8 \rho \alpha^2 \beta^4} \exp \left( -\omega \eta \gamma \right)
$$

(see Ewing, Jardetzky & Press 1957), where $\beta, \gamma$ are the shear and Rayleigh wave-speeds respectively,

$$
\eta = (1 - \gamma^2/\alpha^2)^{\frac{1}{2}}, \quad \eta' = (1 - \gamma^2/\beta^2)^{\frac{1}{2}},
$$

and $\tilde{\phi}(\omega)$ is the Fourier transform of $\phi(t)$.

Given that Poisson's ratio varies very little from one rock to another, the relative amplitudes of waves generated at the same frequency in different materials are governed principally by the factor $\exp \left( -\omega \eta \gamma \right)/\rho \alpha^2 \beta^4$. The exponential term reflects the attenuation of the surface wave amplitudes with depth of source, a process which is magnified in weaker (small $\gamma$) materials.

Apart from this, the amplitudes of the Rayleigh waves are larger in the weaker materials by the factor $(\alpha^2 \beta^4)^{-1}$, a result very similar to that for the body waves.

2. Short period waves in a layered structure

If an explosive source acts in a layered medium, the first part of the pulse radiated into the source region (or, equivalently, the short period end of the spectrum) will be the same as if the source were set in an unbounded solid. The later parts of the pulse will be affected by nearby variations in material properties after a time equal to the time taken for a reflected or diffracted wave to return to the source. At large distances, therefore, the first motion of body waves radiated at steep angles down into the mantle from an explosion in the crust will be similar in amplitude to body waves generated in unbounded source material except in so far as it is modified by the coefficients of

<table>
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<th>Table 1</th>
<th>Crustal models (Werth &amp; Herbst 1963).</th>
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<td>Nevada test site—granite crust</td>
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<tr>
<td>P wave velocity</td>
<td>S wave velocity</td>
</tr>
<tr>
<td>(km s(^{-1}))</td>
<td>(km s(^{-1}))</td>
</tr>
<tr>
<td>1st layer</td>
<td>4.80</td>
</tr>
<tr>
<td>2nd layer</td>
<td>6.15</td>
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<td>Half space</td>
<td>7.81</td>
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</table>

| Nevada test site—alluvium crust | |
| P wave velocity | S wave velocity | Density | Thickness |
| (km s\(^{-1}\)) | (km s\(^{-1}\)) | (g cm\(^{-3}\)) | (km) |
| 1st layer | 1.70 | 1.0 | 1.8 | 0.5 |
| 2nd layer | 2.30 | 1.3 | 2.0 | 0.67 |
| 3rd layer | 5.20 | 3.0 | 2.7 | 1.6 |
| 4th layer | 6.15 | 3.5 | 2.8 | 26.6 |
| Half space | 7.81 | 4.6 | 3.3 | $\infty$ |

Note: The $S$ wave velocity in the 1st layer of the granite crust was taken to be $4.8/\sqrt{3}$ km s\(^{-1}\) and not $3.4$ km s\(^{-1}\) as given by Werth & Herbst (1973). An $S$ velocity of $3.4$ km s\(^{-1}\) combined with a $P$ velocity of $4.8$ km s\(^{-1}\) gives a Poisson's ratio which is negative.
transmission through lower layers. These coefficients give a factor of approximately
\[
\frac{2\rho_2}{\rho_1 + \rho_2 \alpha'}
\]
(5)
at each interface where \(\rho, \alpha\) are parameters of the upper material, and \(\rho', \alpha'\) of the lower material. There is also a factor of \((\alpha/\alpha')^4\) due to geometrical spreading.

The net result is that the amplitude of body waves from an explosive source in the granite crust in Table 1 is less than that for the alluvium crust by a factor of approximately \(1.6 \times 10^{-1}\). The same result will hold for a double-couple source, but the figure for a point force will be around 0.4.

This ratio is not as small as might be expected from the source factor \((\rho_3)^{-1}\) because amplitudes from the low velocity layer are reduced by the high impedance contrast between the upper and lower layers. However, this also means that the reflection at such a contrast travels upwards to reinforce the free surface reflection \((pP)\). The amplitude of the reflection at the layer boundary at depth 1.17 km in the alluvium crust (see Table 1) is about \(\frac{1}{2}\) of that of the initial pulse. Therefore \(pP\), which very often reinforces the second swing of the direct \(P\) wave, is enhanced by a factor of nearly \(\frac{3}{2}\).

Since short period surface waves are confined to a shallow surface layer, we expect that the short period part of the Rayleigh wave spectrum for a source near the surface will have amplitudes similar to those for a source in a homogeneous half-space. Thus, neglecting the effects of source depth, we find that the source in the granite layer generates short period Rayleigh waves of amplitude about \(\frac{1}{3}\) of those generated in the alluvium crust.

However if both sets of Rayleigh waves (one set from a source in granite, and one from a source in alluvium) are recorded at the same station, there will be different impedance changes in the two propagation paths. This may either reduce or enhance the difference in amplitudes.

3. Long period waves

If the body wave pulse is sufficiently long, reflections within the crust become superimposed and calculations of amplitudes becomes rather difficult. Similarly, the Rayleigh waves of long periods interact simultaneously with many layers and with the mantle. Therefore we need a different approach in order to compare amplitudes at the long period end of the spectrum.

The spectrum of body waves radiated into the mantle is given by (Douglas, Hudson & Kembhavi 1971),
\[
\mathbf{u}^P = \mathbf{\bar{\phi}}(\omega)[\text{div}\mathbf{u}^2]_{x = (0, 0, h)},
\]
(7)
where \(\mathbf{\bar{\phi}}\) is the Fourier transform of \(\phi\) and \(\mathbf{u}^2\) is the displacement in a plane \(P\) wave incident from below on the upper surface; the divergence of \(\mathbf{u}^2\) is evaluated at the source point \(x = (0, 0, h)\).

It is clear that at long periods, \(\mathbf{u}^2\) will not be much affected by a thin low velocity layer at the surface, and so the long period body waves are approximately equal in amplitude.

Similarly, the amplitude of Rayleigh waves is given by (Hudson & Douglas 1975)
\[
\mathbf{u}^R = \left(\frac{\kappa}{2\pi r}\right)^{\frac{1}{2}} \mathbf{\bar{\phi}}(\omega) \frac{[\text{div}\mathbf{u}^R]_{x = (0, 0, h)}}{4\mathcal{F}},
\]
(8)
where \(\kappa\) is the wave number, and \(\mathbf{u}^R\) the displacement in a plane Rayleigh wave, and \(\mathcal{F}\)
FIG. 1. Rayleigh wave spectra for unit dilatational point sources at a series of depths in the alluvium and granite crusts (Table 1). Also shown is the spectrum for a source in a half space of the material forming the underlying half space of the alluvium and granite crusts.

FIG. 2. Relative amplitudes of long and short period P waves (as measured from theoretical seismograms) as a function of $(\rho a^3)^{-1}$ where $\rho$ is the density and $a$ the $P$ wave speed in the source medium.
the energy flux in $\bar{u}^R$. Again it follows that, at long periods, the existence of the low velocity layer will barely affect the amplitudes of Rayleigh waves generated from a given source.

We expect therefore that the amplitudes of the long period waves generated by the two sources in different crusts will be the same. This will be so at all periods which allow time for the high-velocity lower layers to interact with the source and prevent the body force from expending as much energy as it would in an unbounded solid of source material.

4. Computed examples

Fig. 1 shows a series of Rayleigh wave spectra for unit dilatational point sources at different depths in the alluvium and granite crusts given in Table 1. At long periods, as predicted, the amplitudes of all the spectra are roughly equal, whereas at short periods the amplitudes generated depend strongly on the properties of the source medium, being much higher for sources in alluvium than for granite. Fig. 2 shows the relative amplitudes of the long and short period $P$ waves (as measured from theoretical seismograms) radiated into the half-space of Table 1 from the same unit sources as used in Fig. 1, displayed as a function of the value of $(\rho x^3)^{-1}$ in the source material. Again as expected, the long period amplitudes show little change with source medium whereas the short period amplitudes vary more or less directly as $(\rho x^3)^{-1}$.

We have thus shown how different crustal structures will affect the radiation spectra of shallow point sources. A source in a low velocity superficial layer gives rise to more high frequency radiation than one in higher velocity material if all other effects are equal. At low frequencies the details of the structure are unimportant and the two crusts given in Table 1 are roughly equivalent for the generation of waves.

The dependence of the $m_b : M_s$ relationship, derived from synthetic seismograms, on the source medium is now explained: $m_b$ is determined at relatively short periods (~1 Hz) and $M_s$ at relatively long periods (~0.05 Hz) and so for an explosive source in alluvium, $m_b$ will be nearly a magnitude higher than that of a similar source in granite although having the same $M_s$.

This is only true, however, if the shape of the spectrum of $P$ waves radiated at source is independent of material. In practice of course, it is not (Werth & Herbst 1963) and differences in the $m_b : M_s$ relationship due to the source medium will be modified accordingly.

References


