Hall Effect in Excitonic Insulator

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The Hall effect in a weak magnetic field of an excitonic insulator in the semimetallic limit is investigated by the use of the Green function formalism developed recently. Correlations between electrons are treated under the Hartree-Fock approximation with only a dominant term and the effect of impurity scattering is considered. The Hall coefficient increases as the temperature is lowered near \( T=0 \), reflecting the reduction of the number of effective charge carriers; it diverges in a pure case and remains at a finite value in a dirty case. Near \( T_c \) it deviates from the normal state value linearly with \( T_c - T \) and, in a pure case, it shows a minimum.

§ 1. Introduction

The possibility of the existence of a new phase in solids at low temperatures has recently been discussed by several authors.\(^1\sim5\) In semiconductors with a relatively small gap in one-electron energy band and semimetals with a small band-overlap, electrons in the conduction band and holes in the valence band interact with one another by the Coulomb force forming excitons. Below a certain temperature we expect a state in which the excitons can exist coherently, making the Bose-Einstein condensate. This phase is called the excitonic phase and the system is called the excitonic insulator, since it will behave like an insulator as far as the electrical conduction is concerned. Therefore, electromagnetic responses may be the most powerful phenomena to investigate such a phase.

In semimetals, especially, the conduction is metallic if such a correlation effect does not play an important role. But as the temperature is lowered, the excitonic phase is shown, under the Hartree-Fock approximation, to have a lower energy than the normal state. The discussions have been quite similar to those in the theory of superconductivity in the following assumptions:

a) the Coulomb forces are included only as exchange energy and the electron-phonon interactions are disregarded,

b) only one conduction band and one valence band are essential for the formation of the exciton,

c) the number of electrons and holes is sufficiently large that the Coulomb interaction is well screened.

Under the assumption a), the lowest energy states of spin singlet and triplet are degenerate, and our following discussions are independent of the spin state.

The electrical conductivity in such simple systems was calculated by Zittartz\(^9\)
under one more approximation of equal masses of electrons and holes, using the
Green function method. The effects by nonmagnetic impurities turned out to
be identical, in a theoretical sense, with those by paramagnetic impurities in
superconductors,\textsuperscript{10} that is, the impurities have pair-breaking effects. This is due
to the fact that, if the impurity potential is repulsive for electrons (holes), it
is attractive for holes (electrons). Zittartz has shown that at the temperatures
just below the transition temperature $T_c$, the conductivity decrease is proportional
to the temperature decrease. In a system with a low impurity concentration the con­
ductivity vanishes exponentially with the temperature, since the electrons and
holes form the exciton condensate, not participating in the current conduction.
In this sense, the systems in the excitonic phase are insulators. On the other
hand, if the impurity concentration is high enough, the conductivity remains at
a non-zero value even at the absolute zero temperature. The pair-breaking effect
is so strong that some finite fraction of electrons and holes are still out of the
condensate and give rise to the conduction.

In the present paper, we shall discuss another transport property, the Hall
effect, in such simple systems. It will be readily seen that the Hall effect di­
rectly gives a measure of asymmetry between the valence and conduction bands.
If the electron mass in the conduction band happens to be identical with the
hole mass in the valence band, the Hall effect is not observed. It just vani­
shes because of the symmetry between electrons and holes. Therefore we will
assume different values for electron and hole masses and calculate the Hall co­
efficient using the formalism developed in our separate paper\textsuperscript{11} (hereafter referred
to as I).

The Hall coefficient\textsuperscript{12} is given in terms of the off-diagonal and diagonal ele­
ments of the conductivity tensor, $\sigma_{xy}$ and $\sigma_{xx}$ respectively, by the equation
$R = \sigma_{xy}/H \sigma_{xx}^0$. The expression for the off-diagonal element, given by the Kubo
formula in the linear response theory, is expanded in terms of the applied mag­
netic field, the first term, linear in the field, being retained. This procedure is
quite satisfactory for the present problem, since in the excitonic insulators we
do not expect strong modifications in the distributions of magnetic field as the
Meissner effect in superconductors. In the present theory we will make use of the
Green function techniques in order to take into account the effects of the correlations
among the particles in a consistent way. Since the correlations are essential for
the formation of the excitonic phase, their effects involved have to be carefully
examined and systematically considered. The problem of the excitonic insulator
provides one of the typical examples for which our new formalism for the Hall
effect can be most effectively applied, since it is not so clear whether or not
we can take the phenomenological approach such as Boltzmann's transport theory
in order to investigate such highly correlated systems.

The one-particle Green function consists of four elements. Two of them
refer to the intraband elements and the other two to the interband elements.
They are conveniently written in a matrix form just as the Nambu notation in the theory of superconductivity. Since the impurity potentials are assumed to be of short range, the vertex corrections of the current do not give any contributions. The Green function was already determined in Zittartz's work. Substitution of his results immediately gives the off-diagonal conductivity, and explicit calculations are carried out at the two temperature regions. One is just below the transition temperature $T_c$ and the other is close to the absolute zero. It is found that the Hall coefficient begins to deviate from its value in the normal phase proportionally to the temperature reduction as it is lowered from $T_c$. It decreases when the impurity concentration is low. On the other hand it increases when the concentration is high enough. As the temperature is lowered to the absolute zero, the coefficient tends exponentially to infinity in the case of low impurity concentration, as the number of effective charge carriers can be expected to decrease exponentially with the temperature. When the impurity concentration is high enough, the coefficient remains finite at the absolute zero, due to the pair-breaking effects.

In §§2 and 3 the model is explicitly presented and the Hall coefficient is determined. In §4, the results are summarized and some discussions are given.

§2. Evaluation of $\sigma_{xy}$

The model and the notation are the same as those in the article of Zittartz. The unperturbed Hamiltonian is

$$\mathcal{H}_0 = \sum_{p,\sigma} \epsilon_a(p) a_{p,\sigma}^\dagger a_{p,\sigma} + \sum_{p,\sigma} \epsilon_b(p) b_{p,\sigma}^\dagger b_{p,\sigma},$$

(2.1)

where $\epsilon(p)$'s are band energies in an effective mass approximation:

$$\epsilon_a(p) = \frac{1}{2m_a} (p^2 - p_0^2), \quad \epsilon_b(p) = \frac{1}{2m_b} (p^2 - p_0^2).$$

(2.2)

The momentum $p$'s are measured from the band extrema respectively and $p_0$ is Fermi momentum. Using the density fluctuation operator of the system

$$\rho(q) = \sum_{p,\sigma} a_{p+q,\sigma}^\dagger a_{p,\sigma} + \sum_{p,\sigma} a_{p+q,\sigma} b_{p,\sigma},$$

(2.3)

the interaction part of the Hamiltonian is given by

$$\mathcal{H}' = \frac{1}{2} \sum_q v(q) \rho(-q) \rho(q) + \sum_q U(q) \rho(q),$$

where the first term describes the mutual interaction via Coulomb forces and the second the interaction with impurities. The screened Coulomb potential $v(q)$ may finally be replaced by a constant $\tilde{v}$.

The thermal Green functions which describe the properties of this system is defined by a matrix form
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\[ G = \begin{pmatrix} G_b & F \\ F & G_a \end{pmatrix} = -\langle T \phi(p, t) \bar{\phi}(p, t') \rangle, \quad (2.4) \]

where \( \phi \) is used in the sense of Nambu's notation with two components for electrons and holes. \( G \), in the weak impurity scattering, can be written as follows:

\[
G(p, i\omega_n) = -\frac{1}{D_n} \begin{pmatrix} i\omega_{n,a} - \epsilon_a & \tilde{\Delta}_a \\ \tilde{\Delta}_a & i\omega_{n,b} - \epsilon_b \end{pmatrix},
\]

\( \tilde{\Delta}_a = \tilde{\omega}_{n,a} + \tilde{\omega}_{n,b} - \epsilon_a \epsilon_b + i(\epsilon_a \tilde{\omega}_{n,a} + \epsilon_b \tilde{\omega}_{n,b} ) \),

\( \tilde{\omega}_{n,a,b} = \omega_n + \frac{1}{2} \Gamma_{n,a,b} / \Omega_n \),

\( \Omega_n = (\tilde{\omega}_a^2 + \tilde{\omega}_b^2)^{1/2} \),

\( \Gamma_{n,a,b} = \frac{m_i \hbar \omega_{n,a,b}}{2 \pi^2} \int d\Omega |u(\theta)|^2 \),

(2.10)

where \( \omega_n = (2n + 1) \pi / \beta \) and \( \Delta \) is the order parameter which has to be determined self-consistently, and \( \tilde{\omega}_a \) and \( \Gamma \) are defined by

\[
\tilde{\omega}_a = (m_a \tilde{\omega}_{n,a} + m_b \tilde{\omega}_{n,b}) / [2(m_a m_b)^{1/2}],
\]

\( \Gamma = (\Gamma_a, \Gamma_b)^{1/2} = 1/\tau \) .

For later application we rewrite the denominator of \( G \), Eq. (2.6), as

\[ D_n = (\xi - i\Omega_n + i\tilde{\omega}_n) (\xi + i\Omega_n + i\tilde{\omega}_n), \]

(2.13)

where

\[
\xi = (p^2 - p_0^2) / [2(m_a m_b)^{1/2}] = (-\epsilon_a \epsilon_b)^{1/2},
\]

\( \tilde{\omega}_n = (m_a \tilde{\omega}_{n,a} - m_b \tilde{\omega}_{n,b}) / [2(m_a m_b)^{1/2}] \).

(2.14)

(2.15)

The only essential difference from reference 9 lies in \( \tilde{\omega}_n \), which vanishes under the assumption of \( m_a = m_b \). The current operator \( J \) is expressed in the present effective mass approximation as

\[ J(q) = e \sum_p \psi^*(p - q/2) \cdot m^{-1} \psi^*(p + q/2), \]

(2.16)

where

\[
m^{-1} = \begin{pmatrix} m_b^{-1} & 0 \\ 0 & -m_a^{-1} \end{pmatrix}.
\]

(2.17)

Proceeding in precisely the same way as in I, we have the following expression for the off-diagonal conductivity in a uniform magnetic field:
\[ \sigma_{xy}(\omega) = \frac{e^3}{c} \left\{ \frac{1}{\omega} \sum_{n} \left[ \sum_{\mathbf{p}} \mathbf{p} \cdot \mathbf{p} \right] \text{Tr} \left[ m_{-1}^{-1} \mathbf{G}(i\omega_n) m_{-1}^{-1} \mathbf{G}(i\omega_{n-}) m_{-1}^{-1} \mathbf{G}(i\omega_{n-}) \right] \right\}_{t_{1a} \sim a + ib}, \tag{2.18} \]

where \( \omega_{n-} = \omega_n - \omega_a \). Here we have no vertex corrections, for we assumed that \( u(\theta) \) is independent of scattering angle \( \theta \). Carrying out some matrix calculations under \( \text{Tr} \) in Eq. (2.18) and performing the summation over \( \mathbf{p} \) as \( \xi \) integration near the Fermi surface as is usually done, we get

\[ \sigma_{xy}(\omega) = \frac{eH}{mc} \frac{2ne^3}{m} Q(\omega), \tag{2.19} \]

\[ Q(i\omega_n) = \frac{1}{\omega} T \sum_{n} \{ \mathcal{F}(\omega_n, \omega_{n-}) - \mathcal{F}(\omega_{n-}, \omega_n) \}, \]

where

\[ \mathcal{F}(\omega_n, \omega_{n-}) = \pi i \frac{\mathbf{Q}_n + \mathbf{Q}_{n-}}{(\mathbf{Q}_n + \mathbf{Q}_{n-})^2 - (\omega_n - \omega_{n-})^2} \left( 1 - \frac{\mathbf{a}_n \mathbf{a}_{n-} + \mathbf{b}_n \mathbf{b}_{n-}}{\mathbf{Q}_n \mathbf{Q}_{n-}} \right), \tag{2.20} \]

and \( m = (m_an_b)^{1/2} \). \( n \) is the number density of electrons or holes. Transforming the summation over \( n \) into a contour integral and performing an analytic continuation, \( i\omega_n \to \omega + i\delta \), we get \( \sigma_{xy} \) in the static limit \( \omega = 0 \) as follows:

\[ \sigma_{xy} = -\frac{2ne^3}{m} \omega_c Q, \]

\[ Q = \frac{\beta}{4\pi i} \int d\omega \text{sech}^{1/2} \frac{\beta\omega}{2} \left[ \mathcal{F}(x+i\delta, x-i\delta) - \mathcal{F}(x-i\delta, x+i\delta) \right], \tag{2.21} \]

where \( \omega_c \) is the gap in the excitation spectrum and \( \omega_c \) is the cyclotron frequency \( |e|H/mc \).

Equation (2.21) is fundamental for our further discussions. We first define the analytic functions which are continued analytically from the functions defined only at \( i\omega_n \)'s to those with the variable \( x+i\delta \) (\( x \): real, \( \delta \): positive infinitesimal) using the following notations,

\[ i\omega_n \rightarrow \xi(x), \quad i\omega_n \rightarrow \xi(x), \]

\[ \mathbf{a}_{n} \rightarrow \mathbf{a}(x), \quad i\omega_n \rightarrow \mathbf{a}(x). \tag{2.22} \]

From these, it can easily be verified that the four functions presented in Eq. (2.22) with variable \( x-i\delta \) are \( -\xi^*(x), \xi^*(x), \mathbf{a}^*(x) \) and \( \mathbf{a}^*(x) \) respectively. Thus \( \mathcal{F} \) in Eq. (2.21) can be written from Eq. (2.20) as

\[ \mathcal{F}(x+i\delta, x-i\delta) = -\mathcal{F}(x-i\delta, x+i\delta) \]

\[ = \frac{\pi i}{2} \frac{\text{Im} \xi(x) \text{Im} \xi(x)}{(\text{Im} \xi)^2 - (\text{Im} \xi)^2} h(x). \tag{2.23} \]
We have introduced the same function as is discussed in reference 9),

\[ h(x) = \frac{1}{2} \left\{ 1 + \frac{|u|^2 - 1}{|u^2 - 1|} \right\}, \]  

(2.24)

where \( u = z(x)/\tilde{A}(x) \). Thus, we have

\[ \sigma_{xy} = -\frac{2ne^2}{m} \omega_c \beta \int_0^\infty dx \sech^2 \left( \frac{\beta x}{2} \right) \frac{\Im \tilde{\epsilon}(x) \Im \tilde{z}(x)}{2} \left[ (\Im \tilde{\epsilon})^2 - (\Im \tilde{z})^2 \right] h(x). \]  

(2.25)

In the first place we will calculate the conductivity tensor in normal state, i.e. \( \delta = 0 \). The quantities in Eq. (2.25) are given in this case as follows:

\[ \tilde{z}(x) = (M/4\mu)^{1/2} x + (M/2\mu - 1)i \Gamma/2, \]

\[ \tilde{z}(x) = g(M/4\mu)^{1/2} x + g(M/2\mu) i \Gamma/2, \]

\[ \tilde{\epsilon}(x) = \tilde{z}(x), \]  

(2.26)

where

\[ M = m_a + m_b, \quad \mu^{-1} = m_a^{-1} + m_b^{-1} \]

and

\[ g = (m_a - m_b)/(m_a + m_b) = (1 - 4\mu/M)^{1/2}. \]

As \( \omega_b = 0 \), the main contribution for the integral comes from the region \( x = 0 \) at low temperatures, and there results, by Eqs. (2.26)

\[ \sigma_{xy} = -\frac{2ne^2}{m} \omega_c \epsilon^2 \frac{M}{\mu} \left( \frac{M}{2\mu} - 1 \right) g = \sigma_{xy}^n. \]  

(2.27)

This expression can also be derived by summing the \( \sigma_{xy}^n \)’s for each band given from the solutions of the usual Boltzmann transport equation,\(^{10,11}\)

\[(ne^3H/m_a^3c)/\Gamma_a^3 = (ne^3H/m_b^3c)/\Gamma_b^3. \]

Next we consider the ordered state and evaluate explicitly Eq. (2.25) for some specific temperature regions.

i) Near \( T = 0 \) and if \( \omega_b = 0 \) (\( \alpha < 1 \), the dominant contribution comes from the region \( x = \omega_b \) in the integration. Expanding in powers of \( (x-\omega_b)/\omega_b \), we obtain to its lowest order (see Appendix)

\[ \Im \tilde{\epsilon}(x) \approx \Im \tilde{\epsilon}(\omega_b) = 4\alpha^{1/3} - \Gamma/2, \]  

(2.28)

\[ \Im \tilde{z}(x) = g \frac{M}{4\mu} \sqrt{\frac{2}{3}} \alpha^{-2/3} (1 - \alpha^{1/3})^{1/3} \Gamma \sqrt{\frac{x-\omega_b}{\omega_b}} \]  

(2.29)

and

\[ h(x) = \frac{2}{3} \alpha^{-1/3} (1 - \alpha^{1/3}) \frac{x-\omega_b}{\omega_b}, \]  

(2.30)

where
By these expressions, we get
\[ \sigma_{xy} = \sigma_{xy}^0 \left( \frac{M}{2\mu} - 1 \right)^{-1} \frac{1}{8} \sqrt{\frac{2\pi}{3}} \frac{\alpha^2/\alpha}{(M/4\mu - \alpha^2/2)^{3/2}} \exp(-\beta\omega_0) \cdot \quad (2.31) \]

ii) In the gapless case, that is, \( \omega_0 = 0 \) at \( T=0 \) (\( \alpha>1 \)), we have
\[ \sigma_{xy} = -\frac{2ne^2}{m} \omega_c \frac{\text{Im} \bar{\epsilon}(x) \text{Im} \bar{\varepsilon}(x)}{2[\text{Im} \bar{\epsilon}(x)(\text{Im} \bar{\varepsilon}(x))]^2} \frac{h(x)}{x=0}. \]

From the Appendix, it follows that
\begin{align*}
\text{Im} \bar{\epsilon}(0) &= \langle M/2\mu - 1 \rangle \Gamma/2, \quad (2.32) \\
\text{Im} \bar{\varepsilon}(0) &= g(M/4\mu) \langle 1 - \alpha^{-2} \rangle \Gamma, \quad (2.33) \\
h(0) &= 1 - \alpha^{-2}, \quad (2.34)
\end{align*}

and we get
\[ \sigma_{xy} = \sigma_{xy}^0 (1 - \alpha^{-2})^{3/2} \frac{[1 + (M/2\mu)^2 g^2 \alpha^{-2}]}{f}, \quad (2.35) \]

This value of \( \sigma_{xy} \) is smaller than \( \sigma_{xy}^0 \) defined by Eq. (2.27) but approaches it as the mean free path gets shorter (i.e. \( \alpha \to \infty \)).

iii) Near \( T_c \) the system is always in the gapless region because of the smallness of \( \Delta \). Expanding \( \text{Im} \bar{\epsilon}, \text{Im} \bar{\varepsilon} \) and \( h(x) \) in terms of \( \delta \) (see Appendix), we obtain
\[ \sigma_{xy} = \sigma_{xy}^0 [1 - B(T_c) \delta], \quad (2.36) \]

where
\[ B(T_c) = \frac{\bar{\epsilon}_c}{2} \int_0^\infty dx \text{sech}^2 \frac{\bar{\epsilon}_c}{2} \frac{\rho x^3 + q T^3}{(x^3 + T^3)^2} > 0, \quad (2.37) \]

\[ \rho = 2M/\mu - 2(M/2\mu - 1)^{-1} - 2\mu/M \geq 11/2, \]

\[ q = 6\mu/M + (2M/\mu) g^2 \frac{3}{2}, \]

\[ T = (\Gamma_+ + \Gamma_-)/2. \]

The equality for \( \rho \) and \( q \) holds if \( m_a = m_b \). As \( \delta \) is proportional to \( T_c - T \), \( \sigma_{xy} \) decreases with a finite gradient at \( T=T_c \). By use of Eqs. (2.31), (2.35) and (2.36) a qualitative plot of \( \sigma_{xy}(T) \) is shown in Fig. 1.
§ 3. The Hall coefficient

In order to express the preceding results in terms of the Hall coefficient, we need the value of $\sigma_{xx}$, and for this purpose we extend the calculation of reference 9) for the case $m_a = m_b$. The expression obtained is

$$\sigma_{xx} = \frac{2ne^2}{m} \left( \frac{\beta}{4} \right) \int_{-\infty}^{\infty} \frac{dx}{\sigma_0} \text{sech}^2 \left( \frac{\beta x}{2} \right) \frac{1}{2(\Im \tilde{z}(x))} \Im \tilde{z}(x).$$  (3.1)

If $m_a = m_b$, then $\tilde{z}(x)$ vanishes and this reduces to that of Zittartz without vertex correction. In the normal state, $\Delta = 0$, Eq. (3.1) gives, by Eq. (2.26),

$$\sigma_{xx} = \frac{(M/2\mu - 1)}{m} \frac{2ne^2}{m} = ne^2 \left( \frac{1}{m_a \Gamma_a} + \frac{1}{m_b \Gamma_b} \right) \sigma^0.$$

Calculations similar to the one for $\sigma_{xy}$ yield the following expressions for $\sigma_{xx}$ in each temperature region, i), ii) and iii) discussed in the preceding section.

i) $\sigma_{xx} = \sigma^0 (M/2\mu - 1)^{-1/2} \frac{1 - \alpha^{2/3}}{3} \frac{\exp(-\beta \omega_0)}{\beta \omega_0},$  (3.2)

ii) $\sigma_{xx} = \sigma^0 (1 - \alpha^{-1}) \left[ 1 + (M/2\mu)^2 \alpha^2 \right]^{-1},$  (3.3)

iii) $\sigma_{xx} = \sigma^0 \left[ 1 - A(T_c) \Delta \right],$  (3.4)

where

$$A(T_c) = \frac{\beta_e}{2} \int_{0}^{\infty} dx \text{sech}^2 \beta_e x \frac{r x^2 + s \Gamma^2}{(x^2 + \Gamma^2)^3},$$  (3.5)

$$r = M/\mu - 2(M/2\mu - 1)^{-1} \geq 2,$$

$s = 4\mu/M + (M/\mu) \alpha^2 \geq 1.$

These $\sigma_{xx}$'s are combined with $\sigma_{xy}$'s to give $R$'s in the three cases.

i) Near $T = 0$ and $\omega_0 \approx 0$

$$R = R_a \frac{9}{32} \sqrt{\frac{2\pi}{3}} \left( \frac{M}{2\mu} - 1 \right) \frac{\alpha^{2/3}}{(M/4\mu - \alpha^{2/3})/2} \left( 1 - \alpha^{2/3} \right)^{1/3} \times \sqrt{\beta \omega_0} \exp(-\beta \omega_0).$$  (3.6)

ii) In the gapless case at $T = 0$

$$R = R_a (1 - \alpha^{-1/2}).$$  (3.7)

iii) Near $T_c$,

$$R = R_a [1 - C(T_c) \Delta],$$  (3.8)

where

\[ Fig. 2. \] Temperature dependence of Hall coefficient.
The schematic representation of \( R \) is given in Fig. 2.

§ 4. Summary and discussion

The Hall coefficient of excitonic insulator in the semimetallic limit is calculated by using the Green function method under the same assumptions as those of the article of Zittartz which are summarized as a), b) and c) in § 1, with the exception that the masses of electrons and holes are not necessarily equal. The temperature dependence of the coefficient is shown in Fig. 2. The increase of \( R \) in very low temperatures indicates the reduction of the number of effective charge carriers by the existence of coherent excitons. However, the impurity scattering affects the behavior of \( R \) very much. Near \( T = 0 \) if the broadening of one-particle energy level in the normal state is smaller than \( \Delta \), \( R \) goes to infinity like that of semiconductors, while for more impure system with \( \alpha > 1 \), \( R \) remains finite even at \( T = 0 \) like that of semimetals but is still larger than the value in the normal state \( R_n \). The temperature dependence of \( (R/R_n)^{-1} \) in the former case is the same as that of Pauli spin susceptibility in a superconductor\(^{17)} \) with a low concentration of paramagnetic impurities. These two quantities reflect the number of unpaired electrons. Near \( T_c \), however, rather curious features are revealed, which may not be attributed only to the change of the number of carriers. The temperature gradient of \( R \) near \( T_c \) is either positive or negative, depending on the concentration of impurities; the latter is the case for a dirty system and the former for a pure case. The critical value of \( T_c \), which gives the zero gradient, is \( \beta_c \Gamma_c \approx 2.9 \) in the limit \( m_a = m_b \). As this value of \( \Gamma_c \) corresponds to \( \alpha > 1 \) at \( T = 0 \), \( R(T) \) diverges as \( T \to 0 \) in this limit. In other limit \( m_a/m_b \gg 1 \) or \( m_a/m_b \ll 1 \), the gradient is negative and \( R(T) \) is larger than \( R_n \) for any temperature.

Recently the Hall coefficient of antiferromagnetic chromium has been measured by Amitin and Kovalevskaya.\(^{18)} \) The temperature dependence resembles the dirty case of our calculation. The theory of itinerant antiferromagnetism\(^{19)} \) is essentially the same as that of excitonic insulator in the semimetallic limit. Although the main scattering mechanism is electron-phonon interaction in the temperature region of experiment, the present calculation may be qualitatively applicable.

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Appendix

The analytic structures of the quantities related to the Green function are given in a way similar to those given in Reference 9. Here we modify those for the case of unequal masses. We get $i\tilde{\omega}_n$ and $i\tilde{\omega}_n$ by insertion of Eq. (2.7) into Eqs. (2.11) and (2.15), and an analytic continuation of these quantities and $\tilde{\omega}_n$, Eq. (2.8), yields

$$\tilde{\omega}(x) = (M/4\mu)^{1/2}x + (M/2\mu - 1) (i\Gamma/2)\tilde{\omega}, \quad (A.1)$$
$$\tilde{\omega}(x) = g(M/4\mu)^{1/2}x + g(M/2\mu) (i\Gamma/2)\tilde{\omega}, \quad (A.2)$$

and

$$\tilde{\omega} = \Delta - (i\Gamma/2)\tilde{\omega}. \quad (A.3)$$

Then the equation for $u(x)$ is obtained as in reference 7):

$$\frac{1}{2} \left( \frac{M}{\mu} \right)^{1/2} x = u \left( 1 - \frac{i\alpha}{(u^2 - 1)^{1/2}} \right). \quad (A.4)$$

From this equation we obtain the gap $\omega_0$ and a corresponding value of $u_0$:

$$u_0 = (1 - \alpha^{1/2})^{1/2}, \quad (A.5)$$
$$\omega_0 = 2(\mu/M)^{1/2}\Delta(1 - \alpha^{1/2}) \quad \text{if } \alpha < 1. \quad (A.6)$$

In terms of $u(x)$ the desired quantities are expressed as

$$\text{Im} \tilde{\omega}(x) = \Delta \text{Im} \sqrt{u^2 - 1 - \Gamma/2}, \quad (A.7)$$
$$\text{Im} \tilde{\omega}(x) = \frac{M}{2\mu} \frac{\Gamma}{2} \frac{\text{Im} u}{\alpha}, \quad (A.8)$$

while $h(x)$ as Eq. (2.24).

For a small value of $x - \omega_0$ in the case $\alpha < 1$, we obtain from Eq. (A.4)

$$u = u_0 + i\sqrt{\frac{2}{3}} \alpha^{1/2} u_0 \sqrt{\frac{x - \omega_0}{\omega_0}}, \quad (A.9)$$

and inserting this equation into Eqs. (A.7), (A.8) and (2.24) we obtain Eqs. (2.28), (2.29) and (2.30).

If $\alpha \geq 1$, $\text{Im} \tilde{\omega}(0)$ is obtained from Eqs. (A.1) as (2.32) and $\text{Im} \tilde{\omega}(0)$ and $h(0)$ from Eqs. (A.4), (A.8) and (2.24) as Eqs. (2.33) and (2.34).

In the gapless region near $T_c$ we expand Eq. (A.4) in terms of $\Delta^2$ and finally obtain

$$\text{Im} \tilde{\omega}(x) = \frac{\Gamma}{2} \left( \frac{M}{2\mu} - 1 \right) \left[ 1 + \frac{2x^2}{(x^2 + \Gamma^2)^2} \right] \frac{\Delta}{(M/2\mu - 1)}, \quad (A.10)$$

$$\text{Im} \tilde{\omega}(x) = \Gamma g \frac{M}{4\mu} \left[ 1 + \frac{x^2 + \Gamma^2}{(x^2 + \Gamma^2)^2} \right] \frac{\Delta}{M}, \quad (A.11)$$
\[ h(x) = 1 - \frac{4\mu}{M} \left( \frac{F^2}{x^2 + F^2} \right)^{\frac{3}{2}}, \tag{A.12} \]

where

\[ F = (\Gamma_a + \Gamma_b)/2. \]

References

10) A. A. Abrikosov and L. P. Gor’kov, JETP 39 (1960), 1781 [Soviet Phys.—JETP 12 (1961), 1243].