APPENDIX—A

Estimation of Optimum Z and β

To estimate the optimum number of vanes and the blade angle of the centrifugal pump having the following dimensions (see reference [15] for the design of main dimensions). $D_1 = 5.8$ cm; $D_2 = 20.8$ cm; $b_1 = 2.2$ cm; $b_2 = 0.7$ cm; $N = 2880$ rpm; $H = 42$ M; $Q = 10.43$ lps; $\lambda = 0.05$.

Equation (25) reduces to (assuming volumetric efficiency of 95.6)

$$1 - x = Z\,\frac{0.0109}{1.39}\left(11.35 + \frac{Z}{2.36}\right)^{1/2} \tag{28}$$

$$m = 4.8;\; n = -(x - x^2)2810 \tag{29}$$

when $x = 0.7925$, $m = 4.8$, and $n = -462$
Equation (18) results in $Z = 6.61 \approx 7$

The value of $x = 0.7925$, $Z = 7$ satisfy the equation (28)

Hence $\cos^2\beta = 0.7925$ or $\beta = 27.3$ deg
$\therefore$ Optimum number of blades = 7 and angle = 27.3 deg

APPENDIX—B

Estimation of Z for Arbitrary Vaned Impeller

Estimation of optimum blade number (Z) for the case of an impeller having arbitrary vanes can be done, by proceeding the same way as that for logarithmic vaned impeller. After some modifications one obtains (reference [15]) the optimum Z equation as given by equation (18), where the simplified values of m and n are given below:

$$m = \frac{\pi}{6}\left[\frac{D_1 \sin^3\beta_2 + D_2 \sin^3\beta_1}{b_1 \sin^3\beta_2 + b_2 \sin^3\beta_1}\right] \tag{30}$$

$$n = -39.5\left(\frac{D_2}{D_1}\right)\frac{(\sin\beta_1 - \sin^3\beta_1)\sin^3\beta_2}{\lambda\,(D_2 - D_1)}.$$

$$\left[\frac{K_{s2}D_2^2 \sin\beta_2 + K_{s1}D_1^2 \sin\beta_1}{D_2 \sin^3\beta_2 + \beta_1 \sin^3\beta_1}\right] \tag{31}$$

DISCUSSION

A. Kovats[2]

The calculation method and the conclusions from the test results are valid only for the particular and rather poorly designed impeller and can not be applied as a general rule for other pumps.

The lowest number of vanes is limited by the vane loading and channel divergence, limits where separation of the flow results, as in the case of the 2 vane impeller described in the paper.

Regarding the performance of the impellers with high number of vanes, as $Z = 8$ and 10, it is evident from Fig. 4, that due to the small D_1 and thick vanes, the entrance passage is reduced so much that excessive losses occur at the suction side, reducing head and efficiency.

The $\beta = 30°$ and $Z = 7$ are surely not the optimum values for a pump with the head and flow characteristics of the test pump. With a pump of the same ψ and $Q/D^3 \times N$, but with smaller dimensions ($D = 18$ cm)), I got 69 percent efficiency and possibly a much lower σ value. The impeller had 5 vanes and $\beta = 21°$.

There are many errors in the theory of the calculation of the optimum vane number and vane angle. The author specifies the head loss as the sum of the decrease of the head due to channel circulation and of friction loss. But the channel circulation is not a loss and does not affect the efficiency and the friction factor is a function of the hydraulic radius, therefore also of the Z, β, and b. It may vary for different designs by as much as 1:1.5. He disregards also the "entrance loss." For the calculation of the head decreases (and indirectly of the vane loading), the author uses the Stodola coefficient = K_s, which he indicates can have values from 0.6 to 1.2. In$^2\beta$ in equation (25) can therefore vary more than 1:2, since it is proportional to λ/K_s and can give, for example "optimum blade angles" from 20° to 30°! In addition entrance vane angle of 30° or higher, produce very high NPSH, a factor that cannot be disregarded in pump design. There are several calculation methods, published in the last 20 years, which give more exact values for optimum Z and β.

Finally it should be noted that the author specifies the head coefficient: $\psi = gH/u^2$, but the values given in Table 1. and Figs. 10, 12, 14, and 15 correspond evidently to $\psi = 2gH/u^2$.

D. G. Zerrer[3]

This paper was of special interest to us because we recently tried to solve the problem of the optimum vane number and angle in connection with the design of a pump impeller. To this end, we calculated the velocity and pressure distributions throughout the impeller channel for various vane numbers. Both circular arc vanes and logarithmic spiral vanes were investigated. It is of interest to note that the conclusions of our study are in close agreement with the authors' results. For the low specific speed pump in question, an impeller with seven vanes of logarithmic shape and a vane angle of 27 appeared to be the best solution.

While our study was directed toward the solution of a specific design problem, certain conclusions of a general nature resulted from our work. They point to the fact that with low specific speed impellers, i.e., impellers of a predominantly radial profile, certain factors come into play which are of little or no account in higher specific speed runners. With a specific speed of N_s = 970 rpm, the impeller tested by Professor Kar and his co-worker falls into this category.

Because of the low flow capacity, velocities in such impellers are relative low. This leads to a pronounced back flow tendency along the pressure face of the vanes, especially if the vanes are highly loaded, i.e., if the channels are short in relation to their circumferential width. For a given discharge angle, the longer logarithmic spiral vanes are therefore superior to circular arc vanes despite an increase in friction due to larger wetted area. The back flow tendency combined with relatively high friction are probably the reasons for the generally low efficiency of low specific speed impellers.

Like other investigators, the authors of this paper consider that the optimum vane number is reached when the beneficial effects of reduced vane loading are offset by the increase in friction losses due to additional wetted area. This model holds under the assumption of zero vane thickness. With finite vane thickness, however, increased vane blockage not only produces higher velocities but changes the geometry of the channel in such a manner that a greater reduction of the relative velocity from inlet to discharge has to take place. This is because additional vanes cause a greater percentage decrease of free flow area at the inlet than at the discharge. Thus, beyond a certain number of vanes, additional vanes actually increase the vane loading rather than decrease it. Small low specific speed impellers such as the one tested by the authors are particularly susceptible to this effect because of the high d_2/d_1 ratio and because the vanes are relatively thick.

The theoretical analysis in Professor Kar's paper is based on the classical approach by Stodola and attempts to account for friction effects by introducing the Darcy equation for noncircular ducts. One should not overlook the fact that both these approaches contain a considerable number of simplifying assumptions. Stodola's work represents an early approach to the problem of circulatory channel flow. The use of Stodola's coefficient K_s also means that the theoretical treatment is not free of empirical input. The Darcy equation, on the other hand, applies

[2] Consultant, Livingston, N. J.
[3] Design and Development, Easton, Md.

strictly only to steady duct flows. It accounts neither for the effect of the adverse pressure gradient present in a pump impeller nor for the fact that energy is transferred from the vane to the fluid. Thus its use implies the assumption that these opposing effects compensate for each other. Modern boundary layer theory may provide the tools for checking the validity of this assumption. Finally it should be noted that the calculation of the hydraulic radius of an impeller passage involves the use of mean values, the determination of which is by no means trivial, and which in the case of very low vane numbers must be problematical.

F. J. Wiesner[4]

In addition to a number of minor typographical errors and apparent conflicts of information, misstatements and omissions on the part of the authors, there appears to be at least one major discrepancy in the mathematical derivations which, when corrected, would tend to leave the major conclusions of this work invalid. While an issue of this type should have arisen in the papers review process, it apparently did not, and we are now faced with the possibility of the publication of dubious material (unless the authors can properly explain or resolve the major objections among the following points.)

1 Slip is regarded as head loss, which is basically an incorrect interpretation (page 412).

2 If equation (7) is taken as the friction head loss per impeller blade channel and the values of λ, L, R_h, and W are taken as representative of a given channel, then equation (11) is incorrect because all channels in an impeller cascade are acting *in parallel.*

3 In equation (12), it can be shown that the terms representing the hydraulic radius of a non-circular channel should be of the form:

$$\left(\frac{1}{b} + \frac{Z}{\pi D \sin \beta}\right),$$

since this equation is referenced to the *relative velocities* in the blade channel for which the approximate "rectangular" dimensions are b on one side and $h = \dfrac{\pi D \sin \beta}{Z}$ on the other side. This means that the additional $\sin \beta$ term must be carried through all of the subsequent derivations.

4 It is not clear how the slip correction term K_s or the friction factor λ are to be assigned in equations (6) and (12). K_s is taken as an empirical modification of the Stodola slip factor (which is not particularly noted for universally accurate practical application). While the proper selection of K_s may indeed compensate for the practical inadequacies of the Stodola slip factor one must know how to go about making this proper assignment of value to K_s.

With regard to the friction factor λ, in a strict sense this should be chosen as representative of the impeller friction loss only since this is the loss represented by equation (12). Later on in the paper, however, it is at least implied that λ is taken as representative of a greater proportion of the head loss in the stage—which includes impeller incidence losses, blade loading losses, impeller exit mixing losses, diffuser losses, etc., to which equation (12) is inapplicable. Thus, an explanation should be given as to how values are assigned to λ.

5 Assuming that equation (12) is admissible, equations (13), (14), (15), and (16) all appear to be correctly derived and there can be no question about the validity of equations (17) and (18). Consistent with all of these steps, however, there appears to be some unexplained assumptions (or perhaps some errors) in obtaining equations (19) and (20–21).

In (17),

$$B = \frac{a_1}{a_0} \quad \text{and} \quad C = -\frac{a_2}{a_0}.$$

Also, the form of the cubic equation, for which (18) is a solution, is:

$$Z^3 + 3bZ^2 + 3cZ + d = 0.$$

Hence,

$$B = 3b = \frac{a_1}{a_0}$$

$$c = 0$$

$$d = -\frac{a_2}{a_0},$$

and it can be shown that $b = m = \dfrac{a_1}{3a_0}$ and that $d = 2n$ or $n = -\dfrac{a_2}{2a_0}$.

When the constants from (16) are substituted directly into these relations, the following rigorous equations are obtained for m and n, and these are *not* universally identical to those shown in the authors' equations (19) and (20):

$$m = \frac{\pi}{6}\,\frac{\left[\dfrac{1}{D_1{}^2b_1{}^3} + \dfrac{1}{D_2{}^2b_2{}^3}\right]}{\left[\dfrac{1}{b_1{}^2D_1{}^3} + \dfrac{1}{b_2{}^2D_2{}^3}\right]} \qquad (19a)$$

$$n = -1.066K_s\left(\frac{N}{Q}\right)^2 \frac{\sin^4\beta}{\lambda}\,\frac{(D_2{}^2 + D_1{}^2)}{(D_2 - D_1)\left[\dfrac{1}{b_1{}^2D_1{}^3} + \dfrac{1}{b_2{}^2D_2{}^3}\right]} \qquad (20a)$$

The differences between these relations and those of the authors will seriously affect the value of the optimum number of impeller blades which may be illustrated by resolving the example of Appendix A.

Although the specific value of K_s was not given in Appendix A, the authors' solution implies that $K_s \cong 0.840$ and this was used (along with all of the other given data) for substitution in the author's equations (16) and (17) with the following result:

$$Z^3 + 11.61Z^2 - 2914 = 0,$$

which is satisfied by $Z \cong$ *11.28*. The values of m and n for the configuration of the experimental impeller would then become 3.87 and -1457, respectively; not $m = 4.8$ and $n = -462$ as stated in the paper (though these do follow correctly from the authors' equations (19) and (20)).

Obviously, with an apparent optimum of between 11 and 12 blades from a more rigorous solution of the cubic equation for Z, there is no longer a correlation with the experimental results and the major conclusions of the paper would become invalid. In order to resolve this situation, the authors must supply a rational explanation for their equations (19) and (20)—otherwise, the results must be viewed as coincidental.

6 It is anticipated that further discrepancies may be expected in equations (24) and (25), especially since β also influences the hydraulic radius as mentioned in item 4.

7 Equation (26) is affected by the same discrepancies that were noted for equation (21), from which it was derived.

8 In Fig. 4, the impeller cover outside diameter at the inlet is incorrectly dimensioned.

9 It should be pointed out somewhere in the text that the test flow coefficient $\phi = \dfrac{Vm_2}{U_2}$ is based upon the gross discharge area of the impeller, i.e. without impeller blade blockage, which increases with Z.

[4] Aerodynamic Specialist, MSG Advanced Development, Carrier Air Conditioning Co., Syracuse, N. Y.

10 In Appendix A, the value of K_s should be stated; $H = 42M$ does not agree with the "optimum" head of $52M$ obtained with the 7 bladed impeller; the use of a "volumetric efficiency" (95.6 percent) in equation (25) should be explained; and $X = 0.7925$, $Z = 7$ satisfies equation (28), not equation (27).

11 In Appendix B, equations (30) and (31) will not be essentially correct when the discrepancies noted in item 6 are properly taken into account.

12 The authors compare their determinations of optimum number of impeller blades with those obtained using empirical methods proposed by Pfleiderer and Stepanoff. In addition to their own tests, those of Varley, Bommes, and Kasai are cited as experimental "proof" of the validity of these methods.

A number of other methods of empirically estimating an optimum number of blades may be found in the works of Eckert [16],[5] Ris [17], Tsitkin [18], Wosika [19], and Kulakov [20] which have been summarized by Anisimov, et al. [21], and there are probably many more. In the experimental *compressor* study conducted by Anisimov, et al., it was determined that $Z = 14$ to 18 provided best performance, while $Z = 10$ to 12 resulted in the best stable range. Considering the criterion of optimum stage performance, these investigators found that the methods of Eckert, Ris, Stepanoff, and Wosika all provided reasonable agreement with their experimental findings. The method of Pfleiderer apparently produced too high an estimate of the optimum number of blades when a "compressor" constant of 11.0 was substituted for the "pump" constant in the (author's) statement of Pfleiderer's expression (equation (27)). When 6.5 is taken as the constant, the optimum number produced by the Pfleiderer equation is then in fair agreement with the Russian experimental compressor determination ($Z \cong 14$). Thus, the "competitive" empirical methods, chosen for comparison by the authors, seem to work well enough for compressors too, as do the other methods mentioned previously. For compressors, however, there appears to be a shift to somewhat lower blade numbers when optimization of stable range is desired.

13 It is questionable that a high degree of precision is really necessary in determining an optimum number of impeller blades because the experimental observations usually provide a range of choice (e.g. $Z \cong 5$ to 7 for the authors' experimental pump), and because other considerations such as blade blockage effects, sympathetic frequencies in other parts of the system and fabrication costs may predominate.

It is now also questionable that the authors' methods will provide any higher degree of precision in this estimate because of the lack of rigor in their "theoretical" development and because of the uncertainties associated with the choices of K_s and λ. Even if the authors should be able to justify their derivations of m and n on a rational basis, these latter factors may shade the results. As an example of this, referring again to Appendix A, only slightly different values of K_s and λ could have resulted in a predicted optimum blade number of $Z \cong 6$. At the same time, due to the apparent scatter in the test results, an experimental optimum could just as well have been interpreted to occur at $Z = 6$, and incidentally, it would have been very helpful in this regard if the authors had added a test configuration with an impeller having five blades. Also, it is expected that a very careful study of the test data contained in Figs. 5 through 7 would reveal that experimentally derived values of K_s and λ really differ from stage to stage and that they do not precisely equal the assumed values for any of the stages.

14 While the authors are to be commended for their efforts in searching for a better basic understanding of the influences which result in achievement of optimum impeller blade numbers and blade angles of centrifugal impellers, it is the opinion of this observer that they have fallen far short of this goal.

[5] Number 16–21 in brackets designate Additional References at end of discussion.

Additional References

16 Eckert, B., "Osevyye i tsentrobezhnyye kompressornyye," Mashgiz, 1959.

17 Ris, V. F., "Tsentrobezhnyye kompressornyye mashiny," Mashgiz, 1951.

18 Tsitkin, S. I., "Tsentrobezhnyye kompressory, gazoduvki i ventilyatory," Mashgiz, 1950.

19 Wosika, L. R., "Radial Flow Compressors and Turbines for Simple Small Gas Turbines," TRANS. ASME, Vol. 74, 1952.

20 Kulakov, V. M., "O chisle lopatok v kolese turbocompressora," Trudy MVTU inn. Baumanax, 1958, No. 75.

21 Anisimov, S. A., Rekstin, F. S., and Seleznev, K. P., "The Influence of the Number of Vanes in the Efficiency of the Centrifugal Wheel with a Single-Stage Cascade," Energomachinostroyeniye, No. 221, 1962; Translation FTD-TT62-1816, Power Machine Construction, Foreign Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, March 4, 1962.

Authors' Closure

We thank the discussers for their general interest in the paper. In reply to Mr. Kovats, we agree that the efficiency might have been improved by one or two points if the vane thickness was reduced from the present value; but this leads to the difficulty of fixing the vanes securely to the impeller shrouds. The efficiencies given in the Paper are overall (energy) efficiencies, and what Mr. Kovats refers to his Pump ($D = 18$ cms) is the hydraulic efficiency. Surely it is not a surprise to get 69 percent hydraulic efficiency for a pump of $D = 18$ cms. However it should not be forgotten that blade number and angle are not two independent parameters. If we reduce the blade number then blade angle also reduces, and hence the lower values quoted by Mr. Kovats for his Pump. This can very easily be seen from equations (18) to (20). Since Mr. Kovats has not given his pump impeller details, it is difficult to calculate the optimum blade number and angle. The entrance loss has been disregarded since its influence was reported to be less (see reference [1]). Stoloda's coefficient "Ks" is a function of the specific speed of the pump. If the selection of Ks is done on the basis of specific speed, there is no choice of getting β values of 1:2. The head coefficient $\left(\psi = \frac{2gH}{U^2}\right)$ used for other's work (Fig. 10 to 15) is twice the value defined in the text. It is also obvious that optimum β value calculated using the present equations simultaneously satisfying optimum blade number will never exceed more than 30 deg for conventional designs and hence his fear of high NPSH is unfounded. It is unfortunate that Mr. Kovat's does not give any references, though he stated that there were several methods of calculating optimum "z" and "β." The test impeller with $z = 7$ and $\beta = 30$ deg is surely the best pump for the duty specified, since both impellers of $z = 6$ and 8 resulted in lower efficiency compared to $z = 7$.

We are glad to note that Mr. Zerrer's conclusions are in close agreement with our results. It is gratifying that for low specific speed pumps (like the present one), he achieved an optimum blade number of seven and optimum blade angle of 27 deg. We agree with Mr. Zerrer regarding the vane loading with increasing the number of blades for low specific speed pumps and also the limitations of λ and Ks.

Mr. Wiesner's statements are all wrong and baseless, further his remarks about the reviewers are mischievous. Let us proceed point by point. Items 2, 8, 10, and 12 are typographical errors.

1 It is well known that slip is a function of number of blades and blade angle. It is also clear that the vane efficiency or vane ineffectiveness is related to slip. But this vane ineffectiveness is more with less number of blades and less with large number of blades. Even though by definition this vane ineffectiveness is not a loss, it is simple to imagine that small number of blades result in large eddy loss. With large number of blades, since the interblade space is small, the eddy losses are small. Thus, greater channel circulation leads to greater slip and also greater eddy loss. Otherwise how can one explain for higher input power with im-

pellers of lower blade number say, $z = 4$ and 6 in Fig. 7. Evidently impeller of $z = 7$ had the lowest input power of all the impellers listed except when $z = 2$.

This in itself suggests that slip is not free from eddy loss. The eddy loss may be due to several reasons which is very difficult to estimate for a real case. However, it is possible to assume that slip and eddy loss are proportional. It is in this sense that slip was brought into picture. In a simple way if we assume that one interblade passage of an impeller is similar to a conventional diffuser, then the slip is assumed equivalent to diffusion or eddy loss. Thus, it is the diffusion losses in the impeller which were taken equivalent to slip.

3 Mr. Wiesner has mistaken the pressure drop to total energy loss across a parallel system. Consider a simple case of a flat plate in a moving stream. If we consider the total energy drop due to friction across the flat plate of unit width for the quantity of flow Q, and if it is equal to say x, then if we put two flat plates of unit width, and length equal to same as the first case, then the total energy drop due to friction across the two plates would be $2x$ and not x, even though the pressure drop is same across each of the plates. Hence equation (11) is perfectly valid and is correct. In other words, one can very easily say that with increase in the wetted surface in a flow system, the frictional losses proportionately increase.

4 Regarding the hydraulic radius, assuming that "b" and "e" represent breadth and interblade distance, respectively, of an impeller, hydraulic radius (R_h) = area/wetted perimeter = $\frac{b.e}{2(b+e)}$.

Since $e = \frac{\pi D}{z}$, $R_h = \frac{1}{\left(\frac{2z}{\pi \cdot D} + \frac{2}{b}\right)}$. Hence what Mr. Wiesner writes for hydraulic radius is wrong and hence his subsequent statements are false (see footnote 6, p. 52).

5 The value of Ks varies from 0.7 to 1.2 for high pressure pumps of the type described in the paper. Assuming the lower and upper limiting values of Ks for the specific speed range of high pressure pumps, one can easily estimate the value of Ks for the pump under consideration. However, λ values are selected on the basis of exit breadth of the impeller and Reynolds number. Excellent tables are available for impeller breadths varying from 0.4 in. to 20 in. and for different materials at different Reynolds numbers (See footnote 6, p. 32).

6 There are no errors in equations (19) and (20)–(21). Since it is not desirable to have sudden acceleration and deceleration for the fluid within the impeller passage it is generally designed such that $b_1D_1 = b_2D_2$. Simplifying equations (19a) and (20a) with the assumption of equal flow areas at inlet and exit one will get equations (19) and (20) as given in the paper. Hence equations (19) and (20) are more rigorous than equations (19a) and (20a), since we have not given any provision in the basic equations for accelerating or decelerating fluid.

For the reason mentioned previously, Mr. Wiesner's cubic equation for the present case is invalid and hence his conclusions thereafter. It is enough to state that using the equations derived in the main paper, we obtained satisfactory comparison with seven test impellers tested in different countries under different system conditions, which, in itself shows that our experimental and theoretical results are not coincidental.

7 Since Mr. Wiesner's equation in his item (4) is wrong, equations (24) and (25) are valid and correct.

9 Equation (26) is not affected since equation (21) is correct.

11 Test flow coefficient ϕ is based upon gross discharge area of impeller as defined in the Nomenclature.

13 In Appendix A, the value of $Ks = 0.85$, and the volumetric efficiency of 95.6 percent was obtained from the following equation

Vol. efficiency $= \frac{1}{1 + 0.68(n_s)^{-2/3}}$ where ns was taken as 65 (M.K.S.) units.

In the preliminary design of the test impellers, value of $H = 42M$ was assumed based on the value obtained with another impeller in the same pump system. However because of better design and a bit more outside diameter and breadth resulted a higher head than assumed.

14 Since Mr. Wiesner's item (6) is incorrect, equations (30) and (31) are still valid.

15 We have listed in our reference [15], as many as 66 references. However for want of space we selected only 14 references which are very relevant. Mr. Wiesner is apparently confusing himself as well as others by mixing the optimum number of blades for pumps and compressors. In general pumps have lower exit blade angles of the order of 30 deg, whereas compressors have higher blade angles of the order of 90 deg (due to strength consideration). Hence higher blade angles for compressors result in higher blade number (10 to 20) and lower blade angles for pumps result lower blade number (5 to 7). The main aim of the present paper is to bring out the interdependence of blade angle and blade number. We think that it will not be out of place to mention that at I.I.T. Bombay, we have successfully designed and tested a blower using the equations reported in the present paper, which resulted in a blade number of 14 for $\beta_2 = 85$ deg.

16 The results presented in the paper are for a single stage end suction centrifugal pump. We do not understand why Mr. Wiesner mentions "stage to stage" in his statements. Apparently he is assuming a multistage pump. We have already made it clear that Ks and λ can reasonably be assumed from the available literature, hence there is no question of interpreting $z = 6$ as the optimum.

17 It is our opinion that we have mostly cleared Mr. Wiesner's dubious thoughts on pumps.

[6] Kovats, A., de, and Desmur, G., *Pumps, Fans and Compressors*, Blackie & Son, Ltd., London, 1958.