Gravitational wave heating of stars and accretion discs

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ABSTRACT

We investigate the electromagnetic counterpart of gravitational waves (GWs) emitted by a supermassive black hole binary (SMBHB) through the viscous dissipation of the GW energy in an accretion disc and stars surrounding the SMBHB. We account for the suppression of the heating rate if the forcing period is shorter than the turnover time of the largest turbulent eddies. We find that the viscous heating luminosity in 0.1 $M_\odot$ stars can be significantly higher than their intrinsic luminosity, but still too low to be detected for extragalactic sources. The relative brightening is small for accretion discs.

Key words: black hole physics – gravitational waves – galaxies: nuclei.

1 INTRODUCTION

The coalescence of supermassive black hole binaries (SMBHBs) generates gravitational waves (GW) which are a primary source for the proposed Laser Interferometric Space Antenna (LISA). SMBHBs are inevitable outcomes of galaxy mergers. Spatially resolved active galactic nuclei have been observed (Komossa et al. 2003; Bianchi et al. 2008; Green et al. 2010; Fabbiano et al. 2011; Koss et al. 2011). In addition, spectroscopic surveys (Comerford et al. 2009; Liu et al. 2010b; Smith et al. 2010) and observations that combine ground-based imaging show numerous systems containing compelling SMBHB candidates with pc to kpc separations (Rodriguez et al. 2006; Liu et al. 2010a; Fu et al. 2011; McGurk et al. 2011; Shen et al. 2011). Hydrodynamic simulations of galaxy mergers also predict SMBHB pair formation (e.g. Escala et al. 2004, 2005; Di Matteo, Springel & Hernquist 2005; Hopkins et al. 2006; Robertson et al. 2006; Callegari et al. 2009; Colpi et al. 2009; Blecha, Loeb & Narayan 2012).

Electromagnetic (EM) counterparts to GW sources complement the GW detection by determining the host galaxy redshift and the environment of the sources (Kocsis et al. 2006; Phinney 2009). A large variety of EM signatures have been proposed to accompany the coalescence of SMBHBs (Haiman et al. 2005; Schnittman 2011). In the pre-merger phase, the torques of the SMBHB excavate a hollow region in the disc and lead to periodic accretion across the gap on the orbital time-scale (Hayasaki, Mineshige & Ho 2008; MacFadyen & Milosavljević 2008; Cuadra et al. 2009). After the merger, the recoil of the black hole remnant and its sudden mass loss due to the GW burst produce shocks in the accretion disc, which lead to EM signals (Bode & Phinney 2007; Lippai, Frei & Haiman 2008; Schnittman & Krolik 2008; Shields & Bonning 2008; O’Neill et al. 2009; Rossi et al. 2010). The recoil of the black hole remnant changes the tidal disruption rate of stars due to the refilling of the loss cone and the wandering of black hole remnant (Stone & Loeb 2011a,b; Li et al. 2012). Finally, the infall of gas on to the black hole remnant produces an EM afterglow (Milosavljević & Phinney 2005; Tanaka & Menou 2010).

In this paper, we consider the viscous dissipation of GWs generated by an SMBHB in a neighbouring gaseous medium. In particular, the velocity shear induced by GWs in the gas is damped by viscosity. The dissipated GW energy turns into heat, and produces an EM flare. Unlike other EM counterparts, the brightening here follows promptly within a few hours to days after the coalescence of the SMBHB (Kocsis & Loeb 2008). The effect provides a unique test of general relativity for the interaction of GWs with matter. In Sections 2 and 3, we investigate GW dissipation in a gaseous accretion disc and stars in the vicinity of the SMBHB. We examine the suppression of the effect if the forcing period is shorter than the turnover time of the largest eddies (Krolik 2010), in analogy to a similar treatment of tidal heating in binary stars (Zahn 1966; Goldreich & Keeley 1977). Finally, we discuss our conclusions and their implications in Section 4.

2 METHOD

We start by presenting our approach for estimating the GW heating inside an accretion disc and stars due to turbulent viscosity. Following Kocsis & Loeb (2008), we approximate the GW luminosity by matching the Newtonian inspiral luminosity prior to merger ($t < 0$), the peak luminosity at the merger ($t = 0$) and the decay luminosity afterwards ($t > t_1$), where $t_1$ can be fixed from this matching procedure. Specifically, in the Newtonian inspiral regime, the luminosity is

$$L_{\text{GW inspiral}} = \frac{32}{5} G^4 M^3 \mu^2 \frac{\dot{a}^5}{c^5},$$

where $M = M_1 + M_2$ is the sum of the masses of the SMBHB members, $\mu = M_1 M_2 / M$ is the reduced mass of the SMBHBs and $a$ is

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is the separation between the SMBHBs, which can be expressed as

\[ a = \left( \frac{256 G^3}{5 c^5} \mu \tilde{M}^2 (t_1 - t) \right)^{1/4}, \]

assuming a circular orbit. The peak luminosity is approximated from numerical simulations (Berti et al. 2007; Buonanno, Cook & Pretorius 2007) as

\[ L_{GW, peak} \approx 10^{-3} \frac{c^5}{G} \left( \frac{\mu}{M} \right)^2, \]

and the ringdown luminosity is set to be

\[ L_{GW, ringdown} = L_{GW, peak} \exp \left( -\frac{c(t_1 - t)}{5 \tau_{GW}^2} \right), \]

where \( \tau_{GW} = GM/a \) is the gravitational radius of the SMBHB. The peak luminosity is modified by a factor of 2 (Berti et al. 2007; Buonanno et al. 2007) due to different magnitudes and orientation of the spin of the SMBHB. In this paper, we assume the masses of the two black holes are the same.

With the approximated expression of GW luminosity as a function of time, the dissipation of GW energy inside a viscous medium can be calculated by solving the weak-field Einstein equation (Hawking 1966; Weinberg 1972):

\[ \dot{e}_{GW} = \frac{16\pi G \eta}{c^2} e_{GW}, \]

where \( e_{GW} \) is the dissipation rate, \( \eta \) is the dynamical viscosity and \( e_{GW} \) is the GW energy density. \( e_{GW} \) can be obtained from \( e_{GW} = Y(\theta)L_{GW}/4\pi c r^2 \), where \( \theta \) is the angle relative to the total angular momentum vector, \( Y(\theta) = \frac{5}{2}[\sin^2(\theta/2) + \cos^2(\theta/2)] \). We use the average value \( \langle Y \rangle = 1 \) below. With \( L_{GW} \) derived, the only unknown parameter is the dynamical viscosity of the medium that the GW passes through. The dissipation rate of the GW energy gives the heating rate of any gaseous medium such as an accretion disc and stars.

Next, we estimate the dynamical viscosity for stars. We use stellar models produced by Modules for Experiments in Stellar Astrophysics (MESA: Paxton et al. 2011), a one-dimensional stellar evolution code, and we consider stellar models whose properties are included in Table 1. We associate the dynamical viscosity with the mixing-length theory diffusion coefficient, which is directly provided in the simulated models by MESA. When the period of the driving force is smaller than the largest eddy turnover time, the eddy viscosity depends on the ratio of the period to the largest eddy turnover time in one of two possible ways:

\[ \eta = \eta_{\text{min}} \left( \frac{\tau_{GW}}{2 \tau_{r}} \right)^{1}, \]

or

\[ \eta = \eta_{\text{min}} \left( \frac{\tau_{GW}}{2 \tau_{r}} \right)^{2}, \]

where \( \eta_{\text{min}} \) is the intrinsic viscosity in the absence of shear force with short period, \( \tau_{r} \) is the largest eddy turnover time-scale and \( \tau_{GW} \) is the shear force period, which is calculated as \( 2\pi/\omega_{GW}, \) where \( \omega_{GW} = 2\sqrt{GM/a^3} \) in the inspiral phase \( a < 6R_{\odot} \) and \( 0.25(GM/c^3) \) after the ringdown, and extrapolates linearly during the transition according to Buonanno et al. (2007). The viscosity scaling given by equation (6) is discussed in Zahn (1966, 1989) and Zahn & Bouchet (1989) and that given by equation (7) in Goldreich & Keeley (1977) and Goldreich & Nicholson (1989). Observations are more consistent with Zahn’s scaling for pulsating stars in the red edge of the instability strip (Gonczi 1982), for tidal circularization of binary stars (Verbunt & Phinney 1995; Melbon & Mathieu 2005), while the damping of the solar p-mode oscillations is more consistent with Goldreich’s scaling (Goldreich & Kumar 1988; Goldreich, Murray & Kumar 1994). Recently, Penev et al. (2009) studied turbulent viscosity in low-mass stars using the perturbative approach of Goodman & Oh (1997), taking into account compressible fluid and anisotropic viscosity. Their simulation suggests a linear scaling. However, Ogilvie & Lesur (2012) found results more consistent with Goldreich’s scaling when studying the limit of a low-amplitude short oscillation period shear. We considered both scalings for stars in this paper.

With the viscosity for stars and \( L_{GW}(t) \) in hand, the GW heating rate can be estimated using equation (5). The EM luminosity increase can be estimated by solving the radiative transfer equations:

\[ t_\circ(r) \frac{d}{dt} \Delta f(r) + \Delta f(r) = e_{\text{heat}}, \]

\[ L_{GW} = \int_{\text{star}} \Delta f(r) dV, \]

where \( \Delta f(r) \) is the excess EM signal produced per unit volume as a function of location in the star. \( L_{GW} \) is the excess EM luminosity associated with GW heating and \( t_\circ(r) \) is the cooling time as a function of the location, which characterizes the time it takes for heat to travel to the surface. We estimate the latter by taking the integral of the minimum of the photon diffusion time, \( dr/c \times [\tau(r) - (R_e - r)dr(r)/d\tau] \), and the turbulent convection time, \( dr/v_t(r) \), in each spherical shell inside the star, where the optical depth, \( \tau(r) \), and the convective velocity, \( v_t(r) \), are obtained from the MESA simulation, and \( R_e \) is the radius of the star.

Finally, we estimate the heating in accretion discs. We adopt the geometrically thin, optically thick standard accretion disc model, where the angular momentum transport is associated with the internal stresses due to turbulence (Novikov & Thorne 1973; Shakura & Sunyaev 1973). Heat is dissipated locally by turbulent viscosity, and transported vertically outwards by photon diffusion or advection. Specifically, the viscosity of the accretion disc is

\[ \eta(r) = \frac{2}{3} \Omega^2(r)^{\alpha}, \]

where \( \Omega^2(r) = GM/r^3 \) is the angular velocity, \( \alpha \) is a constant which we assume to be 0.3 (King, Pringle & Livio 2007), and \( P \) is the turnover time
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3 RESULTS

First, we consider the GW heating of nearby stars. As an example, we examine the GW heating light curve for a 0.1 M\(_\odot\) star (stellar model 2) surrounding an M = 10\(^{6}\) or 10\(^{7}\) M\(_\odot\) SMBHB, respectively. Using equations (8) and (9), we calculate F(t) and plot the GW heating light curve in Fig. 1. We assume that the star is located at \(d = 5\) tidal radii from the SMBHB (corresponding to 320 and 15R\(_g\)) for 10\(^{6}\) and 10\(^{7}\) M\(_\odot\) SMBHBs, respectively. Note that since the GW luminosity is proportional to \((d/R_g)^{-2}\), the GW heating effect is much larger around more massive SMBHBs because the viscosity suppression for a high-mass SMBHB is smaller.

Fig. 1 shows that the excess luminosity of the star surrounding the 10\(^{7}\) M\(_\odot\) SMBHB is much higher than the intrinsic luminosity of this star. The peak luminosity surrounding the 10\(^{7}\) M\(_\odot\) SMBHB is much higher than the intrinsic luminosity of this star.

To examine the influence of the GW heating in different types of stars, we consider stellar models of different stellar masses and ages as included in Table 1. We include the extreme cases with 0.1 and 100 M\(_\odot\) stars. We plot the ratio of the peak heating luminosity to the intrinsic luminosity for different stellar models in Fig. 2 with Zahn’s scaling. We find that the influence of the GW heating is more significant as the metallicity of the star increases, and GW heating is not significant for very massive (M \(\geq\) 100 M\(_\odot\)) stars.

Next, we discuss the heating effects in accretion discs. For \(\alpha\) and \(\beta\) discs, we solve equations (11) and (12) for the heating flux, and plot the heating light curve of the disc due to GW heating in Fig. 3. The accretion disc is punctured with an inner hole. This geometry is essentially ‘frozen’ during the final GW merger time-scale with a gap radius \(\gtrsim 100\) M\(_\odot\) for \(\alpha\) discs (Milosavljevic & Phinney 2005).

Recent MHD simulations by Noble et al. (2012) indicate that the stresses may be enhanced in a binary, such that gap decoupling

is strongly suppressed in the stellar interior (\(\tau_{GW}/\tau_{el}(r) \ll 1\) for \(r \lesssim 0.99R_{\text{rad}}\)), the heating effect is negligible to the star as a whole. In addition, these stars are very faint; the absolute peak GW heating luminosity in the star is typically too faint to be observed outside of the Galaxy.

Since the turnover time of turbulent eddies is much longer in the interior of the star than that at the surface, the energy is mostly dissipated at the surface. Since the cooling time near the surface (~200 s) is short compared to the peak GW time-scale (~10R\(_g\)/c \(\sim 500\) s\(_M\)/M\(_\odot\)/10\(^7\) M\(_\odot\)), the light curve of the star closely tracks the luminosity curve of the GW. When the GW driving period is shorter than the eddy turnover time, the viscosity caused by the eddy depends on the ratio \(\tau_{GW}/\tau_{el}\) where the exact scaling is uncertain as discussed in Section 2. For stars surrounding a 10\(^7\) M\(_\odot\) SMBHB, the differences between the two scalings are smaller as the period of peak GW emission for this SMBHB mass is much more comparable to the surface eddy turnover time in a 0.1 M\(_\odot\) star.

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Figure 2. Ratio of the peak GW heating luminosity to the intrinsic stellar luminosity. The horizontal axis shows the mass of the SMBHB, and the vertical axis plots the distance ($d$) between the star and the SMBHB in units of the tidal radius ($r_t$). First row: models 1, 2, 3; second row: models 4, 5, 6; third row: models 7, 8, 9. Solid black line indicates where the distance between the star and black hole binary is 6$R_g$, the radius of the innermost stable circular orbit around a non-spinning black hole. In the last panel, the points in the figure lie out of 6$R_g$, and so the black line is not shown. The first two rows correspond to 0.1 M$_\odot$ stars with metallicity $Z = 0.16$ and 0.01 respectively, and the last row corresponds to 100 M$_\odot$ stars. GW heating is most significant for high-metallicity low-mass stars.

Figure 3. The excess luminosity relative to the disc luminosity due to GW heating on an accretion disc (inner disc truncated at 20$R_g$) before ($t < 0$) and after ($t > 0$) the binary coalescence event. The time axis is shown on a logarithmic scale at both negative and positive values (in units of $R_g$). The SMBHB mass is 10$^7$ M$_\odot$. Solid lines correspond to the frequency dependence ($\tau_{GW}/2\tau_l$)$^{11/9}$ derived according to the energy spectrum of accretion disc based on MHD simulations by Flock et al. (2011), and the dashed lines correspond to the scaling ($\tau_{GW}/2\tau_l$)$^2$, assuming Kolmogorov turbulence. Occurs further in, at 20$R_g$. We optimistically adopt this value for our estimates, which implies a larger heating rate than that for a larger gap radius. We integrate over the accretion discs between the inner and outer boundary. We set the latter to 2 x 10$^4$R_g, but this value does not influence our result as the heating in the outer accretion disc is negligible. We include the different light travel time from different accretion disc surface elements along the line of sight. Our calculation of the heating in the accretion discs improves the simplified treatment of Kocsis & Loeb (2008) by including the dependence of viscosity on the ratio of the GW driving period to the largest eddy turnover time, which suppresses the dissipation of GWs. We consider two cases in this plot. Following the perturbative turbulence derivation by Goodman & Oh (1997), the power-law index is 2 for Kolmogorov turbulent scaling, and 11/9 according to MHD disc simulation by Flock et al. (2011). The eddy turnover time increases rapidly as the radius increases, and so the suppression of the GW heating is less significant for discs truncated closer to the SMBHB. Therefore, the heating luminosity is more significant for discs that are truncated closer to the SMBHB.

4 DISCUSSION

In this paper, we considered the dissipation of GWs in an accretion disc or stars surrounding an SMBHB. We have found that the GW heating luminosities of the accretion disc and stars are low, and make no significant EM flare relative to their intrinsic luminosity.
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except for low-mass stars ($\sim 0.1 M_\odot$). The integrated excess luminosity from heated low-mass stars is too low to be observed in galactic nuclei as they are faint. Assuming a Bahcall–Wolf distribution of stars or assuming a collision time-scale larger than 1 Myr, we find that only a few stars are expected to be within 5 tidal radii of a coalescing SMBHB, where the GW heating effect is significant. Therefore, the overall brightening of the stellar cluster is negligible.

In order to be heated significantly by GWs, the stars need to be close to the SMBHB. One possible avenue is that stars get caught in mean motion resonances (such as Trojan resonances) and move inwards as the SMBHB merge (Schmittman 2010; Seto & Muto 2010). This is only effective for SMBHB with an unequal mass ratio $q \lesssim 10^{-2}$; otherwise the stars get ejected before the coalescence. Another possibility is for stars to get captured or form in the outer parts of accretion discs, and migrate inwards by processes analogous to planetary migration (Karas & Šubr 2001; Miralda-Escudé & Kollmeier 2005; Levin 2007).

We assumed that GW energy is dissipated locally through turbulent viscosity. The damping of shear stress by eddy viscosity in stars was found to be consistent with observations in the context of the tidal circularization of binaries (Verbunt & Phinney 1995; Meibom & Mathieu 2005). The underlying accretion disc model is uncertain since the disc structure is unstable to both thermal and viscous instabilities. Recently, Blecha et al. (2011) found that radiation-dominated discs differ significantly from the standard disc models, where the dissipation associated with the turbulent cascade and radiative damping dissipate energy non-locally. It remains to be seen whether the GW heating effect is more prominent in alternative disc models.

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