$$D_jP_{ij} = \frac{1}{2\Delta\theta^2}[P_{i+1,j} + P_{i-1,j}]$$

$$+ \frac{3}{4\Delta\theta H_{ij}}\left(\frac{\partial H}{\partial\theta}\right)_{ij}[P_{i+1,j} - P_{i-1,j}] + \frac{R^2{}_j}{2\Delta R^2}[P_{i,j+1} + P_{i,j-1}]$$

$$+ \frac{1}{4\Delta R}\left(\frac{3R^2{}_j}{H_{ij}}\frac{\partial H}{\partial R_{ij}} + R_j\right)[P_{i,j+1} - P_{i,j-1}]$$

$$- \frac{\Delta R^2{}_j}{2H^3{}_{ij}}\left\{\frac{\partial}{\partial R}(U'_r H) + \left(\frac{U'_r H}{R}\right) + \frac{1}{R_j}\frac{\partial}{\partial\theta}(U'_\theta H)\right\}_{ij} \quad (35)$$

$$D_j = \left[\frac{1}{\Delta\theta^2} + R^2{}_j\frac{1}{\Delta R^2}\right] \quad (36)$$

$$H = (1 + a\xi + b\xi^2)^m + \epsilon\cos n\theta + \epsilon_i R\cos(\theta + \psi) + \epsilon_s \quad (37)$$

These equations can be combined as follows:

$$P_{ij} = A_{ij}P_{i+1,j} + A_{2j}P_{i-1,j} + A_{3j}P_{i,j+1} + A_{4j}P_{i,j-1} - A_{5j} \quad (38)$$

where

$$A_{1j} = [(1/2\Delta\theta^2) + (3/4\Delta\theta H_{ij})(\partial H/\partial\theta)_{ij}]/D_j \quad (39)$$

$$A_{2j} = [(1/2\Delta\theta^2) - (3/4\Delta\theta H_{ij})(\partial H/\partial\theta)_{ij}]/D_j \quad (40)$$

$$A_{3j} = \{(R^2{}_j/2\Delta R^2) + (1/4\,\Delta R) \times [(3R^2{}_j/H_{ij})(\partial H/\partial R)_{ij} + R_j]\}/D_j \quad (41)$$

$$A_{4j} = \{(R^2{}_j/2\Delta R^2) - (1/4\,\Delta R) \times [(3R^2{}_j/H_{ij})(\partial H/\partial R)_{ij} + R_j]\}/D_j \quad (42)$$

$$A_{5j} = (\Delta R^2{}_j/2H^3{}_{ij})\left\{\frac{\partial}{\partial R}(U'_rH) + (U'_rH/R) + \frac{1}{R_j}\frac{\partial}{\partial\theta} \times (U'_\theta H)\right\}\Big/D_j \quad (43)$$

$$D_j = 1/\Delta\theta^2 + R^2{}_j/\Delta R^2 \quad (44)$$

If it is assumed that the pressures in the circumferential direction ($P_{i+1,j}$, $P_{i-1,j}$) are known or assumed, then one may start at one of the boundaries of the seal and work across the seal obtaining expressions for P_{ij} in terms of the starting point boundary pressure and $P_{i,j+1}$. Thus, proceeding across the seal to the other boundary pressure, the last $P_{i,j+1}$ is known—i.e., it is the second boundary pressure and working back across the seal, all P_{ij}'s are determined on that radial line. Going on to the next radial line the procedure is repeated.

The equations used for this procedure

$$P_{ij} = X_jP_{i,j+1}, + Y_j \quad (45)$$

where

$$X_j = A_{3j}/(1 - A_{4j}X_{j-1})(j = 2, \ldots\ldots, j_{\max} - 1) \quad (46)$$

$$X_1 = 0 \quad (47)$$

$$Y_j = (A_{1j}P_{i+1,j} + A_{2j}P_{i-1,j} - A_{5j} + A_{4j}Y_{j-1})/ \times (1 - A_{4j}X_{j-1});\ (j = 2, \ldots, j_{\max}) \quad (48)$$

$$Y_1 = P_m \text{ (first boundary pressure)} \quad (49)$$

$$P_{i,j\max} = P_n \text{ (second boundary pressure)} \quad (50)$$

The calculational procedure is as follows:

Step 1: Short-Bearing Approximation (i.e., $\partial P/\partial\theta = 0$)

(*a*) $A_{1j} = A_{2j} = 0$.
(*b*) $D_j = R^2{}_j/\Delta R^2$.
(*c*) Calculate X_j, Y_j, (j = 2,3, ... , $j_{\max} - 1$); i fixed.
(*d*) Calculate P_{ij} ($j = j_{\max-1}$, ... , 3, 2); i fixed.
(*e*) Repeat for all i's.

Step 2: First Correction

(*a*) Calculate A_{1j}, A_{2j} from equations (39) and (40).
(*b*) Use $P_{i+1,j}$ and $P_{i-1,j}$ from step 1.
(*c*) Calculate X_j, Y_j, (j = 2, 3, ... , $j_{\max-1}$); i fixed.
(*d*) Calculate P_{ij}, [equation (45)], ($j = j_{\max-1}$, ... , 3, 2); i fixed.
(*e*) Repeat for all i's.

Steps 3 to N: Additional Corrections

Repeat procedure under step 2, always using the $P_{i+1,j}$ and $P_{i-1,j}$-values from the previous step (or iteration).

This method is now in use on the GE 635 Seal Program and has proved to be very successful. The results obtained thus far indicate that for the full-film seal the short-bearing solution to Reynolds' equation is very close to the two-dimensional solution.

DISCUSSION

J. G. Pape[2]

The paper treats the interesting and so far rather puzzling phenomenon of radial pumping in radial face seals. Although this phenomenon is not a common experience in radial face seal operation and seems to be rather limited concerning the maximum pressure head that can be developed, it may nevertheless have a favorable or a detrimental effect on the actual leakage rate in seals.

In this paper a feasible possibility is presented that might explain the radial pumping phenomenon. According to the author a prerequisite for radial pumping action is a misaligned eccentric seal. Concerning the necessary misalignment the discusser doubts if this condition is always present in a well-designed seal. It was observed in experiments where the film thickness fluctuations were studied that usually two film thickness fluctuations per shaft revolution occur of virtually equal amplitude, compare [1, 15]. In terms of the author this means that $n = 2$, which was stated to be prohibitive for the occurrence of radial pumping. The fact that Denny [1], nevertheless, observed radial pumping seems to contradict this statement.

The analytical solution given in the paper for the case of a wavy eccentric seal, applying the narrow bearing approximation, is most interesting since it enables a quantitative estimate of the maximum pressure head that can be generated, or sealed against. The interesting case where $q = o$ is represented by equation (14) and it is shown by the numerical example that the stalling pressure to be expected is rather small. However, the author says that at smaller average film thicknesses the pumping effect will become relatively more important. It is interesting to examine this conclusion somewhat closer. It should be noted then that the average film thickness cannot be chosen arbitrarily. Due to hydrodynamically and hydrostatically developed pressures in the liquid film between the sealing faces there will be only one definite value for the average film thickness at a certain set of physical parameters. When only the effect of the hydrodynamic pressure generation is considered it was shown [15] that there exists a definite relationship between the average film thickness and the parameter $\mu\omega/p_a$, where p_a is the average film pressure. Introducing the parameter $\mu\omega/p_a$ into equation (14) yields:

$$\frac{p_i - p_o}{p_a} = \frac{3(r_0 - r_i)e\,\epsilon\cos\alpha}{(1 + 1.5\epsilon^2)\,\bar{h}_i^2}\frac{\mu\omega}{p_a} \quad (51)$$

In [15], equation (14), the relation between $\bar{h}_i$ and $\mu\omega/p_a$ is given for $n = 2$. Analogously it can be shown that for $n = 1$ the relation becomes:

$$\bar{h}_i = \left[\frac{(r_0 - r_i)^2}{2\pi}\epsilon\frac{\mu\omega}{p_a}\right]^{1/2} \quad (52)$$

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Substitution of (52) into (51) yields:

$$\frac{p_i - p_0}{p_a} = \frac{6\pi e \cos\alpha}{(1 + 1.5\epsilon^2)(r_0 - r_i)} \tag{53}$$

Relation (53) predicts the maximum achievable value of $(p_i - p_o)/p_a$ at $q = 0$. It appears that this value is about constant at small ϵ and constant values of e and α. Hence the maximum relative importance of the stalling pressure is about constant and is independent of film thickness. A quantitative estimate on basis of (53) says that $(p_i - p_o)/p_a$ is of the order 0.1. It must be noted, however, that this value will not be attained due to the detrimental effect of cavitation on the pumping action. In practical applications usually p_a is of the same order as the pressure to be sealed, therefore the pumping effect will not contribute much to the sealing performance. An exception may be formed by cases where the pressure level is high with respect to the pressure differential across the seal, as in the inboard seal of a high-pressure double seal arrangement.

Author's Closure

I wish to thank J. G. Pape for his thoughtful comments. It is always helpful to have someone else's reflections on your work. In reply to these comments, I would like to make the following observations:

1 For normal seal applications, I do not think that radial pumping is of concern. It should be of considerable concern in those applications where injestion of air into the system is a problem or where the seal separates two different fluids and the leakage must be in a specified direction so as to prevent contamination.

2 For radial pumping to occur, it is necessary to have eccentricity and misalignment *or* once per revolution waviness. Even though Denny measured two waves per revolution, their amplitudes were not *precisely* equal and thus there was a small component of once per revolution waviness.

3 The point made in regard to the average film thickness not being determined arbitrarily is a good one. Also, the relationship for the stalling pressure (53) derived by applying the results of [15] is most interesting. Caution must be exercised in applying this equation, however. First of all, if there were no surface waviness ($\epsilon = 0$), there would be no hydrodynamic load nor any pumping, yet (53) still predicts a pressure differential for $q = 0$. Second, (51) applies only for a full film whereas (52) relies on their being cavities. Therefore two incompatible relationships are combined to obtain (53). When cavities were present it was never possible to reduce the leakage to zero [10]. Unfortunately, this point was not made plain in this paper.