Discussion

The relative merits of the three solutions for the freezing of slab can be observed in part from Fig. 2 where the various \( \phi \)-
functions are given for \( 0 < \lambda \leq 2 \). Except for very small values of
\( \lambda \), \( \phi_0 \), the corresponding term which appears in
the exact solution for a semi-infinite solid. For all values of \( \lambda 
\) considered, \( \phi_0 \) is greater than \( \phi_0 \). This will result in a larger value
for \( \lambda \) which one would expect since the finite slab will freeze (or
melt) faster than a semi-infinite system with corresponding
boundary conditions. The solutions containing the quadratic
and cubic profiles should not be used when \( \phi_0 < \phi_0 \). Thus, for
small values of \( \lambda \), the solution involving the cosine function
should be utilized.

The use of the solution for the semi-infinite body for region 1
(and region 2) can be justified since it satisfies both energy equa-
tions and the boundary conditions in that region.

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DISCUSSION

T. R. Goodman

The authors have done an excellent job of applying the heat
balance integral to the posed problem. In their problem melting
occurs because the front-face of the slab is brought to a tempera-
ture above freezing, while the backface is insulated. This com-
plements a solution which I obtained some years ago in which
the frontface was subjected to a heat flux while the backface was
either insulated or held at constant temperature.

Authors’ Closure

The authors wish to thank Dr. Goodman for reviewing this
paper.

The main difference between the techniques used in this paper
and those developed by Dr. Goodman is the combined use of both
exact and approximate solutions. Thus, since we used exact
temperature distributions in the newly formed phases, our entire
solution becomes exact if the initial temperature of the slab equals
the phase change temperature.

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