streamwise vorticity, which is proportional to the radial gradient of the mainstream flow and to the turning; see reference [10]. The secondary velocities v_s (along the normal to the wall) and w_s (in the tangential direction) are defined by

$$v_s = -\frac{\partial \psi_s}{\partial z} \quad (43a)$$

$$w_s = \frac{\partial \psi_s}{\partial y} \quad (43b)$$

Assuming the vorticity ξ independent of y and z, which should be a valuable approximation at least for untwisted blades, ψ_s will be a symmetrical function of y and z, so that w_s is even and v_s uneven in z, the tangential coordinate when the y-axis is placed along the center line of the blade passage.

Therefore, in a flow field represented by

$$c_a = c_{a_0}(x,y,z) \quad (44a)$$

$$c_y = c_{r_0}(x,y,z) + v_s(x,y,z) \quad (44b)$$

$$c_z = c_{z_0}(x,y,z) + w_s(x,y,z) \quad (44c)$$

one may assume as a good first approximation that $c_0\ (x,y,z)$ is independent of or even in z, whereas $c_{r0}\ (x,y,z)$ will generally not depend on z except through second-order effects of centrifugal and Coriolis forces which can lead to radial velocities symmetrical with respect to the tangential direction.

Hence the secondary stress produced by the inviscid secondary flow $-\overline{c'_a c'_r}$ will be zero in first approximation since it is the integral of an even function times an uneven function, as well as $-\overline{c'_z c'_r}$ except for the contribution of the uneven part of c_z which, however, should be negligible in absence of tip clearance. So, it is to be expected that the main contribution to the total stress will come from the ordinary turbulent stresses.

With tip clearance, the leakage flow will disturb the flow field (44) and probably give rise to appreciable contributions to the secondary stresses. This can be seen for instance on a vortex core model with solid body relation proposed by Laksminarayana [11] who calculated the blade-to-blade velocities v_s and w_s produced by the tip leakage flow. Both components have even as well as uneven parts in z so that significant contributions to the secondary stresses will arise.

Besides, since the radial velocity c'_r plays a dominant role in the secondary momentum transport, where c'_r is a fluctuation of an already low radial velocity distribution c_y, the secondary stresses can therefore be dependent on the complete design of the bladings through the equilibrium of radial pressure gradient, centrifugal forces, and Coriolis forces.

DISCUSSION

G. Mellor[1]

First, let me say that I am gratified to learn that Professor Hirsch has seen fit to expand his intellectual energies on the theory I proposed in 1970. From that 1970 base there were two ways to go; (1) accept the entire theory dogmatically and try to determine the three "constants" to fit overall efficiencies and pressure ratios of a large number of machines or; (2) try and understand the physics of the end-wall problem with the *help* of the 1970 theory and/or compare the theory in detail with detail measurements. In the preceding paper Balsa and I chose course-of-action number 1, whereas Professor Hirsch chose number 2.

Having stated the situation thusly it is evident that Professor Hirsch is to be congratulated whereas I had better defend myself. My defense is that I wasn't at all sure anyone would accept the theory until the theory could be shown to produce useful predictions of multi-stage machines. It is my hope that the previous paper does serve that purpose and will further encourage Professor Hirsch to continue his more fundamental attack on the important end-wall problem. It is, by the way, my understanding that other European investigators are pursuing this line of inquire, although I do not know of any ongoing effort in the U.S.

Professor Hirsch is quite correct in challenging the assertion of equation (7) that—for zero clearance—the exit tangential and momentum thickness are equal (note: this statement is not at all synonymous with a statement that the velocities are everywhere collateral as is evident upon a detailed reading of the Mellor-Wood paper) and I now consider it more than possible that $K\delta_c$ in equation (7) should be replaced by something like $K_1\delta_c + K_2c$ where δ and c are tip clearance and chord. In fact, a careful examination of Figs. 4-6 of our paper indicates that an allowance for a $K_2 \neq 0$ would permit us to lower C_f while maintaining nearly the same predicted efficiencies. Our findings indicate that this might be attributed to the tip to wall relative velocity (scrubbing action?) which still exists as $\delta_c \to 0$ but does not exist when the wall and blade are integral. However, it is my understanding that Professor Hirsch believes that C_f is truly large; perhaps he could elaborate further on these possibilities.

From other attempts to describe the basic integral equations in streamline coordinates, I have in the past felt that they are too complicated to be instructive or useful. However, I note from Professor Hirch's paper that the small compromise of maintaining *axial* velocity in the definition of momentum thicknesses simplifies the equations so that the streamline integral equations are quite simple.

Our final point is, that I am not sure that I agree with Professor Hirsch's assertion that the "secondary stress" is small. From a mathematically linear point of view, this is correct. But considering the nonlinear effect of tilting Bernoulli surfaces introduces a true mainstream –vertical velocity—correlation that looks like a Reynolds stress that might, for example, result from a turbulent structure hypothesis whereby a mean velocity field is superimposed with turbulent streamwise vortices.

[1] Professor, Princeton University, James Forrestal Campus, Princeton, N.J.

Author's Closure

The author would like to thank Professor Mellor for his much appreciated comments. The question of high C_f-values at tip is closely linked to the order of magnitude of the "secondary stresses." As it appears from the Appendix 2 of our paper we only stated that the contribution to the total secondary stresses arising from the symmetrical secondary flow, the inviscid contribution "à la Squire and Winter," is zero in first approximation and also noted the importance of the radial velocity fluctuations in the determination of these secondary stresses. However, we agree with Professor Mellor that the deviations from this symmetrical secondary flow could be large giving rise to non-negligible mainstream --vertical velocity correlations. More particularly, in presence of tip clearance we think that the secondary stresses could be high. Our belief that C_f could be large with tip clearance is the expression of the possibility that, due to the leakage flow and vortices, the secondary stresses increase by a large amount in this region. At rotor hub, we feel however that the secondary stresses should be smaller than at rotor tip but value should depend strongly on the blade configurations.