Observed versus modelled $u$, $g$, $r$, $i$, $z$-band photometry of local galaxies – evaluation of model performance

K. S. Alexander Hansson, Thorsten Lisker and Eva K. Grebel

Astronomisches Rechen-Institut, Zentrum für Astronomie der Universität Heidelberg, Mönchhofstr. 12-14, 69120 Heidelberg, Germany

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ABSTRACT

We test how well available stellar population models can reproduce observed $u$, $g$, $r$, $i$, $z$-band photometry of the local galaxy population ($0.02 \leq z \leq 0.03$) as probed by the Sloan Digital Sky Survey (SDSS). Our study is conducted from the perspective of a user of the models, who has observational data in hand and seeks to convert them into physical quantities. Stellar population models for galaxies are created by synthesizing star formation histories and chemical enrichments using single stellar populations from several groups (STARBURST99, GALAXEV, the Maraston models, GALEV). The role of dust is addressed through a simplistic, but observationally motivated, dust model that couples the amplitude of the extinction to the star formation history, metallicity and the viewing angle. Moreover, the influence of emission lines is considered (for the subset of models for which this component is included). The performance of the models is investigated by (1) comparing their prediction with the observed galaxy population in the SDSS using the $(u-g)-(r-i)$ and $(g-r)-(i-z)$ colour planes, (2) comparing predicted stellar mass and luminosity weighted ages and metallicities, specific star formation rates, mass-to-light ratios and total extinctions with literature values from studies based on spectroscopy. Strong differences between the various models are seen with several models occupying regions in the colour–colour diagrams where no galaxies are observed. We would therefore like to emphasize the importance of the choice of model. Using our preferred model we find that the star formation history, metallicity and also dust content can be constrained over a large part of the parameter space through the use of $u$, $g$, $r$, $i$, $z$-band photometry. However, strong local degeneracies are present due to overlap of models with high and low extinction in certain parts of the colour space.

Key words: dust, extinction – galaxies: evolution – galaxies: formation – galaxies: ISM – galaxies: stellar content.

1 INTRODUCTION

Optical broad-band colours have proven to be a powerful tool in studying galaxies. Their dependence on luminosity and environment has greatly increased our knowledge about these systems (Visvanathan & Sandage 1977; Park et al. 2007; Lisker, Grebel & Binggeli 2008). Colours are also used to derive quantities such as star formation histories (SFHs) and stellar masses (Tinsley 1968; Searle, Sargent & Bagnuolo 1973; Charlot & Bruzual 1991; Bell & de Jong 2001; Bruzual & Charlot 2003; Blanton & Roweis 2007), both of which are key properties for understanding galaxy formation and evolution. For example, strong correlations between stellar mass and galaxy structure (Kauffmann et al. 2003b), SFH (Panter et al. 2007), chemical enrichment (Panter et al. 2008) and gas content (Zhang et al. 2009) have been presented suggesting that mass is the main property governing galaxy evolution. These relations reflect the importance of gravity on galactic scales and, moreover, provide further evidence concerning the expected connection between stellar mass and dark matter (cf. Moster et al. 2010). The derivation of SFHs and stellar masses is made through modelling of the light emission from the galaxy and, if necessary, through modelling of obscuration by dust. The method enables a comparison between observational quantities and galaxy formation models (e.g. Bower et al. 2006; De Lucia et al. 2006; Guo et al. 2011). The quality of the derivation of SFHs and stellar masses from colours naturally depends on the quality of the models. A straightforward test is to check how well the models can reproduce the ensemble of observables. In this paper we therefore take a closer look at how successful various models are in reproducing $u$, $g$, $r$, $i$, $z$-band photometry of the local galaxies.

*E-mail: alexander@x-astro.net

1 By focusing on local galaxies ($0.02 < z < 0.03$) we circumvent the shift in the spectral energy distribution caused by redshift.
The base ingredient of stellar population models of galaxies is single stellar populations (SSPs), which are combined into SFHs by linear combinations (Tinsley 1968; Searle et al. 1973). SSPs can consequently be seen as a basic ingredient of the models having a great impact on the emergent spectral energy distribution. A large number of SSP models are available in the literature, including those of Leitherer et al. (1999), Bruzual & Charlot (2003), Maraston (2005) and Kotulla et al. (2009), which we use in this paper. We do not aim at being complete considering the numerous options available. However, we intend to cover some of the most widely used models predicting $u$, $g$, $r$, $i$, $z$-band photometry.

We treat dust obscuration in a simple way. The amplitude of the extinction is coupled to a galaxy’s SFH, metallicity and viewing angle, as motivated by works of Cid Fernandes et al. (2005), Engblom et al. (2008), da Cunha et al. (2010) and Masters et al. (2010), and a power-law form of the wavelength dependence of the extinction is assumed (see, e.g. Charlot & Fall 2000). This method can be seen as an easy, but simplistic, approach with respect to detailed modelling of dust (e.g. Tuffs et al. 2004; Popescu et al. 2011), though it has a nice feature that it only requires the axis ratio apart from colours.

As spectroscopic data carry a lot more information than $u$, $g$, $r$, $i$, $z$-band photometry, a comparison with these kinds of data can serve as an important test. We therefore carry out a detailed comparison between literature data based on spectroscopy and our photometric model in order to validate the latter.

We note that a different approach to the testing of stellar population models has been performed by Conroy, Gunn & White (2009), Conroy, White & Gunn (2010) and Conroy & Gunn (2010). We refer the readers to this series of papers if they are interested in more details on the impact and uncertainties of various model ingredients rather than the overall performance for optical broad-band photometry.

This paper is organized as follows. In Section 2 the observational sample used for the model evaluation is presented. The SSPs are introduced in Section 3, and how these are combined into SFHs is described in Section 4. How we treat dust is described in Section 5. The performance of the models is presented in Section 6. The results are discussed in Section 7 and summarized in Section 8.

2 OBSERVATIONAL SAMPLE

Our source of observational imaging data is the Sloan Digital Sky Survey (SDSS) Data Release 7 (DR7; Abazajian et al. 2009). Galaxies with measured spectroscopic redshifts from the SDSS in the range $0.02 < z < 0.03$ are chosen to minimize the shifts in the spectral energy distribution caused by redshifts while keeping a large sample with photometry of sufficient quality. [Note that, in particular, nearby galaxies have inaccurate photometric measurements from the SDSS pipeline (Blanton et al. 2005, 2011; Bernardi et al. 2007; Lauer et al. 2007; Lisker et al. 2007).] Our sample consists of 24 120 galaxies. We downloaded model magnitudes (modelMag) for these galaxies, which have been corrected for Galactic extinction using the maps of Schlegel, Finkbeiner & Davis (1998). The sample spans an absolute magnitude range of $-17 \geq M_r \geq -23$, and thus only the brightest of the dwarf galaxies are included. The photometry is $k$-corrected using the code of Chilingarian, Melchior & Zolotukhin (2010), but the small amplitude of the corrections ($\leq 0.1$) suggests that the errors introduced in this step are negligible. In this paper we only make use of magnitudes in the AB system, including the SDSS to AB conversion factors$^2$ of $u_{AB} = u_{SDSS} - 0.04$ mag and $z_{AB} = z_{SDSS} + 0.02$ mag. A quality assessment of the galaxy colours, as described in Appendix A, shows that the distribution of colours for the sample appears to be consistent with independent measurements.

Information about galaxy structure can aid in the interpretation of the modelling and is of interest. Fig. 1 illustrates the properties of our galaxy sample, relying on structural parameters from the SDSS pipeline and the Galaxy Zoo project (Lintott et al. 2008). The SDSS pipeline (Stoughton et al. 2002) models galaxy light profiles through a best-fitting linear combination of a de Vaucouleurs and an exponential profile. The de Vaucouleurs fraction, $fracDeV$, is a measure of the fraction of light in the de Vaucouleurs profile of the two-component fit. We further make use of the ratio of the semimajor axis, $a$, and the semiminor axis, $b$, at the object’s 25 mag arcsec$^{-2}$ isophote which we denote as $ab$. From the Galaxy Zoo project (Lintott et al. 2011) we obtain $P_{ab}$ which gives the probability that the objects are assigned elliptical morphology through a visual classification. We chose the values that were debiased from resolution effects (see Bamford et al. 2009). Moreover, we make use of absolute $r$-band magnitudes, $M_r$, computed from the SDSS modelMags and redshifts using $H_0 = 71$ km s$^{-1}$.

To test the performance of photometric modelling we compare the outcome with results based on spectroscopic data from the SDSS. As the spectra are obtained using optical fibres with a diameter of

\[2 \text{ See www.sdss.org/dr7/algorithms/fluxcal}\]
3 Single Stellar Population Models

The stellar population models we test in this paper are summarized in Table 1. We retrieve SSPs for each of these models which we combine into SFHs as described in the following section. As most of the models have several options regarding specific model ingredients, we decided to constrain the choice to avoid ending up with an impractical number of models. Details of the model subselection are given in Table 1, and the model ingredients are briefly commented below. The total amount of spectral information contained in u-, g-, r-, i-, z-band photometry can be fully represented in four dimensions by, e.g., the \((u - r), (g - r), (i - r)\) and \((z - r)\) colours. However, for practical reasons we make use of 2D projections in the form of colour–colour diagrams, and for simplicity we constrain ourselves to the use of the \((u - g) - (r - i)\) and \((g - r) - (i - z)\) planes. Note that these projections alone cannot capture all available information, though they ought to include the majority since they contain information from all five bands.

3.1 Stellar spectra

The models fall in two different categories regarding how the stellar spectra are generated: theoretical models based on stellar atmosphere models (Lejeune, Cuisinier & Buser 1997) and models based on libraries of observed stellar spectra (Le Borgne et al. 2003).

3.2 Stellar evolution

In terms of stellar evolution several different sets of models are used, including Schaller et al. (1992), Bertelli et al. (1994), Cassisi et al. (2000), Girardi et al. (2000), Marigo & Girardi (2007) and Marigo et al. (2008).

3.3 Initial mass function

In a recent review Bastian, Covey & Meyer (2010) conclude that there is no clear evidence that the initial mass function (IMF) varies strongly and systematically, and that the majority of systems on galactic scales are consistent with having a Kroupa (2001) or Chabrier (2003) IMF. However, an IMF that evolves with time cannot be excluded, and recent studies do suggest a steeper IMF in massive elliptical galaxies (van Dokkum & Conroy 2010, 2011; Spiniello et al. 2011). Here we investigate the behaviour of our models with respect to single-slope IMFs. The Leitherer et al. (1999) model (S99) has the option of freely choosing the IMF. Fig. 2 shows the behaviour of the model boundaries as a function of the IMF slope. Clearly, the S99 models do not reproduce the observed distribution of colours very well regardless of the IMF slope. As expected, the colours only change mildly with the IMF, and we therefore chose to apply either a Kroupa (2001) or a Chabrier (2003) IMF. For each of the models one can choose between one of these two IMF prescriptions.

3.4 Emission lines

Emission lines are prominent features in the optical spectra of star-forming galaxies. It is therefore desirable to include emission lines in the modelling of photometric properties. Two of the models we use incorporate gas emission, namely S99 and GALEV (Kotulla et al. 2009). The former predicts the strength of H\(_{\alpha}\) and H\(_{\beta}\), while

<table>
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<th>Table 1. Sources of SSP models.</th>
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<td>Model ID</td>
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\(^{a}\)Additional references left out, see model reference.  
\(^{b}\)These models have the option of choosing between populations with or without blue horizontal branch. We chose the latter.  
\(^{c}\)Charlot private communication.  
\(^{d}\)Updated SDSS filter responses from Doi et al. (2010) used.  
\(^{e}\)GALEV uses the 1999 version of isochrones from the Padova group.  
\(^{f}\)Magnitudes obtained by convolving the low-resolution spectra with the updated SDSS filter response from Doi et al. (2010).
the latter predicts several other lines in the optical, which are expected to be important in star-forming galaxies (Anders & Fritze-v. Alvensleben 2003). Modelling of line emission from SSPs inevitably suffers from some uncertainty, but can be sufficiently accurate for predicting broad-band colours (Győry et al. 2011). From the colour evolution of the SSPs it is clear that gas emission in the models only has a significant effect on the colours at ages of about $10^7$ yr or younger. Fig. 3 shows how the outlines of the model grids shift with the inclusion of emission line modelling. Hα and Hβ, the only emission lines included in S99, only affect the $r$ and $g$ bands, while additional prominent emission lines included in GALEV also affect $u$, $i$ and $z$ causing large discrepancies between the two models in $(r-i)$. Modelling of line emission as in S99 or GALEV can be included in any of the models we consider by adding the corresponding change in the colours of S99 or GALEV. This can only be done for SSPs of the same age and metallicity under the assumption that the stellar emission in $u$, $g$, $r$, $i$ and $z$ is similar in the two models.

4 STAR FORMATION HISTORY

We synthesize SFHs from the SSPs through the use of smooth models to which some stochastic sampling has been added. Such Monte Carlo libraries of SFHs have previously been employed by several authors (Kauffmann et al. 2003b; da Cunha, Charlot & Elbaz 2008; Zibetti, Charlot & Rix 2009) and have proven versatile in modelling stellar population properties. The stellar population synthesis method we employ is thus not new, but the details differ between our and these previous works. The addition of some random component in the modelling is motivated by the knowledge that star formation to some extent occurs stochastically. Processes such as galaxy merging (Di Matteo et al. 2007) and ram pressure stripping (Gunn & Gott 1972) often occur on short time-scales and can strongly influence a galaxy’s SFH.

Gavazzi et al. (2002), inspired by the work of Sandage (1986), showed that a ‘delayed exponential’ SFH does a better job in reproducing colours of Virgo cluster galaxies than the classical exponential model. We therefore use this model as a starting point for our SFHs,

$$SFR = \frac{T}{\psi^2} \exp\left(-\frac{T^2}{2\psi^2}\right),$$

where SFR is the star formation rate, $T$ is the time from the initial onset of star formation and $\psi$ is a parameter governing the decline of star formation over time. We take the galaxy formation time to be 13.5 Gyr. The exact starting point is not crucial due to the slow evolution of spectral properties at old ages, but it is to some extent motivated by current estimates of the onset of re-ionization in the Universe (Fan, Carilli & Keating 2006).
SFHs are created through sampling of equation (1) using SSPs for values of ψ in the range 1–20 Gyr. The sampling is done logarithmically, since the colour evolution is roughly proportional to the logarithm of the age. Thus, the age of each SSP, tᵢ, is randomly drawn from the following distribution:

\[ \frac{T^2}{ψ^r} \exp \left( -\frac{T^2}{2ψ^2} \right). \]  

(2)

To compensate for the logarithmic sampling the strength, i.e. mass, of each SSP is multiplied by its age. In total, 5ψ SSPs are used for each SFH. [More SSPs are needed to sample the larger range in log(age) spanned by models with higher values of ψ.] Stochasticity is introduced in the model through the sampling and by further modifying the strength of each SSP by multiplying with a random component. The random component is drawn from a distribution which is taken to be the absolute value of a normal distribution with μ = 0 and σ = 1. To include many different metallicity configurations the SSPs constituting each SFH are randomly divided into five groups, and each of these groups is assigned random metallicity from the options available for that particular SSP.

The chosen input SSPs limit the lowest ages that can be included. However, note that in the model by Charlot & Fall (2000) the light from populations younger than 10⁷ yr is heavily obscured by the dust in the birth clouds and does not contribute significantly to the integrated light at optical wavelengths. Therefore, we need not consider these missing models at young ages.

Using optical spectral energy distributions the resolution in terms of the age of a stellar population seems to be limited to at least ~10 per cent (González Delgado & Cid Fernandes 2010). Any gaps introduced by discrete sampling of an SFH can thus safely be neglected if they are smaller than about 10 per cent.

For a range of values we create 1250 realizations of the different SFHs, which results in 50 000 models for each set of SSPs. Contours of the model boundaries in the colour–colour space are created in the following way. 2D histograms in (g − r)–(i − z) and (u − g)–(r − i) are made with bin sizes of 0.0055, 0.0040, 0.0100 and 0.0035 for (g − r), (i − z), (u − g) and (r − i), respectively. These histograms are convolved with a Gaussian kernel (σ = 3 pixels) to remove the noise at the edges after which contours are drawn at a density of 0.2 models pixel⁻¹, is the age of the th SSP, sᵢ is the strength of the th SSP, the summation is done over all n SSPs in the SFH and the power index of 1.1 is taken directly from da Cunha et al. (2010). This equation makes Aᵣ dependent on the fraction of young stars by comparing the strengths of the SSPs weighted by the factor e⁻ᵗᵢ/ₜₒ (numerator) with the corresponding value for a.

3 Note that Maraston (2005) does not offer young ages at the lowest metallicity.

4 ψ = 1.0, 1.5, . . . 3.0, 3.25, . . . 9.0, 10.0, . . . 20.0. A variation in step size turned out to be useful for better coverage in the (u − g)–(r − i) and (g − r)–(i − z) planes.

5 The model boundaries in Figs 2–6 are created in the same way.

**5 DUST**

In this paper we employ a simplistic model for dust extinction in galaxies that has, by construction, no free parameters. Although several assumptions are made, we expect our model to work reasonably well since it is constructed to capture the most important features of dust extinction in galaxies. It is motivated by the following factors:

1. The connection between the presence of dust and star formation/young stellar populations. da Cunha et al. (2010) showed that the dust mass in galaxies can be estimated remarkably well using the average star formation rate during the last 10⁸ yr by comparing the strengths of the SSPs weighted by the factor e⁻ᵗᵢ/ₜₒ (numerator) with the corresponding value for a.

We take the effective r-band extinction, Aᵣ, to be

\[ Aᵣ = A₀ \left( \sum_{i=0}^{n} sᵢ e^{-tᵢ/ₜₒ} \right)^{1.1}, \]  

(3)

where A₀ and tₒ are constants, tᵢ is the age of the th SSP, sᵢ is the strength of the th SSP, the summation is done over all n SSPs in the SFH and the power index of 1.1 is taken directly from da Cunha et al. (2010). This equation makes Aᵣ dependent on the fraction of young stars by comparing the strengths of the SSPs weighted by the factor e⁻ᵗᵢ/ₜₒ (numerator) with the corresponding value for a.

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constant SFH (denominator). This is essentially a smoother version of the $10^8$ yr cut adopted by da Cunha et al. (2010) assuming that $A_0$ is proportional to the dust to stellar mass ratio. $A_0$ is constrained by the reddest galaxies observed. A value of $A_0 = 0.40$ keeps the models within or close to the observed cloud of galaxies. $t_0$ should be chosen in accordance with the work of da Cunha et al. (2010), and we adopt $t_0 = 3 \times 10^8$ yr.

(2) An observed correlation exists between metallicity and the dust to gas mass ratio. We base the metallicity dependence of our dust models on the observational results of Engelbracht et al. (2008). Their relation between the nebular metallicity, $Z$, and H I to dust mass fraction ($M_{\text{H I}}/M_{\text{dust}}$) can be rather well fitted by a power law (the fit has a scatter of 0.5 dex of which at least half can be explained by measurement errors) with low-metallicity galaxies having higher $M_{\text{H I}}/M_{\text{dust}}$. Assuming that the nebular metallicity is the same as the average metallicity of the stellar populations contributing to the dust (see equation 3), $z$, this leads to a modification of $A_r$ according to $A_r = A_t (z/z_0)^\gamma$, (4)

where $m$ is a constant. Here we have further assumed that the metallicity is the only parameter governing $M_{\text{H I}}/M_{\text{dust}}$ for a fixed SFH and that $M_{\text{H I}}/M_{\text{dust}}$ is proportional to $A_r$. We use $m = 1.7$ as measured from the data presented in Engelbracht et al. (2008). We note that De Lucia & Blaizot (2007) also use a power law to model the metallicity dependence of the effective extinction with a power-law index very similar to the one we adopt.

(3) At optical wavelengths, the wavelength dependence of the extinction appears to be rather well modelled by a power law of index $k$ (Charlot & Fall 2000),

$$A_\lambda \propto \lambda^k,$$

where $A_\lambda$ is the effective extinction at a wavelength $\lambda$. Charlot & Fall (2000) use a different proportionality constant for stellar populations older and younger than $10^7$ yr. However, the contribution of optical light from young populations ($< 10^7$ yr) is small due to obscuration from the dust clouds in which the stars were born. We therefore apply a single power law to model the wavelength dependence of the extinction. This should be seen as a simple approach, as we do not know how the detailed effective extinction curve varies over a range of galactic environments. The colours of dusty models depend on the constant $k$. We use $k = -1.1$, which is an appropriate choice considering the width of the model cloud in $(r - i)$ at $(u - g) > 1.3$ as compared to the observations. The effective extinction in $u, g, r, i$ and $z$ is computed using the effective wavelength of these filters. The use of effective wavelengths rather than a flux weighted average only introduces errors in $A$ of a few per cent (as estimated using the S99 models without emission lines).

(4) The extinction is dependent on the angle under which the galaxy is seen. Empirical relations between colours and inclinations for spiral galaxies in the SDSS were studied by Masters et al. (2010). Here we adopt a similar functional form to express how the effective r-band extinction is modified depending on the isophotal axis ratio, $a/b$,

$$A_r = A_t + \gamma_r \log_{10}(a/b),$$

where $\gamma_r$ is a constant. To determine $\gamma_r$, we select spiral galaxies ($P_{\text{ell}} < 0.2$) from our spectroscopic sample and plot $A_r$ from Cid Fernandes et al. (2005) versus $\log_{10}(a/b)$. For four bins in $\text{fracDeV}_r$, we determine $\gamma_r$ (in analogy to $\gamma_r$) through least-squares fit. The bins were chosen such that they contain equal numbers of galaxies, and we find (fracDeV$_r, \gamma_r$) = (0.44–1.00, 0.51), (0.17–0.44, 0.58), (0.02–0.17, 0.57) and (0.00–0.02, 0.58). Given the small bin-to-bin variations we adopt $\gamma_r = 0.47$ (i.e. $\gamma_r = 0.55$) and apply equation (6) for all galaxies having $A_r > 0.05$ mag according to equations (3) and (4). The extinction for galaxies with little or no dust is thus assumed to be inclination independent, while dusty galaxies are treated as spirals.

We have constructed our model so that it does not have any free parameters. The extinction is completely coupled to the galaxy’s SFH, chemical enrichment and the observed axis ratio. The constants included in the model have values that can be inferred from previous studies, but some were adjusted in order to produce as realistic an ensemble of models as possible as judged by comparing the distribution of observed and modelled colours. We adopted $A_0 = 0.40$, $t_0 = 0.30$ Gyr, $m = 1.7$ and $k = -1.1$.

Note that models which do not have any recent ($t < t_0$) star formation are essentially unaffected by extinction and devoid of line emission. These models are simply linear combinations of SSPs.

6 PERFORMANCE

We test the models described above using an observational sample of galaxies from the SDSS DR7 (Abazajian et al. 2008) as described in Section 2. Given the boundaries of the model cloud for dust-free models in Figs 4 and 5 in comparison with the observations, we see large differences between the various models. In particular, the models of Charlot & Bruzual (in preparation) (CB07) and GALEV occupy a large region in $(g - r) - (i - z)$ where no galaxies are found to reside, and the same is true for CB07 in $(u - g) - (r - i)$. The overall best agreement between observations and models is shown by the Bruzual & Charlot (2003) models based on observed stellar spectra (BC03hr) and spectra from stellar atmosphere models (BC03hr). Of these two, BC03hr performs significantly better in the reddest region of both colour-colour diagrams. In the following we have therefore adopted BC03hr as our preferred model. A missing ingredient in this model is the line emission. As shown in Fig. 3, the inclusion of line emission from GALEV only changes the model track slightly. We can therefore safely include line emission from GALEV into the BC03hr models as discussed in Section 3.4.

GALEV is preferred over S99 since the former takes more emission lines into account. Moreover, we apply our dust models to BC03hr.

The resulting change in the model boundaries is shown in Fig. 6. Considering the photometric errors the model performs very well in reproducing the colours of the galaxy sample, except at about $(g - r, i - z) = (0.4, 0.0)$, where models appear to be missing. The colours of the SSPs in Fig. 6 show that galaxies dominated by recent star formation can have $(i - z) < 0$. However, these objects would be very blue in $(g - r)$ and thus cannot explain the ‘missing models’. Considering Fig. 6, our recipe for the SFHs combines the blue points into models within the green contours, which thereafter are shifted to the red contours by dust and line emission. Fig. 6 clearly shows that a mixture of SSPs of different ages and metallicities is needed to explain the locus of observed galaxies. Additionally, dust is needed to explain some of the reddest galaxies.

6.1 Derived parameters

To illustrate the predictions of the models in terms of galaxy properties Figs 7–9 show the stellar mass weighted$^6$ ages, metallicities and total r-band extinction in the colour–colour planes along with their standard deviations. This is shown for the BC03hr models

$^6$ In this work stellar masses are derived using a Chabrier (2003) IMF and they are taken to be the masses in present-day stars and remnants.
with dust and line emission which are in good agreement with the observations (Figs 4 and 5). A strong, rather orthogonal dependence is seen between age and metallicity. However, for a region of the colour space roughly from \((u−g, r−i) = (1.1, 0.2)\) to \((u−g, r−i) = (1.6, 0.4)\) and from \((g−r, i−z) = (0.5, 0.2)\) to \((g−r, i−z) = (0.8, 0.3)\), all properties are rather poorly constrained due to a mix of models with high and low dust extinction. The model library was constructed to cover the \((u−g, r−i)\) and \((u−g, r−i)\) diagrams and does not tell anything about the probability that a specific model is a good representation of the observations. Further improvement can be achieved using a Bayesian approach. With help from the semi-analytic galaxy formation models of Guo et al. (2011) we compute model weights to match the distribution of stellar population models with semi-analytic galaxies in terms of density in stellar mass weighted age versus stellar mass weighted metallicity, see Appendix C. For 10 randomly chosen example galaxies we show the SDSS colour composite image\(^7\) along with an SFH derived through a Bayesian maximum likelihood for the BC03hr models with emission lines and dust, see Fig. 10.

\[^{7}\text{Images are taken from http://cas.sdss.org/dr5/en/tools/chart/list.asp}\]

6.2 Visual checks

As a test we selected random samples of galaxies for three different parts in the \((g−r)−(i−z)\) space, which according to our models have certain special characteristics. The first group contains galaxies that belong to a well-populated region around \((g−r, i−z) = (0.8, 0.3)\) that cannot be reached by any models (Fig. 11, yellow dots). This group seems to be populated by star-forming galaxies that in many cases exhibit Sm-type morphology. The second group at \((g−r, i−z) \sim (0.8, 0.3)\) contains galaxies that, according to our model, are expected to have strong extinction by dust (Fig. 11, green dots). This group contains many edge-on or highly inclined disc galaxies. The last group at \((g−r, i−z) \sim 0.4, 0.0)\) that cannot be reached by any models (Fig. 11, green dots). This group contains many edge-on or highly inclined disc galaxies.
Based on BC03hr with dust. Deviation in total our 50 000 models based on BC03hr with dust. Right-hand panels: standard deviation in total extinction (colour coded pixels) as a function of \((i−z)\) versus \((g−r)\), as well as \((r−i)\) versus \((u−g)\) for our 50 000 models based on BC03hr with dust.

Left-hand panels: total \(r\)-band extinction (colour coded pixels) as a function of \((i−z)\) versus \((g−r)\), as well as \((r−i)\) versus \((u−g)\) for our 50 000 models based on BC03hr with dust.

Figure 9. Left-hand panels: total \(r\)-band extinction (colour coded pixels) as a function of \((i−z)\) versus \((g−r)\), as well as \((r−i)\) versus \((u−g)\) for our 50 000 models based on BC03hr with dust. Right-hand panels: standard deviation in total extinction (colour coded pixels) as a function of \((i−z)\) versus \((g−r)\), as well as \((r−i)\) versus \((u−g)\) for our 50 000 models based on BC03hr with dust.

(0.8, 0.2) is expected to contain dust-free galaxies (Fig. 11, grey dots). With a few exceptions this group contains early-type galaxies with little or no visible dust and star formation. SDSS multi-colour composites of individual representative galaxies are shown in Fig. 11.

6.3 Comparison with spectroscopic studies

There is no question about the superior information content of optical spectra with respect to broad-band photometry. Photometry is nevertheless an important probe for stellar populations of galaxies as it is easy to obtain and because spectroscopic data often suffer from aperture bias. As mentioned in Section 2 we compiled spectroscopically determined galaxy parameters from the literature for testing the performance of our photometric model. The parameters considered are stellar mass weighted ages, \(\text{age}_{\text{mass}}\), luminosity weighted ages, \(\text{age}_{\text{L}}\), stellar mass weighted metallicities, \(\text{Z}_{\text{mass}}\), luminosity weighted metallicities, \(\text{Z}_{\text{L}}\), specific star formation rates, SSFR, \(\text{SSFR}^8\) \(r\)-band mass-to-light ratios, \(\text{M/L}_r\), and \(V\)-band extinctions, \(A_V\). To be able to make a fairer comparison between photometrically and spectroscopically derived quantities, we set the maximum of \(\text{age}_{\text{mass}}\) be able to make a fairer comparison between photometrically and spectroscopically derived quantities at fixed photometrically derived values. The use of \(u\), \(g\), \(r\), \(i\), \(z\)-band photometry alone leads to a low standard deviation in all properties except the SSFR. This problem is solved if a prior as a function of \(M_r\) is introduced, and additionally a slight improvement can be achieved by introducing \(a+b\) in the fitting. Note, however, that \(M_r\) can be estimated using \(u\), \(g\), \(r\), \(i\), \(z\)-band photometry with an accuracy of about 1 mag, a sufficiently good estimate to reach almost the same accuracy as when \(M_r\) itself is used. Using only \(M_z\), or \(M_r\) and \(a+b\), we can still constrain several quantities as the model galaxy properties have a strong mass/luminosity dependence as well as some structural dependences (Guo et al. 2011), but the standard deviation is considerably higher in many of the parameters.

Moreover, with a similar technique we can test whether the exclusion of certain bands in the fitting can improve the quality of the derived parameters. We find that this is the case. Removal of one or two bands can indeed decrease the standard deviation of the comparison with the spectroscopy by 5–10 per cent for each of the parameters tested. However, an increase in the accuracy of one parameter typically comes at the expense of diminished accuracy in another. For example, excluding \(u\) and \(g\) leads to a better estimate in the mass weighted metallicity but makes the accuracy of the SSFR much worse. As no overall improvement is achieved by excluding one or two bands, we prefer to keep all bands in the fitting.

6.4 Insights from spectral fitting

While the main focus of this paper is testing model predictions for \(u\), \(g\), \(r\), \(i\), \(z\)-band photometry, an efficient test can be achieved by comparing observed and modelled spectra instead of colours due to the much larger information content. A drawback is that we already expect that there will be some issues with the modelling because the chemical content of a galaxy is described by a single parameter in our model, i.e. the metallicity, an approximation that is not expected to reproduce all possible line ratios in galaxies.

We randomly select 1000 galaxies in our sample for which we obtain fully reduced flux calibrated SDSS spectra. These have a median signal-to-noise ratio per pixel of 19 in the \(r\) band and are fitted to our model library in the wavelength range 3700–7000 Å (rest frame), as described in Appendix B. Skylines and emission lines are masked out in the fit, and the line-of-sight velocity distribution is taken into account.

The median reduced \(\chi^2\) of the fit to the 1000 galaxies is 1.4 for the BC03hr models with dust. A visual inspection of observations and best-fitting models shows that not only issues with lines but also with the continuum fit exist. We note that the median reduced \(\chi^2\) can

\[8\] SSFRs are taken as the ratio of the average star formation rate during the last 10\(^8\) yr and the galaxy’s stellar mass. An average star formation rate over the last 10\(^9\) yr as determined from the stellar continuum has proven to be successful in reproducing star formation rates as estimated from emission lines (Asari et al. 2007).
be substantially lowered if the fitted region is restricted to a shorter wavelength interval like 5000–5500 Å instead of 3700–7000 Å.

7 DISCUSSION

Before starting the discussion about the model performance and prediction it is worth taking a look at how the models are constructed.

7.1 Model ingredients

As opposed to Conroy et al. (2009, 2010) and Conroy & Gunn (2010) our aim has not been to judge each model ingredient, and we thus find it sufficient to say that some, but certainly not all,

Other spectral fitting methods that do not rely on a set of pre-defined models (in our case 50 000) have been able to produce better fits to SDSS spectra (cf. Cid Fernandes et al. 2005).

SSP models can be successfully used to predict $u$-, $g$-, $r$-, $i$-, $z$-band photometry of the local galaxy population. Caution should be taken in the choice of model. Of the models we tested (see Table 1), we recommend the Bruzual & Charlot (2003) high-resolution models for the purpose of predicting optical broad-band photometry of galaxies.

Does our library of SFHs make sense? Given the distribution of average values for our models it seems that we have covered essentially all possibilities with ages from 0 to 13 Gyr, metallicities from $0.00$ to $0.05$ in $Z$ and with extinctions from 0 to 3 mag. Furthermore, we note that equation (1) produces SFHs that resemble results from modelling of spectra of nearby galaxies by Heavens et al. (2004).

As mentioned in the introduction, the inclusion of some stochasticity is expected to improve the realism of the models. In the context of the amplitude of the fluctuations in star formation over time we
It is well known that optical colours can be used to separate star-forming galaxies from quiescent ones. However, metallicity and dust extinction also have major impact. Our models suggest that, by using optical colours only, it is possible to reduce the age–metallicity degeneracy and constrain ages and metallicities fairly well over a surprisingly large part of the parameter space (see Figs 7–8). Furthermore, an interesting prediction of the model is that, to some extent, the amount of internal extinction can be measured using optical colours (Fig. 9). Is this really the case? We expect extinction to have the strongest effect on edge-on disc galaxies. We therefore selected two subsamples of galaxies with high and low estimated extinction, respectively (Fig. 11). It does indeed look like that the high-extinction bin contains a lot more edge-on disc galaxies than the low-extinction bin and, moreover, dust is seen in many more cases in the high-extinction bin. It thus appears as if our colour-based estimate is reasonable.

7.2 Model predictions

Is the number of models sufficiently large? When determining galaxy properties along with errors from colours using, e.g., a Bayesian maximum likelihood, the density of models in the colour space is of importance. Judging from Figs 7–9 our library is sufficiently large for this purpose except at the very edges of the grid if using $(u - g)$ versus $(r - i)$ or $(g - r)$ versus $(i - z)$. However, if more than two colours are used, the required density of the model library increases and caution has to be taken. The errors in Fig. 10 do not appear to be underestimated, except for one galaxy falling outside the model grid, and the method we use thus seems to work properly even for 10 colours.

The galaxy spectra are rather poorly reproduced with our model library, as mentioned in Section 6.4, both due to issues with lines and the continuum. The fact that the reduced $\chi^2$ can be lowered by considering a shorter wavelength range probably indicates that our model struggles to reproduce the overall continuum shape, but can perform better over shorter wavelength intervals. Spectral lines may be poorly reproduced due to the simple one-parameter treatment of element abundances. We should, on the other hand, be able to adequately model the continuum, especially considering its close connection to colours. However, in the modelling of galaxy spectra it is often assumed that the continuum is unreliable due to, e.g., flux calibration issues, and continuum variations are therefore removed with polynomial fits (cf. Chilingarian et al. 2007; Koleva et al. 2009). It is thus not completely clear whether the poorly fitted spectra mainly reflect problems with the models or the observations.
the $i$ and $z$ bands which have effective wavelengths of 7472 and 8917 Å. The source of the discrepancy in metallicity is thus not resolved.

As the amount of dust present is coupled to a galaxy’s SSFR in our model, galaxies with young stellar populations are often heavily obscured. The introduction of the prior (see Section 6.1) reduces the degeneracy between old and dust-free and young and dusty galaxies. However, a small number of old galaxies (ages determined by spectroscopy) still have young stellar populations according to our photometric model (Fig. 12). Fig. 13 reveals some systematic differences between the parameters derived from photometry and spectroscopy along with some scatter. However, deviations are to be expected given the large difference in the amount of information contained in the two kinds of data (five data points along the spectral energy distribution for $u$, $g$, $r$, $i$, $z$ with respect to $\sim 2000$ for SDSS spectra). To probe galaxy properties with optical photometry thus stands as a viable option, especially useful when limited spectral information is available such as in the case of severe aperture bias. We would also like to point out that spectroscopically derived quantities are not necessarily ‘correct’ as model dependences may be present.

The exclusion of some bands can slightly diminish the differences between photometrically and spectroscopically derived quantities (see Section 6.4). This may be caused by wavelength-dependent luminosity weighting, but could also reflect that certain colours do not trace all parameters we study.

An interesting result is that the newer CB07 models perform worse than the BC03 models in reproducing the colours of the nearby galaxy population. We would like to point out that this is not the same as that the ingredients of the BC03 model are somehow better than the newer version. We speculate that the offsets in the CB07 models may be a backlash of trying to optimize the model for predictions over a larger wavelength range. In BC03 as well as CB07, models based on the empirical library of stellar spectra by Le Borgne et al. (2003) do improve the modelling with respect to models based on the stellar atmosphere models of Lejeune et al. (1997). This suggests that more uncertainties are introduced through complete modelling than through an empirical approach (see also Maraston et al. 2009; Maraston & Strömback 2011; Peacock et al. 2011).

A subclass of galaxies at $(g - r, i - z) = (0.4, 0.0)$ cannot be modelled by any of the models considered (see Fig. 11). Most of these galaxies are rather faint in the $z$ band ($\langle m_z \rangle \sim 17.0$ as compared to $\langle m_z \rangle \sim 15.5$ for the entire sample), which has led us to suspect that a part of this population could simply be an artefact caused by photometric errors. The fraction of galaxies with small $z$-band photometric errors $\sigma_{m_z} < 0.05$ in the region $(0.3 < (g - r) < 0.4, -0.05 < (i - z) < 0.05)$ is a factor of 2 smaller than that for the overall sample. However, this is not sufficient to explain the entire population. Another explanation is that these galaxies are even more metal poor than the most metal-poor models (see Fig. 8) or, alternatively, that the SSP models fail at low metallicity.
Observed versus modelled colours of galaxies

The SFHs, along with average ages, metallicities and $r$-band optical depth shown in Fig. 10, look reasonable at first glance. A quiescent-looking galaxy, which in fact does not have any emission lines in its SDSS spectra, should according to the models have some amount of recent star formation. However, non-star-forming models lie within the errors. We thus conclude that the model output is reasonably reliable – within the limits of broad-band photometry – in providing information about a galaxy’s SFH and dust content.

8 SUMMARY

If an adequate range of SFHs, chemical enrichments and dust obscuration are being considered, some of the currently available stellar
population models perform well in reproducing the integrated optical $u$, $g$, $r$, $i$, $z$-band photometry of nearby galaxies (see Fig. 6). However, strong differences between the various models are seen, and it is therefore necessary to carefully consider the choice of model before proceeding with the analysis. Our preferred model library is available on request. Optical broad-band colours are insensitive to the slope of the IMF but can, on the other hand, put constraints on SFHs, chemical enrichments and dust extinctions as sensitive to the slope of the IMF but can, on the other hand, put constraints on SFHs, chemical enrichments and dust extinctions as

<table>
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<th>Fitting parameters</th>
<th>age$_{mass}$ (Gyr)</th>
<th>age$_{c}$ (Gyr)</th>
<th>$z_{mass}$ (Z$_{\odot}$)</th>
<th>$z_{lum}$ (Z$_{\odot}$)</th>
<th>SSFR</th>
<th>$M/L_r$ (M$<em>{\odot}$/L$</em>{\odot}$)</th>
<th>$A_V$ (mag)</th>
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<td>0.50</td>
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<td>0.44</td>
<td>0.75</td>
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<td>0.34</td>
<td>0.44</td>
<td>0.49</td>
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APPENDIX A: GALAXY COLOURS

We test the colours of the galaxy sample we use in this paper against two independent samples derived from SDSS data. The first sample from Lisker et al. (2008), Janz & Lisker (2009) and Meyer et al. (in preparation), hereafter S1, consists of Virgo cluster galaxies for which colours are measured within two effective radii or one Petrosian radius. The second sample is from Hansson et al. (in preparation), hereafter S2, and consists of an essentially volume-limited sample of galaxies brighter than $M_r \sim -16$ within 50 Mpc. Colours for these galaxies are measured within 2.5 Kron radii, as determined from the $r$-band image using SExtractor (Bertin & Arnouts 1996). For these two samples as well as for the sample we use in this paper, hereafter S3, the $(r - i)$ versus $(u - g)$ and $(i - z)$ versus $(g - r)$ colours are given in Fig. A1. The general agreement

![Figure A1](https://academic.oup.com/mnras/article-abstract/427/3/2376/1102624)
is good, but differences are seen. S3 have on average higher values of \((i - z)\) at \((g - r) = 0.7\). We test whether this can be explained by the difference in how the magnitudes are computed by looking at the change in colours when adopting Petrosian magnitudes (PetroMag) instead of modelMags. The median change in \((i - z)\) is \(-0.06\) mag, and for the other colours it is between \(-0.02\) and 0.00 mag. Because the definition of Kron magnitudes is similar to Petrosian magnitudes, we conclude that this offset can be explained by the difference in magnitude definitions. S1 also have many more Petrosian magnitudes, we therefore conclude that this offset can be explained by the difference in how the magnitudes are computed by looking at the change in colours when adopting Petrosian magnitudes.

\[ \sigma = \frac{\sum \sigma_i \exp \left( -\frac{1}{2} \chi_i^2 \right)}{\sum \exp \left( -\frac{1}{2} \chi_i^2 \right)} , \]  

(B2)

where \(\sigma_i\) is the best-fitting dispersion of the \(i\)th Monte Carlo realization and \(\chi_i^2\) is the lowest reduced \(\chi^2\) of the \(i\)th Monte Carlo realization.

The emission lines we consider in the fit are those 20 emission lines considered in Stoughton et al. (2002). These lines are expected to be the strongest in star-forming galaxies according to the models of Anders & Fritze-v. Alvensleben (2003) and furthermore include the strongest emission lines expected due to active galactic nucleus activity. For simplicity all 20 lines are masked out in the fit.

**APPENDIX C: BAYESIAN MAXIMUM LIKELIHOOD MODELLING**

Assuming that the errors are Gaussian the SFH, stellar mass weighted stellar metallicity and effective extinction can be computed according to

\[ a = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} w_i (\lambda_i - \lambda_j)^2}{\sum_{i=1}^{n} \sum_{j=1}^{m} w_i} , \]  

(C1)

**Figure C1.** Distribution of Bayesian weights, \(w_i\), in the \((i - z)\) versus \((g - r)\) diagram (colour coded pixels) for the seven bins in \(M_r\) adopted.
where $a$ is any of the properties mentioned above, $i$ denotes a model, $o$ an observation, $c_j$ a colour and $\sigma_{co,j}$ the error in an observed colour, and the summations are done over all $n$ models and all $m$ colours. The priors, i.e. the weights, $w_i$, are taken as the ratio of the number of models in the library and the number of galaxies in the Guo et al. (2011) catalogue\(^{12}\) for bins\(^{13}\) in $M_r$ where the ratios are computed for models and galaxies within 0.1 dex in stellar mass weighted age and those within 0.3 dex in stellar mass weighted $z$ of $i$. Fig. C1 illustrates how the Bayesian approach influences the model grid in terms of age and metallicity.

\(^{12}\) We only make use of the part of the simulation box known as milli-millennium. The sample contains more than 50,000 galaxies which ought to be sufficient.

\(^{13}\) $-24.5$–$22.5$, $-23.5$–$21.5$, ..., $-18.5$–$16.5$ mag. Note that even in the absence of spectroscopic distances $M_r$ can be estimated based on the observed galaxy structure.