In a previous communication \(^1\) (hereafter referred to as I) the ring summation for a classical electron gas has been modified in a simple way. What has been done in I is to attach two additional bonds to the rings as shown in Fig. 1(a). In the notation of Abe \(^2\) these diagrams contain three junctions. However, the junctions in Fig. 1 differ from Abe's for two reasons. In Abe's case each bond represents the Mayer function \(f_{ij} = e^{-\beta v_{ij}} - 1\) (where \(\beta = 1/kT\) and \(v_{ij}\) is the Coulomb potential between the particles). Further the potential \(v_{ij}\) is screened in the Debye-Huckel manner and is of the form \((e^2/r_{ij})e^{-q_D q_{ij}}, q_D\) being the Debye parameter. In contrast to this the bonds in Fig. 1 represent the bare Coulomb potential \(e^2/r_{ij}\).

Here we extend the procedure adopted in I for diagrams with four junctions (Fig. 1(b)). We shall calculate the change in free energy \(F\) due to interactions by evaluating the contribution of diagrams in Fig. 1(b) to the quantity \(-\beta \Delta F/N\) (which is equal to \(S/\rho\) in the notation of Abe \(^3\)) by making use of the prescription given by Brout. \(^3\)

The topological factor which multiplies the contribution from each diagram can be obtained as follows: Among \(n\) vertices there are \(n!/(2n)\) distinct graphs; on each of these the first extra bond can be added in \(n\) ways and the second in \((n-3)\) ways. Now since the sequence in which the two

\[ f_{ij} = e^{-\beta v_{ij}} - 1 \]
bonds are added does not alter the topological structure we have to divide it by 2 to avoid double counting. Thus there are \((n!/2n)n(n-3)/2\) distinct graphs for \(n\) vertices. \(n!\) cancels with the corresponding factor in the cluster expansion leaving a factor \((n-3)/4\) with each diagram.

Following the notation in reference 3) we can now write the contribution from diagrams in Fig. 1(b).

\[
-\beta DF = \frac{-\beta^2 \rho}{4} \int \left( \frac{e^{2 \pi \rho n(q)}}{2} \right)^2 \frac{d^3 q}{q} + \frac{-\beta^2 \rho}{4} \int \left( \frac{e^{2 \pi \rho n(q)}}{2} \right)^n \frac{d^3 q}{q} + \ldots + \frac{-\beta^2 \rho}{4} \sum_{n=4}^{\infty} \frac{n(n-3)}{2} \left( \frac{e^{2 \pi \rho n(q)}}{2} \right)^n \frac{d^3 q}{q}.
\]

In order to sum the series we introduce the following Fourier transforms

\[
v(q) = \int \frac{e^{2 \pi \rho q r}}{r^2} d\tau = \frac{4 \pi e^2}{q^2}, \quad w(q) = \int \frac{e^{2 \pi \rho q r}}{r^2} d\tau = \frac{2 \pi e^4}{q^2}.
\]

Using the convolution method employed to sum the usual ring diagrams (see reference 3)) the general term in the series (1) can be easily rewritten in the form

\[
\frac{-\beta^2 \rho}{4} \int \frac{d^3 q}{(2\pi)^3} \left[ w(q) \right]^n \left[ -\beta \rho v(q) \right]^{n-2} \frac{d^3 q}{(2\pi)^3}.
\]

Thus the series (1) can be cast into the form

\[
-\beta DF = \frac{-\beta^2 \rho}{4} \int \frac{d^3 q}{(2\pi)^3} \left[ w(q) \right]^n \left[ -\beta \rho v(q) \right]^{n-2} \frac{d^3 q}{(2\pi)^3}.
\]

Summing the series under the integral sign, as usual, before carrying out the integration we get

\[
-\frac{\beta DF}{N} = \frac{-\beta^2 \rho}{4} \int \frac{d^3 q}{(2\pi)^3} \left[ w(q) \right]^n \left[ -\beta \rho v(q) \right]^{n-2} \frac{d^3 q}{(2\pi)^3}.
\]

We notice that this has the same form as the corresponding result in I but for the opposite sign. This is interesting in view of the fact that the diagrams contributing to (5) have four junctions whereas those considered in I have only three. It shows that there is no simple correlation between the number of junctions and the power of \(\lambda\) in the series expansion for free energy.

It must be pointed out that in deriving (5) we have not made the approximation of low density and high temperature as has been done in Abe's analysis. We have carried out a natural extension of ring summation without invoking screening in an ad hoc manner. However, the method
as it stands is not suitable for handling more complicated graphs.

