Total Effect of any Nearly-Diurnal Wobble of the Earth's Axis of Rotation in Latitude and Time Observations. Application to the Paris Astrolabe Observations from 1956.6 to 1973.8

Nicole Capitaine

(Received 1975 January 6)

Summary

The existence of a non-spherical ideal liquid core in the Earth's interior produces a free nearly-diurnal wobble of the Earth's axis of rotation. The effect of this wobble in latitude and time observations is more complex than has generally been considered before. In previous discussions only the variation of latitude and time due to the neglected wobble has been considered and not the errors arising from the complementary neglected nutation in space. We establish here the theoretical expression for the total effect and we show that the first effect is negligible in comparison with the second. This expression is compared with similar ones due to the diurnal wobble and the corresponding main terms of nutation in space.

As an application we search for such expressions in the Paris astrolabe observations from 1956.6 to 1973.8 using several kinds of analysis. We first evaluate the amplitudes of errors in the main terms of nutation and secondly we show that the amplitude of a free nearly-diurnal wobble, if it exists, is at least 100 times lower than the one claimed by several authors.

Introduction


As we have previously suggested (Capitaine 1974), such a neglected nutation in space would induce errors in latitude and time observations since the equatorial co-ordinates of stars are assumed error-free in the reduction of these observational data. We examine here the formulation of these errors, which are similar to those due to errors in the main terms of nutation. We compare their theoretical amplitudes with the theoretical variations of latitude and time due to the nearly-diurnal wobble and we study the different ways in which these errors appear.
1. Total effect of the nearly-diurnal wobble in latitude and time observations

1. Variation of latitude and time

We consider a retrograde nearly-diurnal wobble of amplitude $\gamma$ and frequency $k\omega$, with $k$ nearly equal to $-1$, $\omega$ being the angular velocity of the Earth. In the case of the wobble induced by the liquid core, $1+k$ is the dynamical ellipticity of this core (Molodensky 1961); it is about $1/400$ (Jeffreys & Vicente 1957). We take the principal axes of inertia GM, GN, GO, as terrestrial reference axes (Fig. 1), we denote by ($\omega_1$, $\omega_2$, $\omega_3$) and by ($m_1$, $m_2$, $m_3$) respectively the components and the direction cosines of the rotation axis relative to them and by $t_0$ the initial epoch. At instant, $t$, this wobble is such that:

$$
\begin{align*}
  m_1 &= \omega_1/\omega = \gamma \cos \left[ k\omega(t-t_0) - \sigma \right] \\
  m_2 &= \omega_2/\omega = \gamma \sin \left[ k\omega(t-t_0) - \sigma \right] \quad \text{(Woolard 1953)}
\end{align*}
$$

It gives variations $d_1\phi$ and $d_1 C$ of latitude and time at a station of latitude $\phi_0$ and longitude $L_0$:

$$
\begin{align*}
  d_1\phi &= \gamma \cos \left[ k\omega(t-t_0) - \sigma + L_0 \right] \\
  d_1 C &= -\gamma \tan \phi_0 \sin \left[ k\omega(t-t_0) - \sigma + L_0 \right]
\end{align*}
$$

We call these variations the 'direct effect' of the wobble in the observations.

---

**Fig. 1.** Definition of the Euler's angles. G is the Earth's centre of mass.
2. **Errors arising from the complementary nutation in space**

We examine here how the nearly-diurnal wobble produces errors in latitude and time data through its complementary neglected nutation in space.

(a) **Forced motion of the axis of maximum inertia in space.** Every wobble of the rotation axis induces a forced nutation of the axis of maximum inertia in space which is obtained by using Euler's kinematical equations:

\[
\begin{pmatrix}
\dot{\psi} \\
\dot{\theta} \\
\dot{\Phi} + \psi \cos \theta
\end{pmatrix} =
\begin{pmatrix}
-\sin \Phi & -\cos \Phi & 0 \\
-\cos \Phi & \sin \Phi & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\omega_1 \\
\omega_2 \\
\omega_3
\end{pmatrix}
\]  

(3)

In these equations the wobble is expressed by its components \(\omega_1, \omega_2, \omega_3\) and the nutation in space by the temporal variations of \(\dot{\psi}, \dot{\theta}, \dot{\Phi}\) defined by the Fig. 1. Following Woolard (1953), \(\dot{\psi}\) is the celestial co-latitude 20 of the pole of inertia 0, \(\dot{\psi}\) the longitude of its descending node measured in the fixed ecliptic, GXY, eastward from the fixed equinox GX and \(\Phi\) the Earth's diurnal rotation.

The forced nutation corresponding to the nearly-diurnal wobble defined by (1) is therefore such that:

\[
\begin{align*}
\dot{\psi} \sin \theta &= -\omega \gamma \sin [\Phi + k \omega (t - t_0) - \sigma] \\
\dot{\theta} &= -\omega \gamma \cos [\Phi + k \omega (t - t_0) - \sigma] \\
\dot{\Phi} &= \omega_3 - \psi \cos \theta.
\end{align*}
\]

(4)

Variations \(A_1\psi\) and \(A_1\theta\) in longitude and obliquity due to this forced motion in space and such that \(\dot{\psi} = \psi_0 + A_1\psi\) and \(\dot{\theta} = \theta_0 + A_1\theta\) are obtained by integrating (4) between instants \(t_0\) (the chosen origin) and \(t\).

For this purpose some simplifications are possible. We first see that \(A_1\theta\) is periodic with an amplitude \(\omega \gamma/(\Phi + k \omega)\) which is certainly much less than \(10^{-3}\) rad, and we know that \(\theta_0\) is about 23° 27', so a simplification consists of neglecting \(\cos \theta_0, A_1\psi\) and \(\sin \theta_0, A_1\theta\) respectively in comparison with \(\sin \theta_0\) and \(\cos \theta_0\). Hence:

\[
\int_{t_0}^{t} \dot{\psi} \sin \theta dt = \int_{t_0}^{t} (\dot{\psi}(\sin \theta_0 + \cos \theta_0 A_1\psi) d\psi = \sin \theta_0 A_1\psi,
\]

and

\[
\int_{t_0}^{t} \dot{\psi} \cos \theta dt = \cos \theta_0 A_1\psi.
\]

The second simplification consists of neglecting \(\dot{\psi} \cos \theta\) in comparison with \((1 + k)\omega_3\), since the larger term of \(\cos \theta\) due to precession is 50°/year or \(10^{-7}\) rad/day, and \((1 + k)\omega_3\) is of the order of \(2\pi(1 + k)\) rad/day, that is \(10^{-3}\) or \(10^{-4}\) rad/day. We can then put \(\Phi + k \omega = (1 + k)\omega\) and obtain an amplitude of \(\gamma/(1 + k)\) for \(A_1\theta\) and \(\sin \theta A_1\psi\). These simplifications give the following results:

\[
\Phi = \omega_3 (t - t_0) - \psi \cos \theta + \Phi_0
\]

(5)

\[
\sin \theta A_1\psi = \frac{\gamma}{(1 + k)} \cos [(1 + k)\omega_3 (t - t_0) - \beta]
\]

(6)

\[
A_1\theta = -\frac{\gamma}{(1 + k)} \sin [(1 + k)\omega_3 (t - t_0) - \beta]
\]

where

\[
\beta = \sigma + \psi \cos \theta - \Phi_0
\]

(7)

(b) **Forced motion of the rotation axis in space.** The forced nutation of the rotation axis in space due to the wobble is obtained by adding the two preceding motions: the motion of the rotation axis with respect to the axis of maximum inertia and the motion of the axis of maximum inertia in space.
The variations $\Delta \eta$ and $\Delta \varepsilon$ in longitude and obliquity referred to the rotation axis are then such that: $\Delta \eta = \delta \psi(t - t_0)$ and $\Delta \varepsilon = \delta \theta(t - t_0)$ where $\sin \theta \delta \psi$ and $\delta \theta$ are the components of the first motion respectively along the equinox line and its east-perpendicular lying in the equatorial plane.

The nearly-diurnal wobble defined by (1) is such that:

$$\sin \theta \delta \psi = -\gamma \cos [(1 + k)\omega_3(t - t_0) - \beta]$$

$$\delta \theta = \gamma \sin [(1 + k)\omega_3(t - t_0) - \beta]$$

and the corresponding forced nutation of the rotation axis in space can hence be expressed as:

$$\sin \theta \Delta \eta = [-k/(1 + k)]\gamma \cos [(1 + k)\omega_3(t - t_0) - \beta]$$

$$\Delta \varepsilon = [k/(1 + k)]\gamma \sin [(1 + k)\omega_3(t - t_0) - \beta]$$

We denote by $A = k\gamma/(1 + k)$ the amplitude of this nutation in space, by $\Gamma = (1 + k)\omega_3$ its frequency and by $P_2 = 1/(1 + k)$ its period expressed in sidereal days.

(c) Corresponding variations in equatorial co-ordinates of a star. Variations $\Delta \alpha$ and $\Delta \delta$ of right ascension $\alpha$ and declination $\delta$ of a star due to a nutation of the rotation axis in space defined by variations $\Delta \eta$ and $\Delta \varepsilon$ in longitude and obliquity are expressed as (Danjon 1959):

$$\Delta \alpha = \Delta \eta \cos \varepsilon + \sin \varepsilon \sin \alpha \tan \delta - \Delta \varepsilon \tan \delta \cos \alpha$$

$$\Delta \delta = \Delta \varepsilon \sin \alpha + \Delta \eta \sin \varepsilon \cos \alpha.$$ 

Then, the variations $\Delta_1 \alpha$ and $\Delta_1 \delta$ corresponding to the forced nutation defined by (9) are:

$$\Delta_1 \alpha = -A \{\cot \varepsilon \cos \alpha \tan \delta \cos \alpha \sin \Gamma (t - t_0 - \beta) + \tan \delta \cos \alpha \sin \Gamma (t - t_0 - \beta)\}$$

$$\Delta_1 \delta = A \{\sin \alpha \sin \Gamma (t - t_0 - \beta) - \cos \alpha \cos \Gamma (t - t_0 - \beta)\}.$$ 

(d) Corresponding errors in latitude and time data. Erroneous equatorial co-ordinates of stars used in the reduction of observations introduce errors in the computed latitude and time which are the same for each star observed at the same station (Débarbat & Guinot 1970). They are hence equal to the errors obtained for zenith stars with $\alpha = T$, $\delta = \phi_0$ ($T$ being the local sidereal time and $\phi_0$ the latitude of the station).

The errors $\Delta_1 \phi$ and $\Delta_1 C$ deduced from (11) are:

$$\Delta_1 \phi = -A \cos [T + \Gamma (t - t_0) - \beta]$$

$$\Delta_1 C = -A \tan \phi \sin [T + \Gamma (t - t_0) - \beta] - A \cot \varepsilon \cos \Gamma (t - t_0 - \beta).$$

We call these errors the 'indirect effect' of the nearly-diurnal wobble as they appear indirectly through the errors in equatorial co-ordinates of stars.

This 'indirect effect' is $R = k/(1 + k)$ times larger than the 'direct effect' and its period if $P_2$ if the observations are always made at the same local sidereal time. The coefficient $R$ and the period $P_2$ increase as $k$ approaches $-1$, which means as the period of the nearly-diurnal wobble approaches a sidereal day.

2. Total effect of the main terms of nutation in latitude and time observations

Every term, $N_i$, of the luni-solar nutation in space of frequency $\Gamma_i = (1 + k_i)\omega$ induces a forced wobble of frequency $k_i\omega$ in the Earth as appears by inverting relation (3) (Woolard 1953). If $N_i$ and $\theta_i$ are respectively the coefficients in longitude
and obliquity of the term, \( i \), the amplitude of the induced wobble is \( \gamma_i = \Delta \omega_i (1 + k_i) \), where \( \omega_i = (\mathcal{N}_i + \varphi_i)/2 \) and \( \Delta \) is the Fedorov's coefficient which takes into account elasticity of the Earth (Fedorov 1963; Melchior 1973). The total forced wobble induced by the luni-solar precession and nutation is called the 'diurnal wobble' as its main term, has a period of one sidereal day; its amplitude is less than 0''01 (Fedorov 1963).

In the same way as in Section 1 we have expressed the 'direct' and the 'indirect effect' of the nearly-diurnal wobble in observations, we can express:

(i) The variations \( d_2 \phi \) and \( d_2 C \) which appear in latitude and time observations if the correction of the 'diurnal wobble' is neglected.

At a station of latitude \( \phi_0 \) and longitude \( L_0 \), they are:

\[
\begin{align*}
 d_2 \phi &= \sum_i \{ \gamma_i \cos [k_i \omega(t-t_0) - \sigma_i + L_0] \} \\
 d_2 C &= -\sum_i \{ \gamma_i \tan \phi_0 \sin [k_i \omega(t-t_0) - \sigma_i + L_0] \}
\end{align*}
\]  

\hspace{1cm} (13)

where \( k_i \) is nearly equal to \(-1\) and \( \sigma_i \) is a constant.

(ii) The errors \( \Delta_2 \phi \) and \( \Delta_2 C \) induced in latitude and time data by the 'indirect effect' of errors \( \Delta_2 \alpha \) and \( \Delta_2 \delta \) in the co-ordinates of stars used in the reduction of observations and due to errors \( N_i \) and \( \Omega_i \) in nutation coefficients \( \mathcal{N}_i \) and \( \varphi_i \).

As the errors in the nutation in space corresponding to the errors \( N_i \) and \( \Omega_i \) are such that:

\[
\begin{align*}
 \sin \varepsilon \Delta_2 \psi &= -\sum_i \{ N_i \sin [\Gamma_i(t-t_0) - \tau_i] \} \\
 \Delta_2 \varepsilon &= -\sum_i \{ \Omega_i \cos [\Gamma_i(t-t_0) - \tau_i] \}
\end{align*}
\]  

\hspace{1cm} (14)

these errors \( \Delta_2 \phi \) and \( \Delta_2 C \) are for a local sidereal time \( T \) at a station of latitude \( \phi_0 \):

\[
\begin{align*}
 \Delta_2 \phi &= \sum_i \{ -N_i \cos [\Gamma_i(t-t_0) - \beta_i] \cos T + \Omega_i \sin [\Gamma_i(t-t_0) - \beta_i] \sin T \} \\
 \Delta_2 C &= -\sum_i \{ (N_i \cot \varepsilon + N_i \tan \phi_0 \sin T) \cos [\Gamma_i(t-t_0) - \beta_i] \\
 &+ \Omega_i \tan \phi_0 \sin [\Gamma_i(t-t_0) - \beta_i] \cos T \}
\end{align*}
\]  

\hspace{1cm} (15)

with \( \beta_i = \tau_i + \pi/2 \).

If we put \( A_i = (N_i + \Omega_i)/2 \) and \( A_{-i} = (N_i - \Omega_i)/2 \):

\[
\begin{align*}
 \Delta_2 \phi &= -\sum_i \{ A_i \cos [\Gamma_i(t-t_0) + T - \beta_i] \} \\
 \Delta_2 C &= -\sum_i \{ N_i \cot \varepsilon \cos [\Gamma_i(t-t_0) - \beta_i] \\
 &- \sum_i \{ A_i \tan \phi_0 \sin [\Gamma_i(t-t_0) + T - \beta_i] \}
\end{align*}
\]  

\hspace{1cm} (16)

The amplitudes \( \gamma_i \) and \( A_i \) are not related and can be of the same order.

3. Observed effects in classical latitude and time data

Comparing expressions (13) and (16) to (2) and (12), we see that the formulations of the effects considered in Sections 1 and 2 are similar, so we group them by the same notations, putting index \( i_0 \) for the particular case considered in Section 1. We have then: \( A_{i_0} = N_{i_0} \) and \( A_{-i_0} = R_{i_0} \gamma_{i_0} \). We denote for each term, \( i_0 \):

(i) By \( P_{0,i_0} = 1/k_i \), the period of the wobble of amplitude \( \gamma_i \) expressed in sidereal days and by \( P'_{0,i_0} \) this period expressed in mean days.
(ii) By $P_{2,i} = 1/(1+k_i)$, the period of the corresponding nutation in space expressed in sidereal days and by $P'_{2,i}$ this period expressed in mean days. Two different kinds of observations can be made: the first consists of observing at a constant mean universal time as is the case with astrolabe and meridian instrument; the second consists of observing at a constant sidereal time as is the case for observations of the same zenith star.

The effects for these two kinds of observations are different.

1. **Observations at a constant mean universal time**

In this case the observations have a period of one mean solar day (md) and the local sidereal time $T$ has a period of 365.25 md, then:

(i) The ‘direct effect’ appears in the observations with the amplitude $\gamma_i$ and the period $P_{1,i}$ alias of the period $P'_{0,i}$ or $P_{1,i} = P'_{0,i}i/(1 + P'_{0,i})$ md.

(ii) The ‘indirect effect’ of argument $\Gamma_i(t-t_0) + T$ appears in the observations with the amplitude $A_i$ and the period $P_{3,i} = (P'_{2,i} \times 365.25)/(P'_{2,i} + 365.25)$ md.

2. **Observations at a constant sidereal time**

In this case the observations have a period of one sidereal day (sd) and the local sidereal time is constant, then:

(i) The ‘direct effect’ appears in the observations with the amplitude $\gamma_i$ and the period $P_{4,i}$ alias of the period $P_{0,i}$ or $P_{4,i} = P_{0,i}i/(1 + P_{0,i}) = 1/(1+k_i)$ sd and therefore $P_{4,i} = P_{2,i}$.

(ii) The ‘indirect effect’ appears in the observations with the amplitude $A_i$ and the same period $P_{2,i}$.

We give in Table 1, for various values of $k_i$:

(i) The periods $P_{0,i}$ (in sidereal hours), $P_{1,i}$, $P_{2,i}$, $P_{3,i}$ (in mean days).

(ii) The ratio $R_i = k_i/(1+k_i)$ of amplitudes of the nutation, $i$, in space and of its corresponding wobble in the Earth.

The notations $N_p$, $S_1$, $\psi_1$, $P_1$, $\phi_1$, $\pi_1$ and $\Sigma_1$ correspond to the main nutations in space and $JV\,I$, $JV\,II$, $JV\,III$, $Mol\,I$, $Mol\,II$ to theoretical models of the nearly-diurnal wobble respectively due to Jeffreys and Vicente and to Molodensky.

In the cases considered here $R_i$ is very large since $k_i$ is nearly equal to $-1$.

In the case of the nearly-diurnal wobble, $R_{40}$ is also the ratio $A_{40}/\gamma_{40}$ of amplitudes of the ‘indirect’ and ‘direct’ effects in the observations. The direct effect is then negligible in comparison with the indirect effect.

4. **Search for these different effects in the Paris astrolabe observations**

1. **Method of analysis**

We have used latitude and time observations of the Paris astrolabe from 1956.6 to 1973.8, that is 5347 values of each kind.

Each pair of values, $\phi_{I,i}$ in latitude, $C_{I,i}$ in time is given by observation of a group, $I$, of stars: 12 groups are observed during the year and we can consider that observations are made at a constant mean universal time (UT).

Observing instruments, star programme, observing techniques and computation method are described by Arbey & Guinot (1961), Billaud (1970), Débarbat & Guinot (1970), Billaud & Guinot (1971) and Billaud (1973).

These data are homogeneous in latitude and we have homogenized them in time by referring them to International Atomic Time TAI.
Nearly-diurnal wobble of the Earth's axis

Table 1

Values of the periods \( P_{0,1}, P_{1,1}, P'_{2,1}, P_{3,1} \) and of the ratio \( R_{1} \) corresponding to various values of the coefficient \( k_{1} \).

<table>
<thead>
<tr>
<th>( k_{1} )</th>
<th>( P_{0,1} ) in sidereal hours</th>
<th>( P_{1,1} ) in mean days</th>
<th>( P'_{2,1} ) in mean days</th>
<th>( P_{3,1} ) in mean days</th>
<th>( R_{1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-0.99002730)</td>
<td>24.2418</td>
<td>138</td>
<td>100</td>
<td>79</td>
<td>99</td>
</tr>
<tr>
<td>(-0.99180883)</td>
<td>24.1982</td>
<td>183</td>
<td>122</td>
<td>91</td>
<td>121</td>
</tr>
<tr>
<td>(-0.99320000)</td>
<td>24.1643</td>
<td>245</td>
<td>147</td>
<td>105</td>
<td>146</td>
</tr>
<tr>
<td>(-0.99335154)</td>
<td>24.1606</td>
<td>255</td>
<td>150</td>
<td>106</td>
<td>149</td>
</tr>
<tr>
<td>(-0.99453913)</td>
<td>24.1318</td>
<td>365</td>
<td>183</td>
<td>122</td>
<td>182</td>
</tr>
<tr>
<td>(-0.99501365)</td>
<td>24.1203</td>
<td>442</td>
<td>200</td>
<td>129</td>
<td>200</td>
</tr>
<tr>
<td>(-0.99601092)</td>
<td>24.0961</td>
<td>792</td>
<td>250</td>
<td>148</td>
<td>250</td>
</tr>
<tr>
<td>(-0.99667577)</td>
<td>24.0800</td>
<td>1679</td>
<td>300</td>
<td>165</td>
<td>300</td>
</tr>
<tr>
<td>(-0.99715066)</td>
<td>24.0686</td>
<td>8387</td>
<td>350</td>
<td>179</td>
<td>350</td>
</tr>
<tr>
<td>(-0.99726970)</td>
<td>24.0657</td>
<td>(\infty)</td>
<td>365</td>
<td>183</td>
<td>365</td>
</tr>
<tr>
<td>(-0.99800546)</td>
<td>24.0480</td>
<td>-1355</td>
<td>500</td>
<td>211</td>
<td>-500</td>
</tr>
<tr>
<td>(-0.99857533)</td>
<td>24.0342</td>
<td>-764</td>
<td>700</td>
<td>240</td>
<td>-701</td>
</tr>
<tr>
<td>(-0.99900273)</td>
<td>24.0240</td>
<td>-575</td>
<td>1000</td>
<td>268</td>
<td>-1002</td>
</tr>
<tr>
<td>(-0.99950137)</td>
<td>24.0120</td>
<td>-447</td>
<td>2000</td>
<td>309</td>
<td>-2004</td>
</tr>
<tr>
<td>(-0.99985331)</td>
<td>24.0035</td>
<td>-386</td>
<td>6798</td>
<td>347</td>
<td>-6816</td>
</tr>
<tr>
<td>(-1.00014669)</td>
<td>23.9965</td>
<td>-347</td>
<td>-6798</td>
<td>386</td>
<td>6818</td>
</tr>
<tr>
<td>(-1.00099727)</td>
<td>23.9761</td>
<td>-268</td>
<td>-1000</td>
<td>575</td>
<td>1004</td>
</tr>
<tr>
<td>(-1.00142467)</td>
<td>23.9659</td>
<td>-240</td>
<td>-700</td>
<td>764</td>
<td>703</td>
</tr>
<tr>
<td>(-1.00199454)</td>
<td>23.9522</td>
<td>-211</td>
<td>-500</td>
<td>1335</td>
<td>502</td>
</tr>
<tr>
<td>(-1.00214057)</td>
<td>23.9487</td>
<td>-205</td>
<td>-466</td>
<td>1691</td>
<td>468</td>
</tr>
<tr>
<td>(-1.00216367)</td>
<td>23.9482</td>
<td>-204</td>
<td>-461</td>
<td>1760</td>
<td>463</td>
</tr>
<tr>
<td>(-1.00224000)</td>
<td>23.9464</td>
<td>-201</td>
<td>-445</td>
<td>2034</td>
<td>447</td>
</tr>
<tr>
<td>(-1.00249317)</td>
<td>23.9403</td>
<td>-191</td>
<td>-400</td>
<td>4204</td>
<td>402</td>
</tr>
<tr>
<td>(-1.00273030)</td>
<td>23.9347</td>
<td>-183</td>
<td>-365</td>
<td>(\infty)</td>
<td>367</td>
</tr>
<tr>
<td>(-1.00284934)</td>
<td>23.9318</td>
<td>-179</td>
<td>-350</td>
<td>-833</td>
<td>352</td>
</tr>
<tr>
<td>(-1.00332423)</td>
<td>23.9205</td>
<td>-165</td>
<td>-300</td>
<td>-1679</td>
<td>302</td>
</tr>
<tr>
<td>(-1.00398908)</td>
<td>23.9046</td>
<td>-148</td>
<td>-250</td>
<td>-792</td>
<td>252</td>
</tr>
<tr>
<td>(-1.00403000)</td>
<td>23.9037</td>
<td>-148</td>
<td>-247</td>
<td>-767</td>
<td>249</td>
</tr>
<tr>
<td>(-1.00498635)</td>
<td>23.8809</td>
<td>-129</td>
<td>-200</td>
<td>-442</td>
<td>202</td>
</tr>
<tr>
<td>(-1.00546087)</td>
<td>23.8697</td>
<td>-122</td>
<td>-183</td>
<td>-365</td>
<td>184</td>
</tr>
<tr>
<td>(-1.00664846)</td>
<td>23.8415</td>
<td>-106</td>
<td>-150</td>
<td>-255</td>
<td>151</td>
</tr>
<tr>
<td>(-1.00819117)</td>
<td>23.8050</td>
<td>-91</td>
<td>-122</td>
<td>-183</td>
<td>123</td>
</tr>
<tr>
<td>(-1.00997270)</td>
<td>23.7630</td>
<td>-79</td>
<td>-100</td>
<td>-138</td>
<td>101</td>
</tr>
</tbody>
</table>

Table 2

Computed values of the corrections to be added to the observed group-differences.

(Origin of time is 1965.0 of Besselian year)

<table>
<thead>
<tr>
<th>Pair</th>
<th>Latitude in (10^{-3})</th>
<th>Time in (10^{-4}) years</th>
<th>Latitude in (10^{-3})</th>
<th>Time in (10^{-4}) years</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_{l+1,l+1})</td>
<td>(b_{l+1,l+1})</td>
<td>(a_{l+1,l+1})</td>
<td>(b_{l+1,l+1})</td>
<td></td>
</tr>
<tr>
<td>1-2</td>
<td>(-48 \pm 5)</td>
<td>(+70 \pm 5)</td>
<td>(-3 \pm 1)</td>
<td>(+3 \pm 1)</td>
</tr>
<tr>
<td>2-3</td>
<td>(-22 \pm 6)</td>
<td>(-55 \pm 7)</td>
<td>(+1 \pm 1)</td>
<td>(+1 \pm 1)</td>
</tr>
<tr>
<td>3-4</td>
<td>(-34 \pm 7)</td>
<td>(+70 \pm 6)</td>
<td>(+1 \pm 1)</td>
<td>(+1 \pm 1)</td>
</tr>
<tr>
<td>4-5</td>
<td>(+53 \pm 6)</td>
<td>(-14 \pm 7)</td>
<td>(-1 \pm 1)</td>
<td>(0 \pm 1)</td>
</tr>
<tr>
<td>5-6</td>
<td>(-38 \pm 5)</td>
<td>(+40 \pm 6)</td>
<td>(+1 \pm 1)</td>
<td>(+2 \pm 1)</td>
</tr>
<tr>
<td>6-7</td>
<td>(+66 \pm 6)</td>
<td>(-15 \pm 6)</td>
<td>(+1 \pm 1)</td>
<td>(+1 \pm 1)</td>
</tr>
<tr>
<td>7-8</td>
<td>(-137 \pm 5)</td>
<td>(+37 \pm 5)</td>
<td>(-4 \pm 1)</td>
<td>(+4 \pm 1)</td>
</tr>
<tr>
<td>8-9</td>
<td>(+100 \pm 6)</td>
<td>(-12 \pm 6)</td>
<td>(+4 \pm 1)</td>
<td>(-1 \pm 1)</td>
</tr>
<tr>
<td>9-10</td>
<td>(+23 \pm 6)</td>
<td>(+12 \pm 6)</td>
<td>(+1 \pm 1)</td>
<td>(+2 \pm 1)</td>
</tr>
<tr>
<td>10-11</td>
<td>(+50 \pm 5)</td>
<td>(-91 \pm 6)</td>
<td>(-2 \pm 1)</td>
<td>(-2 \pm 1)</td>
</tr>
<tr>
<td>11-12</td>
<td>(+6 \pm 6)</td>
<td>(-16 \pm 6)</td>
<td>(+3 \pm 1)</td>
<td>(-1 \pm 1)</td>
</tr>
<tr>
<td>12-1</td>
<td>(+17 \pm 6)</td>
<td>(+22 \pm 6)</td>
<td>(+2 \pm 1)</td>
<td>(-2 \pm 1)</td>
</tr>
</tbody>
</table>
Each pair \( \phi_{I,t}, C_{I,t} \) of a group \( I \) at instant \( t \) has first to be corrected for constant and secular group corrections respectively denoted by \( a_{0,I} \) and \( a_{1,I} \) in latitude and by \( b_{0,I} \) and \( b_{1,I} \) in time.

Then, we look for terms such that:

\[
\phi_{I,t} + a_{0,I} + a_{1,I} (t - t_0) = \phi_{I,0} + \sum_i \gamma_i \cos \left[ \frac{2\pi(t-t_0)}{P_{1,i}} - \sigma_i + L_0 \right] + \sum_i A_i \cos \left[ \frac{2\pi(t-t_0)}{P_{3,i}} - \beta_i \right]
\]

\[
C_{I,t} + b_{0,I} + b_{1,I} (t - t_0) = C_{I,0} - \sum_i \gamma_i \tan\phi_0 \sin \left[ \frac{2\pi(t-t_0)}{P_{1,i}} - \sigma_i + L_0 \right] + \sum_i A_i \tan\phi_0 \sin \left[ \frac{2\pi(t-t_0)}{P_{3,i}} - \beta_i \right]
\]

\[
+ \sum_i N_i \cot\gamma \cos \left[ \frac{2\pi(t-t_0)}{P_{2,i}} - \beta_i \right]
\]

where \( t \) is expressed in mean days, \( \phi_{I,0} \) and \( C_{I,0} \) are the instantaneous values at instant \( t \) of latitude \( \phi \) and time (TU0–TAI), minus the variations due to the nearly-diurnal and diurnal wobbles. All the terms contained in the second member of (17), should appear in the spectral analysis of individual groups. To remove real variations of latitude and time and make evident only terms of amplitude \( A_i \) (or \( A_i \tan\phi_0 \)) such an analysis can be made for group-differences between two consecutive groups observed at 2-hr intervals. However, such a spectral analysis does not distinguish terms due to errors in the nutation coefficients and terms due to the neglected nearly-diurnal wobble except when the periods \( P_{1,i}, P_{2,i}, P_{3,i} \) (Table 1) corresponding to the principal nutation terms are known.

In order to make a better distinction we can find from the differences, by the method of least-squares, terms given by (15) and consider that for the nearly-diurnal wobble we should have \( N_{i0} = \Omega_{i0} \). All these analyses are described together with their results in the following sections.

2. Analysis of the individual groups

Group corrections with secular variations are first applied to individual values of \( \phi \) and \( C \). Mean Chandlerian, annual and semi-annual terms are then removed. Afterwards they are smoothed and interpolated to obtain equally spaced values which are submitted to spectral analysis.

(a) Corrections used

(1) We have used the method of differences (Archinard 1968; Débarbat & Guinot 1970) for computing the group corrections \( a_{0,I}, b_{0,I} \) and their secular variations \( a_{1,I}, b_{1,I} \) added to observations to refer them to the mean group. For this purpose we have first adjusted constant and secular terms among observations of every pair of groups for all the period from 1956.5 to 1973.8. We have thus obtained the group-differences corrections and their secular variations denoted by \( a_{0,I+1}, b_{0,I+1} \) and \( a_{1,I+1}, b_{1,I+1} \). Their values are given in Table 2. Then, we have computed \( a_{0,I}, b_{0,I}, a_{1,I}, b_{1,I} \) from the preceding values through the classical formula of the method of differences and thus obtained the values given in Table 3.

(2) Corrections used to remove the Chandlerian wobble from observations are obtained by interpolation of amplitude (AM) and phase (PH) of this wobble published
Table 3

Computed values of the group corrections. (Origin of time is 1965.0 of Besselian year)

<table>
<thead>
<tr>
<th>Group number</th>
<th>Latitude</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Secular term</td>
<td>Constant term</td>
</tr>
<tr>
<td></td>
<td>$a_1$ in $10^{-3'}$</td>
<td>$a_6$ in $10^{-3's}$</td>
</tr>
<tr>
<td>1</td>
<td>$-2.0 \pm 1$</td>
<td>$-66.0 \pm 6$</td>
</tr>
<tr>
<td>2</td>
<td>$+1.0 \pm 1$</td>
<td>$-16.0 \pm 6$</td>
</tr>
<tr>
<td>3</td>
<td>$0.0 \pm 1$</td>
<td>$+9.0 \pm 6$</td>
</tr>
<tr>
<td>4</td>
<td>$-1.0 \pm 1$</td>
<td>$+45.0 \pm 6$</td>
</tr>
<tr>
<td>5</td>
<td>$0.3 \pm 1$</td>
<td>$-5.0 \pm 6$</td>
</tr>
<tr>
<td>6</td>
<td>$-0.5 \pm 1$</td>
<td>$+5.0 \pm 6$</td>
</tr>
<tr>
<td>7</td>
<td>$-1.5 \pm 1$</td>
<td>$-28.0 \pm 6$</td>
</tr>
<tr>
<td>8</td>
<td>$+3.0 \pm 1$</td>
<td>$+111.0 \pm 6$</td>
</tr>
<tr>
<td>9</td>
<td>$-1.0 \pm 1$</td>
<td>$+14.0 \pm 6$</td>
</tr>
<tr>
<td>10</td>
<td>$0.0 \pm 1$</td>
<td>$-7.0 \pm 6$</td>
</tr>
<tr>
<td>11</td>
<td>$+2.0 \pm 1$</td>
<td>$-54.0 \pm 6$</td>
</tr>
<tr>
<td>12</td>
<td>$+0.5 \pm 1$</td>
<td>$-52.0 \pm 6$</td>
</tr>
</tbody>
</table>

by Guinot (1972) for every 0.6 years. These corrections are in latitude and time:

\[
AM(t) \sin \left\{ 2\pi [t - PH(t)]/1.190 + L_0 \right\}
\]
and

\[
AM(t) \tan \phi_0 \cos \left\{ 2\pi [t - PH(t)]/1.190 + L_0 \right\}/15
\]

where $t$ is the date of observation in years and $AM(t)$, $PH(t)$ the interpolated pole co-ordinates.

(3) The annual and semi-annual terms removed are the values derived from the observations by the method of least-squares. They are respectively:

\[
A_{365} = 0'' \cdot 09, \quad A_{182} = 0'' \cdot 02 \text{ in latitude}
\]

and

\[
A_{182} = 0s \cdot 0275, \quad A_{182} = 0s \cdot 0071 \text{ in time}.
\]

(b) Smoothing and interpolation of the data. We have smoothed these 5347 corrected latitude and time values by the Vondrak's method (Vondrak 1969) to remove short oscillations and also very long oscillations in time which would give noise in the spectral analysis.

These smoothed values have been interpolated to obtain a point every five days, that is 1261 data points.

(c) Spectral analysis. These corrected, smoothed and equally spaced latitude and time values have been submitted to spectral analysis. Computation of spectral density $S(\xi)$ for frequency $\xi$ has been made by Blackmann and Tukey's method (Blackmann & Tukey 1958) using the 'hanning' spectral window for smoothing the spectra.

Fig. 2(a) and (b) give spectra obtained respectively in latitude and time.

We note the presence in the two spectra of peaks corresponding to the following periods (with sometimes a slight deviation in latitude and time): 86 d, 108 d, 121 d, 200 d, 340 d and 390 d. In addition we note the peaks corresponding to 96 d, 170 d, 290 d, 450 d and 875 d in latitude and to 103 d, 185 d, 242 d, 787 d in time. Some of these peaks can be due to real variations of latitude and time due either to polar motion or to local phenomena as is probably the case for the 450 d-peak, the Chandlerian term having not been completely removed. Other peaks may be due to errors in the nutation corrections as is possibly the case of the 121 d peak (corre-
FIG. 2(a). Smoothed spectrum of individual latitude values. (b) Smoothed spectrum of individual time values.
Nearly-diurnal wobble of the Earth's axis

583

sponding to the semi-annual nutation), of the 185 d peak (corresponding in time to the semi-annual nutation at the period \( P_2 \)) and of the 340 d and 390 d peaks (corresponding to the 18.6 yearly nutation).

3. Analysis of the group-differences

2772 pairs of group-differences \( d\phi_{I,I+1}, dC_{I,I+1} \) are obtained for all observations of pairs of consecutive groups \( I \) and \( I+1 \) during the period, the isolated observations being rejected. The differences are corrected for group-differences corrections \( a^0_{I,I+1}, b^0_{I,I+1} \) and their secular variations \( a^1_{I,I+1}, b^1_{I,I+1} \) they are then smoothed by Vondrak’s method and interpolated to obtain a point every five days. The noise level so obtained is greater than among the individual values since the number of values used is smaller.

The corrected, smoothed and interpolated data are then used in two different ways:

(i) in submitting them to spectral analysis by the same method as in Section 4.2
(ii) in deriving, by the method of least-squares, the variations given by (15).

(a) Spectral analysis. Consecutive groups follow each other at 2-hr intervals, so real variations of latitude and time, except the very short-period terms, vanish in group-differences and only terms given by (12) and (16) should appear.

These terms can be written, if we put \( T_A = T_I - T_{I+1} \) and \( T_B = T_I + T_{I+1} \), \( T_I \) being the local sidereal time when the group \( I \) is observed.

\[
\begin{align*}
\Delta(\phi_I - \phi_{I+1}) &= 2\sin T_A \sum_I \{ A_i \sin [\Gamma_i(t-t_0)+T_B-\beta_i]\} \\
\Delta(C_I - C_{I+1}) &= -2\sin T_A \sum_I \{ A_i \tg_0 \cos [\Gamma_i(t-t_0)+T_B-\beta_i]\}
\end{align*}
\]

\( T_A = -15^\circ \), so \( 2\sin T_A = -0.52 \).

Therefore, every term of amplitude \( A_i \), frequency \( \Gamma_i \) and phase \( \beta_i \) appearing in individual group analysis and corresponding to an error in the position of the rotation axis in space should be found with the same frequency \( \Gamma_i \), amplitude 0.52 \( A_i \) and phase \( \pi/2-\beta_i \). Results of the spectral analysis by Blackmann & Tukey’s method using the ‘hanning’ spectral window are given in Fig. 3(a) and (b).

The noise level is greater than in the preceding spectra but we notice that some peaks are found here again with sometimes a slight change in period from the former or between latitude and time. Such is the case for peaks corresponding to periods of 108 d, 120 d, 340 d and 390 d (the last ones being separated in latitude differences only in the unsmoothed spectrum and with a narrower step in frequency). Some peaks found in one of the preceding spectra are found again here in the two spectra: there is a peak corresponding to the period of 96 d which appeared in latitude, and a peak corresponding to the period of 145 d which appeared in time. A peak is considered as interesting if it appears in at least three analyses.

(b) Least-squares fit of periodic terms corresponding to errors in the position of the rotation axis in space. Theoretical terms given by (15) can be written for group-differences

\[
\begin{align*}
\Delta_i(\phi_I - \phi_{I+1}) &= -N_i(\cos T_I - \cos T_{I+1}) \cos [\Gamma_i(t-t_0)-\beta_i] \\
&+ \Omega_i(\sin T_I - \sin T_{I+1}) \sin [\Gamma_i(t-t_0)-\beta_i]
\end{align*}
\]

\[
\begin{align*}
\Delta_i(C_I - C_{I+1}) &= \{-N_i(\sin T_I - \sin T_{I+1}) \cos [\Gamma_i(t-t_0)-\beta_i] \\
&- \Omega_i(\cos T_I - \cos T_{I+1}) \sin [\Gamma_i(t-t_0)-\beta_i]\tg_0}
\end{align*}
\]
Fig. 3(a). Smoothed spectrum of latitude group-differences. (b) Smoothed spectrum of time group-differences.

Or:

$$\Delta_i(\phi_1-\phi_{i+1}) = X_i B_1 \sin \frac{2\pi(t-t_0)}{P_{2,i}} - Y_i B_1 \cos \frac{2\pi(t-t_0)}{P_{2,i}}$$

$$-Z_i B_2 \cos \frac{2\pi(t-t_0)}{P_{2,i}} - W_i B_2 \sin \frac{2\pi(t-t_0)}{P_{2,i}}$$

$$\Delta_i(C_1-C_{i+1})/\tan \phi_0 = -X_i B_2 \sin \frac{2\pi(t-t_0)}{P_{2,i}} + Y_i B_2 \cos \frac{2\pi(t-t_0)}{P_{2,i}}$$

$$-Z_i B_1 \cos \frac{2\pi(t-t_0)}{P_{2,i}} - W_i B_1 \sin \frac{2\pi(t-t_0)}{P_{2,i}}$$

where

$$B_1 = \sin T_i - \sin T_{i+1}$$

$$B_2 = \cos T_i - \cos T_{i+1}$$

$$X_i = \Omega_i \cos \beta_i, \quad Y_i = \Omega_i \sin \beta_i$$

$$Z_i = N_i \cos \beta_i, \quad W_i = N_i \sin \beta_i$$

$$\Gamma_i = 2\pi/P_{2,i}$$

(21)
Nearly-diurnal wobble of the Earth's axis

Table 4

<table>
<thead>
<tr>
<th>Latitude: ((\phi_G - \phi_{G+1}))</th>
<th>Time: ((C_G - C_{G+1}))(\tan\phi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amp 1</td>
<td>Amp 2</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>25</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>17</td>
</tr>
<tr>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
</tr>
</tbody>
</table>

By a least-squares fit of a term given by (21), corresponding to a chosen period \(P_{2,1}\) among the values of \((\phi_{r} - \phi_{r+1})\) and \((C_{r} - C_{r+1})/\tan\phi_{0}\) we can compute \(X_{1}, Y_{1}, Z_{1}, W_{1}\), that is:

\[
\text{Amp 1} = |\Omega| = \sqrt{(X_{1}^{2} + Y_{1}^{2})}, \quad \text{Amp 2} = |N| = \sqrt{(Z_{1}^{2} + W_{1}^{2})},
\]

\[
\beta_{1} = \arctan(Y_{1}/X_{1}) = \arctan\left(\frac{W_{1}}{Z_{1}} - \frac{\pi}{2}\right).
\]

We have made these computations for each period \(P'_{2,1}\) between 100 days and 20 years with a chosen step. We give in Table 4 the periods \(P'_{2,1}\) and their corresponding amplitudes Amp 1 and Amp 2 for which these amplitudes are both large. We note in the right column the corresponding periods \(P'_{3,1}\) and \(P'_{3,-1}\).

We can deduce from this that there are errors in the corrections used for the semi-annual, annual and 18.6-yr nutations as they appear in the analysis through the coefficients \(A_{i}\) and \(A_{-i}\). The different ways in which the nutation reveals itself explains why the annual nutation for which the error found here is such that \(|\Omega| = |N| = 0''\cdot02\) (consistent with \(A_{i} = 0\) (with the 183.2 day period) and \(A_{-i} = 0''\cdot02\), with an infinite period), did not appear in the spectral analysis and, on the other hand, that the 18.6-yr nutation, for which the error found here is such that \(|\Omega| = 0''\cdot004\) and \(|N| = 0''\cdot02\), appeared with the same order at the two periods: \(P'_{3,1} = 346\) d and \(P'_{3,-1} = 386\) d.

Assuming that no other kind of error appears with the form (21), we can deduce the following errors in the corrections used for the main terms of nutation:

\[
|\Omega| = 0''\cdot010 \pm 0''\cdot005, \quad |N| = 0''\cdot018 \pm 0''\cdot005 \text{ in } \phi \text{ at } 183 \text{ d}
\]

\[
|\Omega| = 0''\cdot020 \pm 0''\cdot005, \quad |N| = 0''\cdot020 \pm 0''\cdot005 \text{ in } C \text{ and at } 366 \text{ d}
\]

\[
|\Omega| = 0''\cdot004 \pm 0''\cdot005, \quad |N| = 0''\cdot020 \pm 0''\cdot005 \text{ in } \phi \text{ and } C \text{ at } 18.6 \text{ yr}.
\]
On the other hand if the nearly-diurnal wobble produces an important nutation in space it should be such that $N_{i0} = \Omega_{i0}$, so this nutation could have, from Table 4, a period $P_{2, i}$ equal to:

$$\pm 106 \text{ d}, \pm 160 \text{ d} \text{ or} \pm 284 \text{ d}$$

This would correspond to periods $P_{3, i}$ respectively: 82 d or $-150 \text{ d}$, 110 d or $-290 \text{ d}$, 160 d or $-1286 \text{ d}$.

Peaks corresponding to the 108 d period exist in both spectral analyses. In time, where the period $P_{2, i}$ must also appear, a peak exists at 160 d. But a definite conclusion is difficult as we do not know a priori the period either in the Earth or in space, of the effect for which we are looking.

5. Interpretation of the results

1. Main nutations in space

The spectral analyses presented here show that some error exists in the correction for the principal term of the 18.6-yr nutation. This conclusion is confirmed by finding the term of the form (21) in the group-differences at the 18.6-yr period.

However, the spectral analysis would be disturbed by the presence of an annual term, and since we have only 17 years of data at one observatory we cannot be very confident of the amplitude found for this error. We have found that it was five times greater in longitude (0’-02) than in obliquity (0’-004) whereas other authors (Fedorov 1959; Taradii 1969) have found nearly the same value (0’-015); but the order is in agreement with the one provided by theoretical models (Jeffreys 1959).

We also found by fitting an expression of the form (21) to the group-differences an error of the order of 0’-02 in the annual nutation correction, that is bigger than the one provided by theory (Melchior 1973), and an error in the semi-annual nutation correction of the order of 0’-015, somewhat smaller than the 0’-025 found by Popov (1959).

Other similar analyses of a very long homogeneous series of observations would be necessary to confirm our results. We have principally examined here a suitable method of analysis for this kind of search and have shown that errors in the main nutation corrections can thus be revealed.

2. Nearly-diurnal wobble

We have shown in the preceding Sections 1 and 3 that a nearly-diurnal wobble of amplitude $\gamma_{i0}$ and period $P_{0, i0}$, appears in latitude and time observations, made at a constant mean UT, first, by the ‘direct effect’ of amplitude $\gamma_{i0}$ and period $P_{1, i0}$ and secondly, by the ‘indirect effect’ of amplitude $A_{i0}$ and period $P_{3, i0}$, $\gamma_{i0}$ being negligible in comparison with $A_{i0}$.

Thomas (1964), Débarbat (1969, 1971), Sugawa & Ooe (1970) with harmonic analysis, have found peaks which they considered to be the direct effect of the nearly-diurnal wobble. But, if correct, there should exist a further term, with an amplitude $R_{i0} = k_{i0} / (1 + k_{i0})$ times larger, at the $P_{3, i0}$ period due to the indirect effect. These authors find that $\gamma_{i0}$ is of the order of 0’-01 and $P_{1, i0}$ is either $-170 \text{ d}$, or $-204 \text{ d}$; then, from Table 1 we can conclude that $R_{i0} \approx 400$ and that the corresponding periods $P_{3, i0}$ are respectively $-2500 \text{ d}$ and $-1700 \text{ d}$. Though these periods are very long, it is probable that a term of amplitude 4’ should have been revealed in the observations. Therefore it seems unlikely that these peaks are due to the direct effect of the nearly-diurnal wobble. Maybe these peaks are due to the indirect effect of the nearly-diurnal wobble; this would change the period $P_{0, i0}$ to $-23.82$ sidereal hours (sh) and the amplitude $\gamma_{i0}$ would be 0’-01/400.
Nearly-diurnal wobble of the Earth's axis

However, in the case of latitude and time analysis of the Paris astrolabe observations (Débarbat 1971; Chollet & Débarbat 1972) the peaks found by the authors at 170 d in latitude and 203 d in time (which have been found again in the spectral analyses of this work) have not given the indirect terms of periods $P'_{2, i_0}$:

\[ P'_{2, i_0} = 320 \text{ d} \quad \text{if} \quad P_{3, i_0} = +170 \text{ d} \quad P'_{2, i_0} = -116 \text{ d} \quad \text{if} \quad P_{3, i_0} = -170 \text{ d} \]
\[ P'_{2, i_0} = 463 \text{ d} \quad \text{if} \quad P_{3, i_0} = +203 \text{ d} \quad P'_{2, i_0} = -130 \text{ d} \quad \text{if} \quad P_{3, i_0} = -203 \text{ d} \]

which suggests strongly that these peaks are not due to the nearly-diurnal wobble.

We have seen in Section 3 that in the case of observations made at a constant sidereal time, the direct and the indirect effect of the nearly-diurnal wobble appear at the same period $P_{2, i_0}$. Now, Popov analysed observations of two bright zenith stars of the same declination (Popov 1963) and found in the observed latitude differences a term of amplitude $0'' \cdot 016$ and period 463 d. He deduced that this term was due to the direct effect of the nearly-diurnal wobble and that $\gamma_{i_0} = 0'' \cdot 016$ for $P_{2, i_0} = 463$ d. But as we have shown that the indirect effect must appear at the same period $P_{2, i_0}$, may be that in fact $A_{i_0} = 0'' \cdot 016$ and hence $\gamma_{i_0} = 0'' \cdot 016 / 463$, the period $P_{0, i_0}$ deduced for the nearly-diurnal wobble being unchanged.

Moreover, the results obtained by our analysis show that there do not exist terms of the form (12) with amplitudes $A_{i_0}$ bigger than $0'' \cdot 015$; they would give an amplitude $\gamma_{i_0}$ smaller than $10^{-4}$ and therefore 100 times lower than the one found by all these authors.

Our results do not permit any conclusion on the period of the nearly-diurnal wobble. We can only note the period $P_{3, i}$ near 108 d which would correspond to the $P_{2, 1}$ period near 155 d, these errors appearing respectively in the four spectral analyses and in the least-squares fit of expressions (21). They would correspond to a period $P_{0, i}$ of $-24.16 \text{sh}$ near the $-24.164 \text{sh}$ period given by the third model of Jeffreys & Vicente (1964).

\[ \text{References} \]


