On the Universal $V-A$ Current-Current Interaction with $CP$ Violation

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(Received October 27, 1969)

A general universal $V-A$ current-current interaction with $CP$ violation proposed by complete analogy with the ordinary weak interaction is analyzed. We propose a test of checking mutual consistency of the various components involved in the interaction; the test consists of the requirements that both the magnitude of the parity violating nuclear potential and that of the $\Delta S=2$ transition do not affect those obtained from the ordinary weak interaction. It is shown that there is still but a little possibility for this kind of interaction to satisfy our requirements and that the $CP$ violating phase difference in the amplitude of $K\to 2\pi$ decay is at most $10^{-3}$ in magnitude which results in $\gamma_{+-}\approx \gamma_{00}$. A discussion is given for comparison with other similar proposals.

§1. Introduction and summary

Theory of current-current interaction, either of local interaction or mediated by boson, seems to be useful in providing a unified view on the whole scheme of elementary particle interaction with $CP$ conservation and non-conservation. In theories so far proposed, there are two alternatives in constructing the $CP$ violating interaction; one is to introduce a phase difference between the vector and the axial vector currents, which gives rise to an odd term under the $CP$ transformation, and the other is to introduce a phase difference between different components of unitary spin of the $V\pm A$ current. Because of as yet insufficient data, we could not discriminate these two methods on the present experimental data. However, we here prefer the second method from purely theoretical grounds since this method leads to a universal interaction on the complete analogy of the ordinary weak interaction. In addition to this, the universal interaction thus constructed can be compared with experiment, as we shall do in the following, so as to give a test of this kind of interactions which are assumed to be of $V-A$ type as well as the ordinary weak interaction.

The universal interaction with $CP$ violation is formed by a “universal” $SU(3)$ current, namely, a current generated from a “primary” current by a unitary transformation just in analogy with the Cabibbo current. This class

1) The main context of this paper was presented at the Symposium on Weak Interaction of Elementary Particles held at the Institute for Nuclear Study, University of Tokyo, March 1969.

§) Research supported in part by the Takeda foundation.
of interaction, however, involves new components which are not met in the ordinary weak interaction: (a) The $\Delta S=0$ transition arises from a $T=1$ component with a strength of significant magnitude in addition to the one from a $T=0$ component. (b) A $\Delta S=2$ transition arises in the lowest order of the interaction. As to the $\Delta S=0$ transition, McKellar has recently pointed out that the parity violating nuclear potential yields too large circular polarization of $\gamma$-ray in nuclear transition, which suggests failure of Oakes' interaction.

In this paper, we examine a universal current-current interaction with $CP$ violation and propose a test of this interaction independent of particular choice of the value of parameters which appear in the interaction. Our test consists of the following requirements that (i) the $\Delta S=0$ portion of the interaction must be compatible with experiment in view of McKellar's result, (ii) the contribution of $\Delta S=2$ transition does not exceed the one from the ordinary weak interaction, and (iii) the $CP$-violation effect has a correct order of magnitude.

In the following sections, we give upper limits on the real and the imaginary parts of the coupling constant of $\Delta S=2$ transition due to the universal interaction and then give an upper limit on the $CP$-violating portion of the $\Delta S=1$ transition. Our conclusions are the following: (a) There is still but a little possibility that the universal current-current interaction is consistent with experiment. (b) The interaction must be a new "independent" interaction in the sense that the over all strength of coupling constant is (one or two order) smaller in magnitude than that of the ordinary weak interaction. (c) The $CP$-violating phase difference between the amplitude of $K \to 2\pi$ decay with $\Delta I=1/2$ and that with $\Delta I=3/2$ is at most $10^{-3}$ in our interaction, which results in $\eta^{+} \approx \eta^{-}$. (d) Precise determination of the parity violating nuclear potential will be vital for discrimination of our theory from similar proposals.

§ 2. Currents and interactions

In constructing the currents, we assume the $V-A$ primary currents $i_-$ and $u_-$ for the weak and the new interactions as $\Delta S=0$, $\Delta Q=1$ and $\Delta S=1$, $\Delta Q=0$ components of an octet of $SU(3)$. The general unitary transformation in $SU(3)$ which is commutable with charge operator, that is, a $U$-spin rotation is written as

$$\mathcal{R}(\alpha, \beta, \gamma) = \exp(2i\alpha U_3) \exp(2i\beta U_2) \exp(2i\gamma U_3),$$

where $U_3=F_8$, $U_2=F_7$, and $U_2=\frac{i}{2}(-F_3+\sqrt{3}F_6)$ are $U$-spin generators or generators of $SU(3)$. Then we assume that the effective hadronic currents are the following rotated currents $[i_-]$ and $[u_-]$: $\mathcal{R}^{-1}(\alpha, \beta, \gamma) i_- \mathcal{R}(\alpha, \beta, \gamma)$

* After submitting this paper for publication, we learned that a similar analysis was done by Zachariasen and Zweig.
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\[ \exp(i\alpha_w) \left[ \exp(i\gamma_w) \cos \beta_w \cdot i_- + \exp(-i\gamma_w) \sin \beta_w \cdot v_+ \right] \]

and

\[ [u_-] = R^{-1}(\alpha_N, \beta_N, \gamma_N) u_- \mathcal{R}(\alpha_N, \beta_N, \gamma_N) \]
\[ = \exp(-2i\alpha_N) \left[ \exp(-2i\gamma_N) \cos^2 \beta_N \cdot u_- - \frac{1}{\sqrt{2}} \sin 2\beta_N \cdot u_0 \right. \]
\[ \left. - \exp(2i\gamma_N) \sin^2 \beta_N \cdot u_+ \right]. \]  

The weak and the new hadronic interactions are postulated to be

\[ \mathcal{H}_w = \frac{G_w}{\sqrt{2}} [i_-] \cdot [i_+] \]

and

\[ \mathcal{H}_N = \frac{G_N}{\sqrt{2}} [u_-] \cdot [u_+]. \]

Here \( G_w \) and \( G_N \) are constants.

In the presence of only the weak interaction, Eq. (4), together with the strong and electromagnetic interactions, the dynamical system is still CP conserving for arbitrary \( \alpha_w \) and \( \gamma_w \). Therefore we take the phase convention for this system as

\[ \alpha_w = \gamma_w = 0. \]

Then the parameter \( \beta_w \) is the usual weak interaction angle. The contents of the interactions, Eqs. (4) and (5), are summarized in Table I. Here we write the conventional notations \( \theta_w, \theta_N \) and \( \phi_N \) in place of \( \beta_w, \beta_N \) and \( 2\gamma_N \).

Table I. Components of the interactions, Eqs. (4) and (5).

<table>
<thead>
<tr>
<th>( \mathcal{H}_w )</th>
<th>( \mathcal{H}_N )</th>
</tr>
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<tbody>
<tr>
<td>( 4S=0 ) ( \frac{G_w}{\sqrt{2}} \left( \cos^2 \theta_w \cdot i_- \right) )</td>
<td>( \frac{G_N}{\sqrt{2}} \left( \cos \theta_N + \sin \theta_N \right) \cdot u_- u_+ )</td>
</tr>
<tr>
<td>( 4S=1 ) ( \frac{G_w}{\sqrt{2}} \left( \frac{1}{2} \sin 2\theta_w \cdot i_- \right) )</td>
<td>( \frac{G_N}{\sqrt{2}} \exp(i\phi_N) \frac{1}{2\sqrt{2}} \sin 4\theta_N \cdot u_- u_+ )</td>
</tr>
<tr>
<td>( 4S=2 ) (2nd order)</td>
<td>( \frac{G_N}{\sqrt{2}} \exp(2i\phi_N) \frac{1}{4} \sin^2 2\theta_N \cdot u_- u_+ )</td>
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§ 3. Comparison with experiment

(1) The parity violating nuclear potential due to one pion exchange (PVOPE) arises from the \( T=1 \) component of the combined interaction of \( \mathcal{H}_w \) and \( \mathcal{H}_N \). Then the requirement (i) given in the previous section sets a
condition on the quantity

\[ r = \frac{(\text{PVOPE})_w + (\text{PVOPE})_N}{(\text{PVOPE})_w} = 1 + \frac{G_N \cos^2 \theta_N}{G_w \sin^3 \theta_w} \]  

(7)
as

\[ |r| \ll \text{order of unity} \quad (\text{say} < 10). \]  

(8)
Note that Oakes’ interaction gives \( r \approx 3 \) for \( |r| \).

(2) **Magnitude of the \( \Delta S=2 \) transition**

The best recipe for estimation of this magnitude would be the mass difference of \( K_L \) and \( K_S \). Define

\[ \langle K | \mathcal{K}_N | K \rangle = \frac{G_N}{\sqrt{2}} \exp \left( 2i\phi_N \right) \sin^2 \theta_N \cdot a_{N(3)} \phi_K \phi_K, \]  

(9)

\[ \langle \pi^0 | \mathcal{K}_w | K \rangle = \frac{G_W}{\sqrt{2}} \sin \theta_W \cos \theta_W \cdot a_W \phi_K \phi_K. \]  

(10)
The mass difference \( \Delta m(K_L - K_S) \) gives an upper limit on the real part of the matrix element of Eq. (9) as

\[ |\text{Re} \langle K | \mathcal{K}_N | K \rangle| \ll m_K \Delta m \approx (0.77 \pm 0.20) \times 10^{-11} m_K^2, \]  

(11)where we used the experimental value \( \Delta m \approx 1/2 \cdot \Gamma_S \) with the decay rate of \( K_S \): \( \Gamma_S = (7.6 \pm 1.8) \times 10^{-13} \text{MeV} \). On the other hand, by using the current algebra with PCAC,\(^{10}\) we obtain

\[ |\langle \pi^0 | \mathcal{K}_w | K \rangle| \approx (1.8 \pm 0.5) \times 10^{-7} m_K^2. \]  

(12)
Then the requirement (ii) sets a condition on the quantity

\[ \rho = \frac{G_N \cos 2\phi_N \sin^2 \theta_N \cos^2 \theta_N}{G_W \sin \theta_W \cos \theta_W} \]  

(13)as

\[ |\rho| \ll \frac{m_K \Delta m}{(\sqrt{2}g_A |M_N|/g_s)^3} \left| \frac{\langle \pi^0 \rho \rangle |T| K_S \rangle \phi_{K} \right| |a_{N(3)}| \right| \]

\[ \approx (0.43 \pm 0.10) \times 10^{-7} \left| \frac{a_W}{a_{N(3)}} \right|. \]  

(14)
Next we give an upper limit on the imaginary part of the matrix element of Eq. (9) from the CP-mixing parameter\(^{13}\) \( \epsilon \) in a \( K-K \) complex. The parameter \( \epsilon \) arising from \( \mathcal{K}_N(\Delta S=2) \) is given by

\[ \epsilon = \frac{G_N}{\sqrt{2}} \sin 2\phi_N \sin^2 \theta_N \cos^2 \theta_N \cdot \frac{a_{N(3)}}{m_K \{2(m_S - m_L) - i(\Gamma_S - \Gamma_L)\}}. \]  

(15)
Defining a quantity

\[ \sigma = \frac{G_N \sin \phi_N \sin^2 \theta_N \cos^2 \theta_N}{G_W \sin \theta_W \cos \theta_W}, \]  

we get

\[ |\sigma| = \frac{m_K (2 (m_N - m_L)) - i (\Gamma_Z - \Gamma_L)}{(\sqrt{2} g_A M_N / g_w) \langle f \bar{f} \rangle (\pi_0 | T | K_0)^{-1} \phi^{-1} \phi^{-1}^{(-1)}} \cdot \frac{a_w}{a_N^{(3)}} \]

\[ \simeq (1.2 \pm 0.3) \times 10^{-7} \cdot \epsilon^{(3)} \cdot \frac{a_w}{a_N^{(3)}}. \]  

Therefore we obtain

\[ |\sigma| \ll |\rho| \]  

since

\[ |\epsilon^{(3)}| < |\epsilon| \simeq 10^{-5}. \]  

(3) **Magnitude of CP violation.**

Let us define a quantity

\[ \chi = \frac{G_N \sin \phi_N \sin \theta_N \cos \theta_N \cos 2\theta_N}{G_W \sin \theta_W \cos \theta_W} \]

as a measure of the magnitude of CP violation, which is the ratio of the imaginary part of the coupling constant of \( \mathcal{L}_N (\Delta S = 1) \) to the coupling constant of \( \mathcal{L}_W (\Delta S = 1) \). We have then an interrelation of \( r, \rho, \sigma \) and \( \chi \) from the definitions Eqs. (7), (13), (16) and (19) as follows:

\[ \chi^2 = \sin^2 \phi_N \tan \theta_W \cdot (r - 1) \sqrt{\rho^2 + \sigma^2}. \]  

Using the limits on \( \rho \) and \( \sigma \) as given in Eqs. (14) and (18), we get

\[ |\chi| < \sin \phi_N \sqrt{r - 1} \cdot (1.0 \pm 0.2) \cdot 10^{-4} |a_w/a_N^{(3)}|^{1/2}, \]  

where we have used the experimental value \([10] \) \( \tan \theta_W \approx 0.23 \). In calculating the \( a \)'s, we express these amplitudes in terms of reduced amplitudes of the unitary symmetries \( SU(2) \) and \( SU(3) \). The result is shown in Table II. The reduced amplitudes in \( SU(2) \) means \( a_M \) and \( b_M \) and those in \( SU(3) \) are assigned by the \( SU(3) \) dimensionality. In \( SU(2) \) assuming the dominance of \( \Delta I = 1/2 \) over \( \Delta I = 3/2 \) for the weak transition, we get from Eq. (21)

\[ |\chi| < \sin \phi_N \sqrt{r - 1} \cdot (0.8 \pm 0.2) \cdot 10^{-4} |a_{1/2}/b_1|^{1/3}. \]  

\(^*\) It is possible to express further the unknown factor \( \sin^2 \phi_N \) appearing in Eq. (20) in terms of \( \rho \) and \( \sigma \) as follows:

\[ \sin^2 \phi_N = \frac{1}{2} \left[ 1 \pm \frac{1}{\sqrt{1 + (\sigma/\rho)^2}} \right]. \]

But this is not necessary as will be seen shortly.
In $SU(3)$ if we assume the octet dominance over $27$, we obtain

$$|\chi| \leq |\sin \phi_N| \sqrt{r-1} \cdot (0.6 \pm 0.1) \cdot 10^{-4} \cdot |a_8/a_{27}|^{\eta_I}.$$ (23)

In both cases, $\chi$ could possibly be of an order of magnitude of $10^{-3}$ if we assume the condition on $r$, Eq. (8), and take $a_8/a_{27}$ or $a_{112}/b_1$ to be $10^{-100}$. Although, at present, there is no reliable dynamical calculation of the $CP$-violating matrix element, our result encourages us in getting a correct order of the $CP$ violation as is observed in the $K$-meson decay.

We now examine the Wu-Yang parameters, $\eta_\pm = \epsilon + \epsilon'$ and $\eta_0 = \epsilon - 2\epsilon'$. The $CP$-phase difference between $A_0$ and $A_2$ of $K \to 2\pi$ decay amplitudes is expressed in terms of our parameters as

$$\tan(\phi_0 - \phi_N) = 3(1 - \sqrt{6}/4 \cdot e_{28}) \cdot \chi,$$ (24)

where $c_{28}$ denotes the ratio between two amplitudes with $\Delta I=1/2$ which arise in the combined interaction of $\mathcal{M}_w(DS=1)$ and $\mathcal{M}_N(DS=1)$. Then assuming the factor $(1 - \sqrt{6}/4 \cdot c_{28})$ as an order of magnitude of unity, we get

$$|\epsilon'| = |\sin(\phi_0 - \phi_N) \cdot A_2/A_0| \simeq 1 \times 10^{-4},$$ (25)

where we have used the value $\chi \simeq 10^{-3}$, and the experimental value $|A_2/A_0| \simeq (3.1 \pm 0.1) \cdot 10^{-3}$. Thus $\epsilon'$ appears to be one order of magnitude smaller than the observed value of $|\eta_{\pm}|$. Therefore our theory leads to an approximate equality $\eta_{\pm} \simeq \eta_0$ which is not inconsistent with the present experimental results. This equality is also the prediction of the super weak model.

§ 4. Discussion

(1) We here make a crude estimate of the magnitude of coupling constant and angles. Firstly the results, Eq. (18) and Eq. (22) or (23), mean that $\phi_N$ must be almost $\pi/2$. Next from Eqs. (7), (13) and (14) we get

$$|\rho/(r-1)| = 1/4 \cdot |\cos 2\phi_N \tan^2 \theta_N \tan \theta_w|$$

$$\lesssim (1.7 \pm 0.4) \cdot 10^{-4} \cdot |(r-1)^{-1}a_8/a_{27}|.$$ (26)

This means that $\theta_N$ is at most of an order of magnitude of $10^{-2} \sim 10^{-3}$. Then from Eq. (7) we obtain

$$G_N/(G_N \sin^2 \theta_N) = r-1 \simeq \text{order of unity}.$$ (27)
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Thus the overall coupling constant $G_N$ of our interaction should be one or two orders of magnitude smaller than $G_W$ since $\sin^2 \theta_W \approx 0.05$. This makes a considerable contrast with the assumption made in Oakes' theory.

It should be noted that our interaction can also accommodate the super weak model, that is, $\epsilon = \epsilon_{(2)}$ as defined in Eq. (15). In this case we have, from Eqs. (16) and (17),

$$\sigma \approx 10^{-8} \sim 10^{-9}. \quad (28)$$

It would then be easy to take the parameters $G_N$, $\phi_N$, $\theta_N$ as small as possible so that they are irrelevant to the observed quantities other than $\epsilon$. We shall not enter into the details of this possibility in this paper.

(2) Our result on $\chi$ seems to be larger than what is usually expected from McKellar's result, Eq. (8). This situation can be traced back to our estimate of $\rho$, Eq. (14), on the basis of the PCAC method. In fact calculating the quantity of Eq. (12) by means of a factorization method\(^\text{18}\) instead of the PCAC method, we obtain, in terms of the form factors of $\pi \to \mu \nu$ and $K \to \mu \nu$ decays,

$$\frac{G_W}{\sqrt{2}} \sin \theta_W \cos \theta_W \cdot \frac{1}{\sqrt{2}} F_{\pi}\ F_{K} m_{K}^2 \approx 0.3 \cdot 10^{-7} m_{K}^2, \quad (29)$$

which is about one order of magnitude smaller than that of Eq. (12). However, it is known that the factorization method fails to give an octet or $\Delta I = 1/2$ enhancement. Thus the enhancement dynamics would be very important in the calculations of all the quantities relevant to the theory. In this respect, we recall that the $K \to 2\pi$ decay is forbidden in the lowest order of $\mathcal{M}_w$ in the exact $SU(3)$ limit. It is not known whether the breakdown of the $SU(3)$ arises in the $K \to \pi$ transition as well. This situation leaves us still indefinite in choosing Eq. (22) or (23) for the determination of $\chi$. Even when the breakdown is large enough to choose the case of Eq. (22), the calculation of $a_{1/2}/b_1$ would be still beyond the present scope because there is no physical process available for fixing $b_1$ alone. Therefore the final presentation of our theory on the dynamical level would need further clarification of the $K$-meson decay amplitude which is caused by the primary four-Fermion interaction.

(3) It is to be noted that our interaction is one of the most general interactions with universal octet currents. In general there are four neutral currents in an octet current of $SU(3)$; $U$-spin singlet and $U$-spin triplet. Among these currents, only $u_-$ or $u_0$ is to be taken as primary current, as an analogue with that of the Cabibbo current which is a rotated current of $U$-spin doublet started from a primary current $i_-$ or $v_+$. Our theory has used the $u_-$ current, while Oakes' theory used the $u_0$ current as the primary current. Let us briefly compare the $u_-$-theory with the $u_0$-theory. The rotated current in the $u_0$-theory is given in the form
\[ [u_0] = R^{-1}(\alpha_N', \beta_N', \gamma_N') u_0 R(\alpha_N', \beta_N', \gamma_N') \]
\[ = \cos 2\beta_N' \cdot u_0 + \frac{1}{\sqrt{2}} \sin 2\beta_N' \cdot [\exp(-2i\gamma_N') \cdot u_- + \exp(2i\gamma_N') \cdot u_+] \]  
(30)

It can be seen that the \( u_- \)-theory is essentially different from the \( u_0 \)-theory. In the three-dimensional Euclidean space of \( U \) spin, \( u_0 \) is a real vector, while \( u_\pm \) are complex vectors. The vector \( u_0 \) is able to be rotated into \( u_\pm \), but not into \( u_\pm \). This shows non-equivalence between these theories. The interaction in the \( u_0 \)-theory is assumed to be as follows:

\[ \mathcal{H}_N' = \frac{G_N'}{\sqrt{2}} [u_0] \cdot [u_0]^\dagger \]
\[ = \frac{G_N'}{\sqrt{2}} \left[ \cos^2 2\theta_N' \cdot u_0 u_0^\dagger + \sin^2 2\theta_N' \cdot u_- u_-^\dagger \right. \]
\[ + \frac{1}{\sqrt{2}} \sin 4\theta_N' \{ \exp(-i\phi_N') \cdot u_- u_-^\dagger + \exp(i\phi_N') \cdot u_0 u_0^\dagger \} \]
\[ + \frac{1}{2} \sin^2 2\theta_N' \{ \exp(-2i\phi_N') \cdot u_- u_-^\dagger + \exp(2i\phi_N') \cdot u_+ u_+^\dagger \} \].  
(31)

Here again we write angles \( \theta_N', \phi_N' \) for \( \beta_N', 2\gamma_N' \).

Comparing Eq. (31) with Table I, both interactions have exactly the same structure for the \( \Delta S=1 \) and \( \Delta S=2 \) portions except a factor \(-2\) in the \( u_0 \)-theory. Difference is mainly in the \( \Delta S=0 \) portion; the parameter \( r \) given in Eq. (7) is written in the \( u_0 \)-theory as

\[ (r)_{u_0\text{-theory}} = 1 - \frac{G_N' \cos 4\theta_N'}{G_W \sin^2 \theta_W} \]  
(32)

instead of Eq. (7). If we assume that both \( G_W \) and \( G_N \) are of positive quantities that would be a natural consequence of the intermediate boson theory, the parameter \( r \) in \( u_- \)-theory must be larger than unity whereas \( r \) in \( u_0 \)-theory can take any value. Therefore a crucial test for discrimination of two theories would be a precise determination of \( r \); if \( r \) is found to be smaller than unity, the \( u_- \)-theory must be invalid.

Oakes’ theory gives \( r \approx -30 \). However both the \( u_- \) and the \( u_0 \)-theories\(^*\) yield \( |r| \approx 15 \) if the coupling constants \( G_N \) and \( G_N' \) were taken as the same with \( G_W \).

(4) We have studied the implication of the \( CP \) violating universal \( V-A \) current-current interaction. We also proposed a test of the whole scheme of this kind.

\(^*\) The discrepancy between \( u_0 \)-theory and Oakes’ theory comes from a difference of normalization. We have taken \( i_1, u_\pm \) and \( u_\theta \) as components in an octet current of \( SU(3) \). In our notation, Oakes used \( i_1 \) and \( \sqrt{2} u_0 \) as the primary currents and the coupling constants \( G_{N'}=G_W \) as can be seen in the comparison of Eqs. (1) and (4) in reference 4) with Eqs. (30) and (31) in this paper.
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of interaction, that is, an interrelation, Eq. (20), among quantities characteristic of the structure of the interaction. This consists of the tests of (a) the magnitude of parity violating nuclear potential due to one pion exchange (PVOPE), and (b) the magnitude of $\Delta S=2$ transition. As for other tests, e.g., an electric dipole moment of nucleon is not relevant to our theory since it does not arise in the lowest order of the interaction. This is consistent with the present experimental data. The other weak processes, especially, hyperon's non-leptonic decays would show up still small CP violation, which is expected to be an order of magnitude of $10^{-3}$. Finally the leptonic process is not altered any more from the presently accepted argument as long as the new interaction is confined to the hadronic interaction alone.

References

8) S. Y. Tsai, Prog. Theor. Phys. 40 (1968), 914.
14) Data are from Particle Data group, Review of Particle Properties; Rev. Mod. Phys. 41 (1969), 109.