A new theory of high energy hadronic collisions is proposed on the basis of the experimental evidences for the so-called H-quantum which was introduced by Hasegawa.

§ 1. Introduction

We shall report here a possible new avenue to a further understanding of the properties of hadronic collisions at very high energy.

It was shown from the accelerator and the cosmic ray experiments that (1) the distribution of the transverse momenta of the emitted mesons is kept almost constant ($\langle p_T \rangle \approx 300 \text{ MeV/c}$) in a wide energy region from a few tens BeV to an extremely relativistic energy ($\approx 1000 \text{ BeV}$) and (2) it is almost independent of the attributes of the incident hadrons. It has also been inferred from some indications that (3) the average value $\langle p_T \rangle$ has a little tendency toward increasing in higher energy reactions and (4) there appears a group of mesons which have high transverse momenta, in extremely high energy collisions. We shall develop a theory in asymptotic energy region simply on the basis of the experimental evidences (1) and (2). The other evidences (3) and (4) will not be discussed here though they might have an important meaning. Our guiding principle based on (1) and (2) is as follows.

The principle of separability

The matrix elements of high energy hadronic collisions consist of two factors, the transverse and longitudinal ones. The former contains only the transverse momenta of the particles concerned while the latter can be described by a one-dimensional dynamics in the longitudinal direction.

It has been pointed out by many authors that the angular distribution of the secondary particles averaged over many events is consistent with the theory of the one-dimensional motion of the relativistic matter continuum as in the present guiding principle. On the other hand, the individual angular distribution frequently shows a specific pattern which may be due to the production of the strongly correlated particles. The correlation can be referred to either usual resonances like $\rho, \omega$ and so on or some other objects whose properties have not
completely been understood yet. In the latter case, it is required from the sup­pression of the transverse momenta of the emitted mesons that the objects radiate mostly soft pions (kaons). In fact, Koshiha et al.\textsuperscript{5} considered the object as the \( \varphi \)-meson which is bound in a nucleon with small \( Q \)-value (so-called aleph baryon with mass \( \approx 2 \text{ BeV} \)). The accelerator experiments also indicate that the emission of soft mesons is the dominant process in the multiple meson production.\textsuperscript{6} We consider that the empirical evidences thus found in the neighbourhood of the leading particles are due to such objects as aleph.

If we turn our attention to the overall features of the jet interactions, we find that the pionization part (fire ball) also has the similar properties. In fact, prior to the above evidences, Hasegawa identified the objects with the so-called \( H \)-quanta with average mass \( \approx 2 \text{ BeV} \).\textsuperscript{7} It was indicated that the velocity distribution of these objects has an enhancement in the Lorentz factors \( \approx 1.5, \approx 7 \) and \( \approx 40 \) in barycentric system. The group of the \( H \)-quanta which are produced in a reaction is called the system "\( L \)".

In the present analysis, we shall take the following description for nucleon-nucleon collision.

\[
\text{nucleon} + \text{nucleon} \rightarrow \text{two aleph like states} + \text{\ L}.
\]

The interaction should satisfy the principle of separability.

In § 2, we shall discuss on the longitudinal factor of the matrix element and on the mechanism of the production of \( H \)-quanta giving a new interpretation for system \( L \). We also analyse the relation between the asymptotic behavior of the inelastic cross section and the number of the fundamental particles.

It is shown in § 3 that the wave function of the quarks in hadron is uniquely determined from the principle of separability, when we take the quark model which was introduced by Gell-Mann.\textsuperscript{8}

The transverse factor of the matrix element is discussed in § 4.

§ 2. Inelastic cross section and the number of the fundamental particles

The direction of the incident hadron has a specific meaning in the present theory. The momentum \( q \) is divided into the transverse and longitudinal components \( q_\perp \) and \( q_\parallel \). We use a space \( H = (q_\perp, \theta, m) \) instead of the space of four-momentum \( (q, q_0) \), where \( \theta \) represents the Lorentz angle in the longitudinal direction, \( m \) and \( q_0 \) the rest mass and the energy, respectively.

\[
\theta = \tanh^{-1}(q_\perp/q_0),
\]

\[
m = \sqrt{q_0^2 - q^2}.
\]

We propose a theory of particle production in the one-dimensional space \( \theta \) on the basis of the principle of separability. Let us introduce a set \( L \) of the levels in the \( \theta \)-space with equal interval as shown in Fig. 1.
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Fig. 1. The diagram of the system $L$. The circles represent the levels which are occupied by some $\lambda$-particles. They are observed as the $H$-quanta. The crosses and arrow represent the unoccupied levels and the position of the entrance.

\[
L = (\cdots, \theta_{-1}, \theta_0, \theta_1, \theta_2, \cdots),
\]

where

\[
\theta_s = (s - s_0) \bar{s}.
\]

Notations $\bar{s}$ and $s_0$ represent constants. It should be considered that the particles in the system $L$ do not have the transverse momentum.\(^{*)}\) We shall take the following fundamental interpretation.

The $H$-quantum is an assembly of the fundamental particles $\lambda$ in a level of the $L$-system.

In Fig. 1, each circle represents the assembly of the $\lambda$-particles at the level of the $L$-system, which is observed as the $H$-quantum. Since the average mass of the $H$-quantum is at most twice as large as the nucleon mass, the Pauli principle should work at these levels and, therefore, the $\lambda$-particle must be a fermion. If the $\lambda$-particle is of $N$-multiplet including the antiparticle and the spin states, each level can contain $N$ $\lambda$-particles at most.

The longitudinal momentum which is transferred to the $L$-system vanishes at an appropriate point of the $\theta$-space which is called the "entrance of energy" or simply the entrance. According to Hasegawa, the production of $H$-quanta is symmetrical in the $\theta$-space and the center appears in the middle of two successive levels as shown in Fig. 1. Conversely, if only the middle of two levels can be the entrance, the symmetrical production is required from the momentum conservation law except for some special cases. Hereafter, we shall take the latter interpretation.

We consider that the hadronic reaction occurs via the interaction between the incident hadrons $a_{1,2}$ and the $L$-system as illustrated in Fig. 2. The energy and momentum of $L$-system is expressed by the coordinate $(0, \theta_L, T)$

\[\text{Fig. 2. The structure of hadronic collision. Particles } a_{1,2} \text{ and } a'_{1,2} \text{ represent the incoming and outgoing hadrons, respectively.}\]

\(^{*)\} This is the simplest and most faithful interpretation. But, there may be many other possible interpretations for the transverse momentum in the $L$-system.
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in the \(H\)-space. From the definition, the \(L\)-system does not have the transverse momentum. The notations \(E, k_{1,2}\) and \(E_{1,2}\) are used for the total energy, the transverse momenta and the energies of the outgoing hadrons \(a_{1,2}\) in the barycentric system. Since the outgoing hadrons have sufficiently high energy and, experimentally, the transverse momenta \(k_{1,2}\) are restricted within a small region \(\leq 300\) MeV/\(c\), we can use the energies \(E_{1,2}\) instead of the longitudinal momenta.

From the guiding principle, the correlation between the transverse momenta \(k_{1,2}\) must be factorized in the matrix element. Then, the inelastic cross section can be written as

\[
\frac{d\sigma^{\text{in}}}{d^3k_1 d^3k_2 dE_1 dE_2} = \frac{C}{E_1 E_2 E^2} \mu(\theta_L) \frac{S(T)}{T} \omega(E, E_1) \omega(E, E_2) W(k_1, k_2), \tag{2.3}
\]

where

\[
W(k_1, k_2) = w(k_1) \delta^4(k_1 + k_2), \tag{2.4}
\]

\[
T = \sqrt{(E - 2E_1)(E - 2E_2)},
\]

\[
\theta_L = \sinh^{-1}((E_2 - E_1)/T). \tag{2.5}
\]

The notation \(\mu(\theta_L)\) represents the distribution of available entrances, \(S(T) dT\) the transition probability for the \(L\)-system from one entrance with energy \(T \sim T + dT\), \(\omega(E, E_{1,2})\) the transition probability from hadrons \(a_{1,2}\) to an entrance of the \(L\)-system. The function \(W(k_1, k_2)\) represents the correlation between the transverse momenta. The energy denominators \(E_1 E_2 E^2\) and \(T\) come from the kinematic factors of hadrons \(a_{1,2}\), \(a'_{1,2}\) and from the Jacobian of the transformation \((\theta_L, T) \rightarrow (E_1, E_2)\).

We consider that the function \(\omega(E, E_{1,2})\) has the form

\[
\omega(E, E_{1,2}) = \bar{\omega}(2E_{1,2}/E). \tag{2.6}
\]

As will be seen later, this is a reasonable assumption in the composite particle model of hadrons.

The transition probability \(S(T) dT\) is assumed to be proportional to the state number in the energy region \(T \sim T + dT\). The primary interaction in the usual theory is of the Yukawa type or the four-fermion type and produces only one pair of particle-antiparticle at a time. The multiple production occurs by their successive operation. However, the direct multiple production can occur in the theory of the non-linear interaction and there is no reason to specialize in the one pair production. In particular, if there is a strong coupling between the levels, the energetic equilibrium is reached very fast and every possible state appears with equal weight. Therefore, it may be a plausible assumption that the primary interaction produces every possible state equally and, thus, the transition probability is proportional to the allowed state number.

The allowed level number \(\pi(E)\) for a \(\lambda\)-particle which has the energy smaller than \(E\) is given by
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\[ \pi(E) \approx \frac{2}{\theta} \log(E/M_\lambda), \]  

(2.7)

where \( M_\lambda \) represents the rest mass of \( \lambda \)-particle. Experimentally, the production of baryon-antibaryon pair is rather few in high energy hadronic collisions. We refer this to that the \( \lambda \)-particle is always accompanied with the antiparticle \( \bar{\lambda} \) at its level. Since the production of \( \lambda\bar{\lambda} \)-pair is symmetrical in the \( \theta \)-space from the momentum conservation law, each level pairs with the counter one. The possible state number for one of the such pairs is given by

\[ \sum_{r=0}^{N/2} (N/2) \gamma^r = sC_{N/2}, \]  

(2.8)

where \( r \) represents the number of \( \lambda\bar{\lambda} \)-pair at the level. Then, the allowed state number \( \nu(T) \) for the energy smaller than \( T \) is given by

\[ \nu(T) \propto (sC_{N/2})^{\gamma(T)/2} \propto T^\gamma, \]  

(2.9)

where

\[ \gamma = \frac{1}{\theta} \log(sC_{N/2}). \]  

(2.10)

From the relation

\[ S(T') = \frac{d}{dT'} \nu(T'), \]  

(2.11)

and a simple analysis of the dimension, we get

\[ \sigma^{\text{in}}(E) \propto E^{r-4}. \]  

(2.12)

This relation is true except for some special cases.

Experimentally, the energy dependence of \( \sigma^{\text{in}} \) is very weak suggesting \( \gamma \approx 4 \). From formula (2.10) and the experimental value \( \theta \approx 1.7 \), we get

\[ N \approx 12. \]  

(2.13)

The positions of levels in the \( \theta \)-space for \( \gamma = 4, N = 12 \) are given in Table I.

We have counted the antiparticle state and the spin states in the definition of number \( N \). The spin must be a half integer as the \( \lambda \)-particle is a fermion. Therefore, the \( \lambda \)-particle must be of 3-multiplet with spin 1/2 or of singlet with spin 5/2. Thus, the \( \lambda \)-particle can be interpreted as the proton, neutron and \( \Lambda \)-particle in the Sakata model\(^9\) or the three quarks in the quark model.\(^9\) In both models, three

<table>
<thead>
<tr>
<th>( s )</th>
<th>( \theta_s(\text{Th.}) )</th>
<th>( \theta_s(\text{Exp.}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8536</td>
<td>( \leq 0.97 )</td>
</tr>
<tr>
<td>2</td>
<td>2.5608</td>
<td>( \approx 2.6 )</td>
</tr>
<tr>
<td>3</td>
<td>4.2880</td>
<td>( \approx 4.4 )</td>
</tr>
</tbody>
</table>
$\bar{\lambda}\bar{\lambda}$-pairs are produced at a level on an average and may be transformed to three heavy mesons. Then, the average effective mass of $H$-quantum is $\sim 2$ BeV. This is quite consistent with Hasegawa’s analysis. In particular, if we assume a small binding energy for quarks in forming hadrons, the rest mass of quark is about one third of the nucleon mass. Therefore, the effective mass of $H$-quantum must be, on an average, almost twice the nucleon mass.

Let us turn to the problem of the inelasticity. Since the entrances distribute with equal interval in the $\theta$-space, we can take the uniform distribution $\mu(\theta_L) = 1$ in the approximation. Then, we get

$$\frac{d\sigma^{in}}{dy_1dy_2} = C' E^{r-1} e(y_1) e(y_2), \quad (2.14)$$

where

$$e(y) = \bar{\sigma}(1-y) \frac{y^{(r-1)}}{1-y}. \quad (2.15)$$

The notations $y_{1,2}$ represent the inelasticity for the outgoing hadrons $a_1, a_2$, i.e.

$$y_{1,2} = 1 - 2E_{1,2}/E. \quad (2.16)$$

This result means that the distribution of inelasticity is energy independent, which is quite consistent with the experimental evidences. The inelasticity of hadron $a_1(a_2)$ is independent of the situation of the counter hadron $a_1(a_2)$ in the present theory.

As proposed by Kato and Mori, the interaction between hadron and $L$-system may be classified by the number of the colliding fundamental particles inside the hadron as illustrated in Figs. 3a, b, c. Let us consider the case of nucleon-nucleon collision in the quark model. Each quark in the incident nucleon has energy $\approx E/6$ and can transfer the energy less than $E/6$.
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to the $L$-system. Therefore, the function $\overline{\omega}$ may consist of three terms corresponding to the processes Figs. 3(a), (b), (c), i.e.

$$\overline{\omega}(1-y) = a_1 \eta(\frac{1}{2} - y) + a_2 \eta(\frac{3}{2} - y) + a_3,$$

(2.17)

where

$$\eta(x) = \begin{cases} 0 & \text{for } x < 0, \\ 1 & \text{for } x > 0. \end{cases}$$

As illustrated in Fig. 4, the present data are not accurate enough to deduce a definite conclusion but still suggest the validity of formulas (2.14) ~ (2.17).

§ 3. The internal structure of hadrons

If the hadrons have the internal structure, it may take part in the reaction. This indicates that our guiding principle relates to the internal structure of hadrons.

We shall adopt the quark model in this section as an example. Our principle requires that the transverse component can be factorized in the wave function of the quarks in a hadron. This is satisfied when the wave function is of the harmonic oscillator. Since the quarks are in the ground state in the case of nucleon, the wave function $\psi$ in the momentum representation is given by

$$\psi(q_1, q_2, q_3) = e^{-\beta(q_1^2 + q_2^2 + q_3^2)},$$

(3.1)

where

$$q_1 + q_2 + q_3 = 0.$$

The notations $q_{1,2,3}$ represent the momenta of the three quarks.

Such internal structure induces a form factor

$$F(q) = e^{-\beta q^2/2},$$

(3.2)

for the interaction of nucleon. A part of the electromagnetic form factor of nucleon is attributed to the internal structure. From the experiments of Hofstadter et al., we get

$$\beta \leq 9.0/\text{BeV}^2.$$

(3.3)

Since the quarks behave as the harmonic oscillator, the mass level of the excited states of nucleon is given by

$$M = M_N + \frac{J}{2m\beta},$$

(3.4)

where $m$ and $M_N(\approx 3m)$ represent the masses of quark and nucleon, respectively, $J$ the orbital angular momentum. However, this is a non-relativistic expression and may not be applicable to the higher excited states. We shall take the qu-
adric form and neglect the $J^2$-term which may be small in the non-relativistic limit.

$$M^2 = M^2_0 + \frac{3}{\beta} J .$$

(3·5)

This formula may be of a relativistic form and gives a good fit to the empirical mass levels when we take

$$\beta \approx 3.0/\text{BeV}^2 .$$

(3·6)

This is consistent with (3·3).

In the present model, the aleph baryon must be considered as an excited state of nucleon which radiates the soft $\varphi$-meson. We simply assume that the $\varphi$-meson behaves like a photon. Since the excitation occurs mainly in the longitudinal direction and the radiation can be considered as a dipole one, the distribution of the emitted $\varphi$-meson is given by

$$\rho(\theta) \sim \sin^2 \theta ,$$

(3·7)

where the polar angle $\theta$ is measured from the longitudinal axis in the rest system of the aleph baryon. This will be examined through a future experimental analysis.

§ 4. The transverse factor in the matrix element

In this section, we shall discuss the properties of the transverse factor of the matrix element.

We define matrix $R$ by the formula

$$S = 1 - i R ,$$

(4·1)

where $S$ represents the $S$-matrix. From the guiding principle, the matrix $R$ of the hadronic collision consists of the transverse and longitudinal factors $R_t$ and $R_l$.

$$R = R_t R_l .$$

(4·2)

From the definition of the transverse momentum, the incoming hadrons $a_1, 2$ in Fig. 2 did not have the transverse momentum. But, in this section, we remove this restriction in order to discuss the unitary condition for the $S$-matrix, i.e. we consider that the incoming hadrons can have the transverse momenta.

From the unitary condition

$$S^* S = 1 ,$$

(4·3)

we get

$$i (R - R^*) = R^* R .$$

(4·4)

Since the matrix $R_t$ is determined from a dynamics in one-dimensional space in
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the longitudinal direction, it also satisfies the unitary condition

\[ i(R_t - R_t^*) = R_t^+ R_t. \]  

(4·5)

Then, we get

\[ R_t^+ = R_t, \]

\[ R_t = (R_t)^2. \]  

(4·6)

This means that the matrix \( R_t \) is a projection operator.

Let \( k_f(-k_f) \) and \( k_i(-k_i) \) be the transverse momenta of the outgoing hadron \( \alpha'_1(\alpha'_2) \) and the incoming one \( \alpha_1(\alpha_2) \) in Fig. 2, respectively. If there is no transition to the excited hadron state in the transverse direction, the matrix \( R_t \) can be represented by one function \( r(k_f, k_i) \),

\[ R_t = r(k_f, k_i). \]  

(4·7)

The elastic and inelastic differential cross sections \( d\sigma^{el} \) and \( d\sigma^{in} \) are given by

\[ d\sigma^{el} = C^{el} |r(k_f, 0)|^2 d^3k_f, \]

\[ d\sigma^{in} = C^{in} |r(k_f, 0)|^2 d^3k_f, \]  

(4·8)

where \( C^{el} \) and \( C^{in} \) represent constants. Evidently, the function \( |r(k_f, 0)|^2 \) corresponds to the factor \( w(k_f) \) in formula (2·4). This indicates that 1) the distribution of the outgoing hadrons is independent of the excitation of the hadrons concerned and 2) it is also independent of the particle production from the L-system (fire ball). This is consistent with the present experimental analyses though the data are not yet accurate enough for the inelastic processes.

§ 5. Discussion

We have introduced some new ideas in the theory of high energy hadronic collisions such as the L-system and the principle of separability. Experimental evidences for the \( P_T \) constancy and the emission of \( H \)-quanta lead us to impose a guiding principle on the possible theory.

Here we discuss briefly the constant \( s_0 \) in formula (2·2), which represents the position of the levels and the entrances in the \( \theta \)-space. Let us take the barycentric system. There are two plausible cases \( s_0 = 1/2 \) and \( s_0 = 0 \). In the low energy emission of \( H \)-quanta, the momentum distribution of the produced mesons may show a specific pattern. In the former case, two groups of mesons with equal speed are produced in an opposite direction while one of the groups halts in the latter case. In both cases, a periodic fluctuation should be observed in plotting the energy dependence of the inelastic cross section, because the production of \( \lambda \)-pair has a definite threshold energy. However, if the parameter \( s_0 \) is determined statistically, the observation of such a fluctuation may not be easy.
It may be worthwhile to mention that we can geometrize the principle of separability using the theory of the non-symmetric space which was introduced by Einstein. The metric tensor $g_{\mu\nu}$ in the non-symmetric space is written as

$$g_{\mu\nu} = h_{\mu\nu} + k_{\mu\nu}, \quad (5\cdot1)$$

where the first and second terms represent the symmetric and skew symmetric components, respectively. Except for a special case (the third case), the skew symmetric tensor $k_{\mu\nu}$ is expressed in terms of four basic null vectors $u_\mu^i (i=1,2,3,4)$ and two basic scalars $\alpha, \beta$.

$$k_{\mu\nu} = 2\alpha u_{\nu}^i u_{\nu}^i + 2\beta u_{\nu}^i u_{\nu}^i, \quad (5\cdot2)$$

where

$$h^{\mu\nu} u_{\mu}^i u_{\nu}^i = 1, \quad h^{\mu\nu} u_{\mu}^i u_{\nu}^i = 1. \quad (5\cdot3)$$

The vectors $u_\mu^1, u_\mu^2$ are perpendicular to $u_\mu^3, u_\mu^4$ and span a two-dimensional Minkowski space $M_2$, while $u_\mu^3, u_\mu^4$ span a two-dimensional Euclid space $E_2$. The spaces $M_2$ and $E_2$ have the meaning as the planes in the tangential space.

We take the following principle. The world lines of the incoming two hadrons span the plane $M_2$ as illustrated in Fig. 5. The skew symmetric tensor $k_{\mu\nu}$ should be considered as a field which is determined from the matter distribution in the initial state in space-time. Evidently, the field theory in the two-dimensional Minkowski space $M_2$ corresponds to the one-dimensional dynamics in the longitudinal direction in the principle of separability. The transverse factor of the matrix element is determined by the theory for the two-dimensional Euclid space $E_2$, as was discussed briefly in § 4.

The geometrization will be discussed in a forthcoming article.

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