On the basis of the field-current identity and the universal current-current interaction a divergence-free model of parity-violating nonleptonic decays is constructed. Octet enhancement is a natural consequence of this model and the decay amplitudes are evaluated without introducing any arbitrary parameter. The theoretical results show an excellent agreement with the corresponding experimental ones. These results render a support for the current-current picture of nonleptonic decays.

§1. Introduction

The universal current-current interaction successfully accounts for both leptonic and semileptonic decays. It has not yet been settled, however, whether or not nonleptonic decays of hadrons are also described by the current-current interaction due to various difficulties: (1) The strong final-state interaction is supposed to modify the primary interaction so strongly that it becomes difficult to trace back to the original Hamiltonian. (2) When we try to evaluate the nonleptonic decay amplitudes starting from the current-current interaction we encounter divergences arising from closed loops. For these reasons we have not yet a reliable interpretation of nonleptonic decays based on the current-current interaction.

In order to dissolve the divergence difficulty mentioned above it is instructive to recall the derivation of the Goldberger-Treiman relation. In their original derivation Goldberger and Treiman\(^1\) assumed that the charged pion decays into a lepton pair through a nucleon-antinucleon pair. This generally introduces a divergent closed-loop contribution into the decay amplitude, but a close examination shows that we also obtain an equally divergent denominator representing the wave function renormalization of the pion field; as a result we find a finite ratio. In this connection we should emphasize the fact that an equivalent result can be obtained by the so-called PCAC hypothesis\(^2\) without going through such a cumbersome procedure. The essence of the PCAC hypothesis consists in replacing the divergence of an axial vector current by an interpolating field. In this way we can derive the Goldberger-Treiman relation without encountering divergent integral in the intermediate steps.

The success of the PCAC hypothesis in semileptonic decays suggests that a similar replacement of a current by an interpolating field be useful in nonleptonic decays.
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decays in eliminating divergences resulting from closed loops. Thus we shall employ the field-current identity\(^{2}\) in this paper. An explicit definition of this identity will be given in the next section.

As has been pointed out by Lee, Weinberg and Zumino\(^{3}\) the field-current identity is satisfied in the Yang-Mills type vector meson theories, so that we shall employ, as a model of strong interactions, chiral dynamics with gauge fields. This choice makes it also possible to overcome the first difficulty related to the strong final state interaction, since chiral dynamics\(^{4}\) is considered to be a phenomenological theory of strong interactions in which perturbation theory is valid at low energies.

Having chosen a model for strong interactions, we shall then fix the primary weak interaction responsible for nonleptonic decays. Its most popular form is given by

\[ \mathcal{L}_{NL} = \frac{G}{\sqrt{2}} J_\mu J^\mu, \quad (1.1) \]

where \( G \) is the Fermi coupling constant and \( J_\mu \) represents the hadronic Cabibbo current. The interaction responsible for nonleptonic decays is given by that part of (1.1) changing strangeness by one unit. This part, however, is known to consist of an octet part and a 27-plet part. The latter violates the empirical selection rule \( |\Delta I| = 1/2 \).

Now that all the ingredients of our theory have been prepared we shall discuss qualitative features of nonleptonic decays in our scheme. When we take the field-current identity for granted, the current-current interaction is bilinear in the vector and axial vector fields; we can regard them as non-diagonal mass terms. In particular, the octet part of the current-current interaction can be renormalized into the \( SU(3) \)-breaking mass terms for the gauge fields by means of a chiral transformation being a member of the \( SU(3)_L \) group. As we shall clarify in §2 this observation leads to the desired octet enhancement mechanism. The field-current identity suggests the importance of the vector-meson-dominance\(^{5}\) since the effects of weak interactions are expected to occur only through the vector meson-channel. From phenomenological analyses of the \( S \)-wave decays of hyperons we have learned that the vector-meson-dominance model\(^{6}\) leads to experimentally favorable relative decay amplitudes.\(^{7}\) Thus the above-mentioned characteristics of our scheme are very encouraging, and we have decided to discuss the problem of the \( S \)-wave decays in this paper.

In §3 we shall discuss the mass renormalization that causes a mixing between vector and axial vector fields and leads to octet enhancement. In addition to the mixing due to weak interactions we have to study the mixing between axial vector and pseudoscalar fields characteristic of chiral dynamics. Combination of these two renormalization procedures leads to an effective weak interaction coupling of the pion fields with the \( K^* \)-meson fields.

Finally in §4 we evaluate the \( S \)-wave decay amplitudes of hyperons on the
basis of the effective interaction obtained in the preceding section. The final expressions for them depend only on the Fermi coupling constant \( G \), the pion decay constant \( F_\pi \), and the rest masses of hadrons. The agreement between theory and experiment is unexpectedly good. Similar results are obtained for the \( K_\pi^0 \to 2\pi \) decays and we find a reasonable agreement. Finally we have estimated the \( K^+ \) decay amplitude from the 27-plet part of the Fermi interaction. The theoretical amplitude is about three times as large as the experimental one suggesting the possible existence of a suppression mechanism for the 27-plet contributions.

§2. Octet enhancement in the current-current theory

In this section we shall clarify an octet enhancement mechanism in the current-current theory on the basis of the field-current identity, and we shall also show the equivalence of the present approach to the quark density model\(^\text{10}\) provided that the chiral symmetry is broken in a specified manner.

First we postulate that the weak interaction Lagrangian is of the universal current-current type:

\[
\mathcal{L} = \frac{G}{\sqrt{2}} J_a^\alpha J_\alpha^a,
\]

where \( G \) is the Fermi coupling constant and \( J_\alpha^a \) represents the Cabibbo current. Since we are interested only in nonleptonic decays we shall let \( J_\alpha^a \) represent the hadronic Cabibbo current in what follows. The interaction responsible for nonleptonic decays is that part of (2.1) obeying the selection rule \( |\Delta S| = 1 \), and this part consists of an octet part and a 27-plet part:

\[
\mathcal{L}_{|\Delta S|=1} = \mathcal{L}^{(8)} + \mathcal{L}^{(27)}_{1/2} + \mathcal{L}^{(27)}_{\bar{3}/2}.
\]

The octet part \( \mathcal{L}^{(8)} \) is given explicitly by

\[
\mathcal{L}^{(8)} = \frac{3}{5} \frac{G}{\sqrt{2}} \sin \theta \cos \theta \left[ (J_\mu^4 - iJ_\mu^5)(J_\mu^1 + iJ_\mu^2) + (J_\mu^1 - iJ_\mu^5)(J_\mu^4 + iJ_\mu^5) \right.

- 2J_\mu^6 \left\{ J_\mu^3 + \frac{1}{3} J_\mu^8 \right\} \right]
\]

\[
= \frac{3}{5} \frac{G}{\sqrt{2}} \sin \theta \cos \theta \sum_{\alpha,\beta=1}^8 d_{\alpha\beta} J_\alpha^a J_\beta^a.
\]

The 27-plet part is further divided into two parts; one transforming as an isospin doublet and the other as an isospin quadruplet:

\[
\mathcal{L}^{(27)}_{1/2} = \frac{1}{15} \frac{G}{\sqrt{2}} \sin \theta \cos \theta \left[ (J_\mu^4 - iJ_\mu^5)(J_\mu^1 + iJ_\mu^2) + (J_\mu^1 - iJ_\mu^5)(J_\mu^4 + iJ_\mu^5) \right.

- 2J_\mu^6 \left\{ J_\mu^3 - 3\sqrt{3} J_\mu^8 \right\} \right],
\]

\[
\mathcal{L}^{(27)}_{\bar{3}/2} = \frac{1}{15} \frac{G}{\sqrt{2}} \sin \theta \cos \theta \left[ (J_\mu^4 - iJ_\mu^5)(J_\mu^1 + iJ_\mu^2) + (J_\mu^1 - iJ_\mu^5)(J_\mu^4 + iJ_\mu^5) \right.

- 2J_\mu^6 \left\{ J_\mu^3 + 3\sqrt{3} J_\mu^8 \right\} \right].
\]
The angle $\theta$ denotes the Cabibbo angle and each $J_{\mu}^a$ consists of the vector and axial vector parts:

$$J_{\mu}^a = V_{\mu}^a + A_{\mu}^a.$$  \hfill (2.7)

These two parts are related to the vector and axial vector fields, respectively, through the field-current identity.

As a model of the field-current identity we shall consider a chiral-invariant Lagrangian with massive Yang-Mills gauge fields. A simple Lagrangian invariant under the chiral $SU(3) \times SU(3)$ group may be written as

$$\mathcal{L}_0 = -\frac{1}{4} \text{Tr} \left[ (\partial_{\mu} \mathcal{V}_\nu^a)^T \mathcal{V}_\nu^a + (\partial_{\mu} \mathcal{A}_\nu^a)^T \mathcal{A}_\nu^a \right]$$  \hfill (2.8)

and

$$\mathcal{V}_\mu^a = \frac{1}{\sqrt{2}} (\mathcal{V}_\mu^0 \pm \mathcal{A}_\mu^0)$$  \hfill (2.9)

where $\mathcal{V}_\mu^a$ and $\mathcal{A}_\mu^a$ represent the octet vector and axial vector fields, respectively. The matrices $\lambda_\alpha$ denote Gell-Mann's $3 \times 3$ unitary spin matrices. The Lagrangian $\mathcal{L}_m$ is the matter field Lagrangian that is invariant under chiral transformations, and we shall specify its form in the next section since we shall not need it in the present section.

Since the mass terms of the gauge fields are the only terms in $\mathcal{L}_0$ that violate the local gauge invariance, the currents associated with $\mathcal{L}_0$ are given by

$$V_{\mu}^a = \frac{m_0^a}{g} \mathcal{V}_\mu^a$$  \hfill (2.10)

and

$$A_{\mu}^a = \frac{m_0^a}{g} \mathcal{A}_{\mu}^a.$$  \hfill (2.11)
Thus the Lagrangian (2.8) provides a model of the field-current identity. On the basis of the field-current identity (2.12) and (2.13) the current-current interaction reduces to a bilinear form in the gauge fields, namely, the octet part of the Fermi interaction $\mathcal{L}^{(\alpha)}$ may be written as

$$\mathcal{L}^{(\alpha)} = \frac{3}{\sqrt{2}} \frac{G}{3} \sin \theta \cos \theta \left( \frac{m_\alpha}{g} \right)^2 \text{Tr}(\lambda_\alpha \{ \mathcal{V}_A, \mathcal{V}_A \}) \hspace{1cm} (2.14)$$

and we have similar expressions for $\mathcal{L}^{(\gamma)}$.

Next we shall clarify the octet enhancement mechanism on the basis of the field-current identity. For this purpose it is necessary to break the $SU(3) \times SU(3)$ symmetry and we shall write the total Hamiltonian for strong interaction in the following form:

$$H = H_0 + H_a + H_s \hspace{1cm} (2.15)$$

where $H_0$ denotes the invariant part of the Hamiltonian corresponding to $\mathcal{L}_0$, $H_a$ is the part responsible for the mass splitting among the members of the octet vector mesons, and $H_s$ denotes the other symmetry-breaking term. We shall write $H_s$ as

$$H_s = -\frac{1}{2\sqrt{3}} \Delta m^2 \left[ \text{Tr}(\lambda_6 \{ \mathcal{V}_A^L, \mathcal{V}_A^R \}) + \text{Tr}(\lambda_8 \{ \mathcal{V}_A^L, \mathcal{V}_A^R \}) \right] \hspace{1cm} (2.16)$$

where

$$\Delta m^2 = m_\alpha^2 - m_\beta^2.$$ 

Because of the presence of the symmetry-breaking terms the currents are generally not conserved, and the divergence of a current is expressed in terms of an equal-time commutator:

$$\partial_\mu J_{\mu}^{aL,R} = -i [F_a^{L,R}, H] \hspace{1cm} (2.17)$$

where $F_a^{L,R}$ denotes the $a$-th charge of the group $SU(3)_{L,R}$, and is given by

$$F_a^{L,R} = -\frac{i}{2} \int \left( V_A^a \pm A_4^a \right) dx = \frac{1}{2} (F_a \pm F_a^*) \hspace{1cm} (2.18)$$

In particular for the seventh component we have

$$\partial_\mu J_{\mu}^{7L} = -i [F_7^L, H] = -i [F_7^L, H_s] - i [F_7^L, H_0] = -\frac{1}{4} \Delta m^2 \text{Tr}(\lambda_6 \{ \mathcal{V}_A^L, \mathcal{V}_A^L \}) - i [F_7^L, H_s] + O((\Delta m^2)^2). \hspace{1cm} (2.19)$$

It should be emphasized here that once the symmetry breaking term $H_s$ is introduced the relations (2.12) and (2.13) become approximate because of the mass splitting among the members of the vector meson octet, and this is the reason for the third term on the right-hand side of Eq. (2.19). In what follows,
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However, we shall neglect this term. Then we have

$$\langle f | \text{Tr}(\lambda_6 \{ C V_{\mu}^L, C V_{\mu}^R \} ) | i \rangle \approx -\frac{A_f}{\Delta m^2} \langle f | [ F^L, H_s ] | i \rangle ,$$  \hspace{1cm} (2.20)

provided $P_f = P_i$, since the matrix element of a divergence between two states of equal energy-momentum vanishes identically. The left-hand side of Eq. (2.20) is then proportional to the octet Lagrangian $L^{(8)}$ in (2.14), and we immediately obtain

$$\langle f | L^{(8)} | i \rangle \approx -\frac{6}{5} G \sin \theta \cos \theta \left( \frac{m_{\alpha}^2}{g} \right)^2 \frac{1}{\Delta m^2} \langle f | [ F^L, H_s ] | i \rangle .$$  \hspace{1cm} (2.21)

The mass splitting for the vector meson octet is relatively small compared with those for other multiplets and the appearance of $\Delta m^2$ in the denominator provides an octet enhancement mechanism.

For the 27-plet, however, we cannot find a similar argument so that the contributions from the 27-plet are not enhanced, and this explains the origin of the phenomenological selection rule $| \Delta I | = 1/2$ in nonleptonic decays. A detailed argument on this point is given in §4.

The presence of the mass splitting term $H_s$ modifies the field-current identity (2.12) and (2.13) as

$$V_{\mu}^a = \frac{m_{\alpha}^2}{g} C^{\alpha a} \rho_{\mu}^a$$  \hspace{1cm} (2.22)

and

$$A_{\mu}^a = \frac{m_{\alpha}^2}{g} C_{\mu}^{\alpha a} ,$$  \hspace{1cm} (2.23)

where $m_{\alpha}$ denotes the observed mass of the $\alpha$-th vector meson, and the octet part of the weak interaction $L^{(8)}$ is given by

$$L^{(8)} = \frac{3}{5} \sqrt{2} G \sin \theta \cos \theta \sum_{\alpha, \beta = 1}^{8} d_{\alpha \beta} \left( \frac{m_{\alpha}^2}{g} \right) \left( \frac{m_{\beta}^2}{g} \right) C_{\mu}^{\alpha a} C_{\mu}^{\beta a} .$$  \hspace{1cm} (2.24)

In §4 we shall use this form rather than the commutator form (2.21) since the decomposition of the total Hamiltonian into the symmetric part and symmetry-breaking part is not necessarily practical. The commutator form, however, is convenient to exhibit octet enhancement as well as to show the equivalence of the present model to the quark density model.

We shall assume, as done by Gell-Mann, Oakes and Renner, that the symmetry-breaking Hamiltonian $H_s$ transforms according to the representation $(3, 3^*) + (3^*, 3)$ under the chiral SU(3) $\times$ SU(3) group, and put

$$H_s = \epsilon_{\alpha \beta} m_{\alpha} + \epsilon_{\beta \alpha} m_{\beta} ,$$  \hspace{1cm} (2.25)

where the $u$'s denote scalar densities introduced by Gell-Mann. By substituting Eq. (2.25) into Eq. (2.24) and using the commutation relations for the $u$'s we
\[ \langle f \mid \mathcal{L}^{(2)} \mid i \rangle = \frac{6\sqrt{2}}{5} G \left( \frac{m_{\pi}^2}{g} \right)^2 \frac{\sin \theta \cos \theta}{m_{\pi}^2 - m_{\pi}^2} \langle f \mid c_1 v_i + c_2 u_i \mid i \rangle, \tag{2.26} \]

where

\[ c_1 = \frac{1}{2\sqrt{3}} e_8 - \sqrt{\frac{2}{3}} e_8, \quad c_2 = \frac{\sqrt{3}}{2} e_8. \]

This equation shows that the octet part of the current-current interaction is approximately equal to that given by the quark density model provided that the field-current identity is applied.

§3. The renormalization problem

In the preceding section we have presented a possible mechanism for octet enhancement in a formal manner, but in order to exhibit this mechanism explicitly we have to solve some related problems.

Equation (2.19) shows that the octet part of the current-current interaction can be completely renormalized into \( H_4 \) by means of a rotation in the chiral space if higher order corrections in \( H_4 \) are discarded. Therefore, we shall study this transformation in detail and carry out the necessary renormalization procedure in this section.

We shall start from the Lagrangian (2.8) and assume that \( \mathcal{L}_m \) is given explicitly by

\[ \mathcal{L}_m = -\frac{1}{8f_{\pi}^2} \text{Tr}(D^a M D_a M^*), \quad + \mathcal{L}_s(\psi, D_\mu \psi) + \mathcal{L}_{\text{int}}(\psi, M, C V_\mu, \ldots), \tag{3.1} \]

where \( D^a M \) denotes the covariant derivative:

\[ D^a M = \partial^a M - i\frac{\theta}{\sqrt{2}} [\lambda^a, M], \tag{3.2} \]

\[ \partial^a M = \partial_a M - i\frac{\theta}{\sqrt{2}} [C V^a, M]. \tag{3.3} \]

The meson field \( M \) is \( 3 \times 3 \) matrix field defined by

\[ M = \frac{1 + i f_{\psi}}{1 - i f_{\psi}}, \quad \psi = \frac{1}{\sqrt{2} f_{\psi}} \sum_{a=1}^{8} \lambda_a \varphi^a, \tag{3.4} \]

where \( \varphi^a \) denotes the field operator of the \( a \)-th member of the pseudoscalar octet. The baryon part \( \mathcal{L}_b \) and the interaction part \( \mathcal{L}_{\text{int}} \) are irrelevant for the arguments in this section. As for the weak interaction we shall retain only the parity-violating part responsible for the \( S \) wave decays of hyperons, since the \( S \) wave problem is much easier than the \( P \) wave problem.
Thus the total Lagrangian including the mass splitting term for the vector meson octet reads

\[ \mathcal{L}_{\text{meson}} = -\frac{1}{8f_0^2} \text{Tr}(D_\mu^a M D_\mu^a M^a) \]

\[ -\frac{1}{4} \text{Tr}(C V_{\mu a} C V_{\mu b} + C V_{\mu b} C V_{\mu a}) \]

\[ -\frac{1}{2} \sum_{a=1}^{8} (m_{VA})^2 C V_{\mu a} C V_{\mu a} - \frac{1}{2} \sum_{a=1}^{8} (m_{AA})^2 \bar{A}_\mu^a A_\mu^a \]

\[ + \frac{6v_2}{5} G \sin\theta \cos\theta \sum_{a,b=1}^{8} d_{ab} \frac{(m_{VA})^2}{g} \frac{(m_{AA})^2}{g} C V_{\mu a} A_\mu^a \]

(3·5)

In this equation we have introduced the \( SU(3) \)-breaking effects by distinguishing the masses of the gauge fields. It should be mentioned, however, that until now the masses of the vector and axial vector mesons are degenerate, that is, \( m_V = m_A \), but this degeneracy will be lifted by means of mass renormalization in the last stage of this program. This is the reason why we have introduced the labels \( V \) and \( A \) for the masses in Eq. (3·5).

The last three terms are bilinear in the gauge fields and are interpreted as the mass terms. We shall diagonalize them by means of a unitary transformation belonging to the \( SU(3) \times SU(3) \) group. Since the last term violates parity this transformation mixes \( C V_\mu^a \) and \( A_\mu^a \), and this transformation is given, to first order in \( G \), by

\[ C V_{\mu a} = C V_{\mu a} \frac{6v_2}{5} G \sin\theta \cos\theta \sum_{b=1}^{8} d_{ab} \frac{1}{m_{va}^2 - m_{VB}^2} \frac{(m_{VA})^2}{g} \frac{(m_{AA})^2}{g} A_\mu^b \]

(3·6)

\[ A_\mu^a = A_\mu^a \frac{6v_2}{5} G \sin\theta \cos\theta \sum_{b=1}^{8} d_{ab} \frac{1}{m_{VA}^2 - m_{Vb}^2} \frac{(m_{VA})^2}{g} \frac{(m_{AA})^2}{g} C V_{\mu b} \]

(3·7)

One can easily verify that this infinitesimal transformation is generated by \( F_7^a \) provided that the masses of the vector mesons satisfy the Gell-Mann-Okubo mass formula.

By this transformation the mass terms are diagonalized and the weak interaction disappears on the surface. The effects of the weak interaction, however, still remain in the form of mixing. By expressing the first term in Eq. (3·5) in terms of the new fields \( C V_\mu^a \) and \( A_\mu^a \), we find

\[ -\frac{1}{8f_0^2} \text{Tr}(D_\mu^a M D_\mu^a M^a) \]

\[ = -\frac{1}{8f_0^2} \text{Tr}(D_\mu^a M D_\mu^a M^a) \]

\[ + \frac{g}{\sqrt{2} f_0^2} \frac{6v_2}{5} G \sin\theta \cos\theta \sum_{a,b=1}^{8} d_{ab} \frac{1}{m_{va}^2 - m_{VB}^2} \frac{(m_{VA})^2}{g} \frac{(m_{AA})^2}{g} C V_{\mu a} A_\mu^b \]

\[ + \cdots \]

(3·8)
where $D_n M$ is obtained from $D^a_n M$ by replacing the original gauge fields by the transformed ones in Eqs. (3·2) and (3·3). In deriving Eq. (3·8) we have expanded $M$ in powers of the meson field $\phi$ and have kept only the lowest order. We have also neglected terms of higher order in $G$.

Thus the total meson Lagrangian is given, in terms of the new gauge fields, by

$$\mathcal{L}_{\text{meson}} = -\frac{1}{8f_0^2} \text{Tr}(D_n M D_n M')$$

$$- \frac{1}{4} \text{Tr}(CV^L_{\alpha\beta}CV^L_{\alpha\beta} + CV^R_{\alpha\beta}CV^R_{\alpha\beta})$$

$$- \frac{1}{2} \sum_{a=1}^8 (m_{\alpha a})^2 \partial C_{\alpha a} \partial C_{\alpha a} - \frac{1}{2} \sum_{a=1}^8 (m_{\alpha a})^2 \partial a \partial a$$

$$+ \ldots + \mathcal{L}_{\text{PV}}^{(8)} .$$

(3·9)

It should be mentioned that the mass shift by the transformation (3·6) and (3·7) is of the second order in $G$ and has been neglected here. $\mathcal{L}_{\text{PV}}^{(8)}$ denotes the parity-violating part of $\mathcal{L}_{\text{PV}}$ and its transformed form is given by

$$\mathcal{L}_{\text{PV}}^{(8)} = \frac{6}{5} \frac{Gg}{f_0^2} \sin \theta \cos \theta \sum_{a,b=1}^8 \frac{1}{m_{a b}^2} \left( \frac{m_{\alpha a}^2}{g} \right) \partial a \partial b a$$

(3·10)

So far we have considered only the mass renormalization implied by the weak interaction, but the strong interaction Lagrangian $\mathcal{L}_m$ by itself needs renormalization as shown below. The Lagrangian $\mathcal{L}_m$ leads to a mixing between the axial vector and pseudoscalar fields as seen from

$$-\frac{1}{8f_0^2} \text{Tr}(D_n M D_n M') = -\frac{1}{2} \text{Tr} \left( \partial_{\alpha \beta} - \frac{g}{\sqrt{2} f_0} \mathcal{A}_{\alpha} \right)^2 + \ldots .$$

(3·11)

Elimination of the cross terms is effected by the following transformation:

$$\mathcal{A}_{\alpha} = \frac{g}{\sqrt{2} f_0 m_{\alpha}} \left( \frac{m_{\alpha a}^2}{g} \right) \partial a \partial \phi,$$

(3·12)

where

$$Z_\alpha = 1 + \frac{\theta^2}{2f_0^2 m_{\alpha}^2} .$$

(3·13)

After this transformation the coefficients of the kinetic energy part of the pseudoscalar fields are altered and we must introduce the renormalized pseudoscalar fields $\bar{\phi}$ so as to restore the correct coefficient. We are then led to

$$\bar{\phi} = Z_\alpha^{-1/2} \phi .$$

(3·14)

The new masses of the axial vector mesons are given by
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\[ m_{A_k}^2 = Z_\alpha m_\alpha^2 = Z_\alpha m_\nu^2 \]  
(3.15)

Within the framework of the chiral \( SU(2) \times SU(2) \) group, we have the KSFR relation \(^{12}\)

\[ g^2 = 2m_\rho^2 f_0^2 \]  
(3.16)

and Weinberg's mass relation \(^{13}\)

\[ m_{A_1}^2 = 2m_\rho^2 \]  
(3.17)

provided that we choose \( Z_\alpha = 2 \) for \( \alpha = 1, 2, 3 \).

Thus we have completed all the diagonalizations or renormalizations of the meson fields that are needed for the present purpose. The Lagrangian \( \mathcal{L}_{PV}^{(0)} \) is expressed in terms of renormalized quantities by

\[ \mathcal{L}_{PV}^{(0)} = \frac{6}{5} \frac{Gg}{f_0} \sin \theta \cos \theta \times \sum_{\alpha, \beta = 1}^{3} d_{\alpha \beta} \frac{1}{m_{\rho \beta}^2 - m_{\rho \alpha}^2} \left( \frac{m_{A_\alpha}^2}{g} \right) \left( \frac{m_{A_\beta}^2}{g} \right) \mathcal{C} \frac{\varphi^{\alpha}}{\varphi^{\beta}} \]  
(3.18)

where

\[ g_{\alpha \beta} = \left( \frac{m_{A_\alpha}}{m_{A_\beta}} \right)^2 g = \left( \frac{m_{A_\alpha}}{m_{A_\nu}} \right)^2 g = Z_\alpha g \]  
(3.19)

The renormalized Lagrangian (3.18) has the same enhancement factor as that in Eq. (2.21) and provides the effective interaction to evaluate the parity-violating nonleptonic decay amplitudes.

§4. Parity-violating nonleptonic decays of hadrons

On the basis of the Lagrangian (3.18) we shall evaluate the octet part of the parity-violating nonleptonic decay amplitudes of hadrons. From the Lagrangian (3.18) we shall pick out those terms which are responsible for pion emission and find its effective form also changing the overall sign for convenience.

\[ \mathcal{L}_{\alpha \mu \nu}^{(0)} = \frac{3}{5} \sqrt{2} G \sin \theta \cos \theta \frac{m_{K^*}}{g} \frac{m_{A_1}}{m_{K^*} - m_\rho} F_{\pi} \times \left[ K_\mu^* \partial_{\rho} \partial_{\mu} + K_{1*}^* \partial_{\rho} \partial_{1*} - \frac{1}{\sqrt{2}} K_{1}^* \partial_{\rho} \partial_{1} - \frac{1}{\sqrt{2}} K_{1}^* \partial_{\rho} \partial_{1} \right] , \]  
(4.1)

where we have already used the KSFR relation and the following relation:

\[ \frac{m_{A_1}}{g_\rho} = \frac{1}{2f_0} = F_{\pi} \]  
(4.2)

This relation follows from the KSFR relation and Weinberg's first sum rule: \(^{13}\)

\[ \frac{m_{A_1}^2}{g_\rho^2} + \frac{1}{4f_0^2} = \frac{m_{\rho}^2}{g^2} \]  
(4.3)
F_\pi denotes the pion decay constant, numerically equal to 94 MeV.

The original Lagrangian (2·2) involves also the 27-plet part. As was discussed in §2, we find no enhancement mechanism. Furthermore, \( \mathcal{L}_{\pi + \frac{1}{2}}^{(27)} \) has a coefficient which is smaller than that of \( \mathcal{L}_{\pi + \frac{1}{2}}^{(27)} \) by a factor of 5, so that we shall discard \( \mathcal{L}_{\pi + \frac{1}{2}}^{(27)} \) in what follows. In the absence of an enhancement mechanism we do not know exactly how to evaluate the contribution from \( \mathcal{L}_{\pi + \frac{1}{2}}^{(27)} \) so that we shall simply take its direct matrix elements. Then the effective form of its parity-violating part is given by

\[
\mathcal{L}_{\text{eff}}^{(27)} = \frac{1}{3} \sqrt{2} G \sin \theta \cos \theta \frac{m_{\pi^*}^2}{g} F_\pi^2
\]

\[
\times \left[ K_{\pi^*}^* \partial_{\pi^*} + K_\pi^* \partial_\pi + \sqrt{2} K_{\pi^*} \partial_{\pi} + \sqrt{2} K_{\pi^*} \partial_\pi^* \right],
\]

where we have used the field-current identity and the relation (3·12).

First we shall evaluate the contributions from \( \mathcal{L}_{\text{eff}}^{(27)} \) and then estimate those from \( \mathcal{L}_{\text{eff}}^{(27)} \).

a) hyperon decays

We first evaluate the amplitudes for the nonleptonic decays of hyperons \( B_i \rightarrow B_f + \pi \). The decay amplitudes are defined by

\[
\langle B_i, \pi | S | B_f \rangle = i(2\pi)^4 \delta^4(p_i - p_f - q) \langle B_i, \pi | \mathcal{L}_{\text{eff}}(0) | B_f \rangle
\]

\[
= i(2\pi)^4 \delta^4(p_i - p_f - q) \frac{1}{\sqrt{2} q} \bar{u}(p_f) u(p_i) A(B_i \rightarrow B_f + \pi),
\]

where \( u \) is normalized by \( u'u = 1 \). Combination of the weak interaction (4·1) with the universal interaction between the baryon and vector meson fields gives rise to the hyperon decay illustrated in Fig. 1. The latter interaction is determined on the basis of gauge invariance, so that the coupling is of the \( F \) type and the coupling constant is given by \( g \).

Then we can readily evaluate the decay amplitudes \( A \), and for the octet interaction (4·1) we have

\[
A(B_i \rightarrow B_f + \pi) \bar{u}(p_f) u(p_i)
\]

\[
= g h C(B_i \rightarrow B_f + \pi) \bar{u}(p_f) \gamma_5 u(p_i) \left( \partial_{\pi^*} + \frac{q_\pi q_\pi}{m_{\pi^*}} \right) \frac{q_\pi}{m_{\pi^*} - m_\pi}
\]

\[
= M_i - M_f h C(B_i \rightarrow B_f + \pi) \bar{u}(p_f) u(p_i),
\]

where

\[
h = \frac{3\sqrt{2}}{5} GF_\pi \sin \theta \cos \theta \frac{m_{\pi^*}^2}{g} \frac{m_{\pi}^2}{m_{\pi^*}^2 - m_\pi^2}.
\]
The constant \(C(B_i \rightarrow B_f + \pi)\) denotes the product of two Clebsch-Gordan coefficients, one associated with the strong vertex and the other with the weak one, and has been tabulated in Table I.

Thus the decay amplitude reads

\[
A(B_i \rightarrow B_f + \pi) = \frac{3}{5} \sqrt{2} G_F \sin \theta \cos \theta \frac{m_{A_1}}{m_{K^*} - m_{\pi}} \times (M_i - M_f) C(B_i \rightarrow B_f + \pi). \tag{4.8}
\]

It is interesting to see that the above expression is independent of the universal coupling constant \(g\). Furthermore, the above formula involves only known fundamental parameters such as the Fermi coupling constant \(G\), the pion decay constant \(F_\pi\), and the rest masses of known hadrons. It is also worth while to emphasize that the amplitude (4.8) leads to the Lee-Sugawara sum rule

\[
A(A^0 \rightarrow p + \pi^-) - \sqrt{3} A(S^+ \rightarrow p + \pi^0) = 2A(B^- \rightarrow A^0 + \pi^-), \tag{4.9}
\]

provided that the Gell-Mann-Okubo mass formula is satisfied for the baryon octet. The numerical values of the decay amplitudes \(A\) are tabulated in Table I along with the experimental ones. The agreement between theory and experiment is excellent.

<table>
<thead>
<tr>
<th>(C(B_i \rightarrow B_f + \pi))</th>
<th>(A(B_i \rightarrow B_f + \pi)_{\text{exp}} \times 10^7)</th>
<th>(A(B_i \rightarrow B_f + \pi)_{\text{theor}} \times 10^7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A^0 \rightarrow p + \pi^-)</td>
<td>(\sqrt{3})</td>
<td>(3.37 \pm 0.05)</td>
</tr>
<tr>
<td>(S^- \rightarrow A^0 + \pi^-)</td>
<td>(\sqrt{3})</td>
<td>(4.39 \pm 0.06)</td>
</tr>
<tr>
<td>(S^+ \rightarrow p + \pi^0)</td>
<td>(\sqrt{2})</td>
<td>(4.04 \pm 0.04)</td>
</tr>
<tr>
<td>(S^- \rightarrow n + \pi^0)</td>
<td>(0)</td>
<td>(0.02 \pm 0.07)</td>
</tr>
<tr>
<td>(S^+ \rightarrow p + \pi^0)</td>
<td>(-1)</td>
<td>(-3.38 \pm 0.31)</td>
</tr>
</tbody>
</table>

b) \(K \rightarrow 2\pi\) decays

Next we consider the \(K \rightarrow 2\pi\) decays. These processes are realized as con-
sequences of the combination of the weak interaction (4·1) and the universal interaction between the pseudoscalar and vector meson fields and are illustrated in Fig. 2. The decay amplitudes, defined by a formula similar to Eq. (4·5), are given by

\[ A(K_s^0 \to \pi^+ + \pi^-) = \frac{6}{5} \sqrt{2} G F \sin \theta \cos \theta \frac{m_{\pi}^2}{m_{K^*}^2 - m_{\pi}^2} (m_{\pi}^2 - m_{\pi^*}^2) \] (4·10)

and

\[ A(K_s^0 \to 2\pi^0) = A(K_s^0 \to \pi^+ + \pi^-). \] (4·11)

Because of the Bose statistics for the neutral pions the latter relation leads to

\[ \Gamma(K_s^0 \to \pi^+ + \pi^-) = 2\Gamma(K_s^0 \to 2\pi^0), \] (4·12)

so that it is more convenient for comparison between theory and experiment to introduce \( A'(K_s^0 \to 2\pi^0) \) defined by

\[ A(K_s^0 \to 2\pi^0) = \sqrt{2} A'(K_s^0 \to 2\pi^0). \] (4·13)

These theoretical values as well as the corresponding experimental ones \(^{16}\) are tabulated in Table II. This time the agreement is reasonable.

<table>
<thead>
<tr>
<th>( K_s^0 \to \pi^+ + \pi^- )</th>
<th>( A_{\text{exp}} \times 10^4 )</th>
<th>( A_{\text{theor}} \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_s^0 \to 2\pi^0 )</td>
<td>3.98±0.12</td>
<td>4.8</td>
</tr>
<tr>
<td>( K_s^0 \to 2\pi^0 )</td>
<td>2.70±0.12</td>
<td>3.4</td>
</tr>
</tbody>
</table>

For \( K_s^0 \to 2\pi^0 \), \( A \) actually denotes \( A' \) defined by (4·13).

The amplitudes are given in units of Mev.

Finally we shall investigate the \( K^+ \to \pi^+ + \pi^0 \) decay. This process violates the selection rule \( |A| = 1/2 \) so that it can take place only through the interaction (4·4). By taking directly the matrix element of (4·4) we obtain

\[ A(K^+ \to \pi^+ + \pi^0) = \sqrt{2} G F \sin \theta \cos \theta (m_{K^*}^2 - m_{\pi}^2), \] (4·14)

where we have neglected the \( \pi^+ - \pi^0 \) mass difference. The ratio of (4·14) to (4·10) amounts to

\[ A(K^+ \to \pi^+ + \pi^0) / A(K_s^0 \to \pi^+ + \pi^-) = \frac{5}{6} \frac{m_{K^*}^2 - m_{\pi}^2}{m_{\pi}^2} \approx \frac{1}{7}, \] (4·15)

which should be compared with the corresponding experimental value of about 1/20. This result suggests the presence of a mechanism of suppressing the 27-plet contributions by a factor of about 3. Assuming that this suppression mechanism is at work the corrections from the suppressed interaction \( L_{123}^{\gamma, \eta, \bar{\eta}} \) to the hyperon decays and \( K_s^0 \to 2\pi \) decays are estimated to be at most several percent.
Thus the 27-plet contributions do not essentially alter the octet contributions. It still remains to be clarified, however, how the 27-plet contributions are suppressed.

We may conclude that the description of nonleptonic decays in terms of the universal current-current interaction is promising. It is desirable to generalize the present approach to parity-conserving nonleptonic decays of hadrons.

References

2) M. Gell-Mann and M. Lévy, Nuovo Cim. 16 (1960), 705.
   See also
9) We follow here, as closely as possible, the notation adopted by K. Kawarabayashi, Lectures in Theoretical Physics (Univ. of Colorado, Boulder), Vol. XI-A (1968) p. 227.
15) J. D. Berge, Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966, p. 46. The numerical values quoted in Table I have been obtained from those in this report by a change of units.