Effects of data time-step on the accuracy of calibrated rainfall–streamflow model parameters: practical aspects of uncertainty reduction

Ian G. Littlewood and Barry F. W. Croke

ABSTRACT

The effects of data time-step on the accuracy of calibrated parameters in a discrete-time conceptual rainfall–streamflow model are reviewed and further investigated. A quick-flow decay time constant of 19.9 hr, calibrated for the 10.6 km^2 Wye at Cefn Brwyn using daily data, massively overestimates a reference value of 3.76 hr calibrated using hourly data (an inaccuracy of 16.1 hr or 429%). About 42 and 58% of the inaccuracy are accounted for by loss of information in the effective rainfall and streamflow data, respectively. A slow-flow decay time constant is inaccurate by about +111%, of which about 94 and 17 percentage points (85 and 15% of the absolute inaccuracy) are due to loss of information in the effective rainfall and streamflow data, respectively. Discrete-time rainfall–streamflow model parameter inaccuracy caused by data time-step effects is discussed in terms of its implications for parameter regionalisation (including database aspects) and catchment-scale process studies.

Key words | accuracy, conceptual models, precision, regionalisation, uncertainty

Ian G. Littlewood (corresponding author)
IGL Environment,
Didcot,
Oxfordshire OX11 7XN,
UK
E-mail: *ianlittlewood505@btinternet.com*

Barry F. W. Croke
Department of Mathematics,
Fenner School of Environment and Society and
National Centre for Groundwater Research and
Training,
The Australian National University,
Canberra,
Australia

INTRODUCTION

Conceptual catchment-scale rainfall–streamflow model parameters are often calibrated using time series of streamflow, basin rainfall and other hydrometeorological variables sampled at a common frequency, e.g. discrete-time data at hourly or daily or monthly time-steps (Δt).

The loss of information in the data as Δt increases can cause calibrated model parameters to become increasingly inaccurate. Although this point is well known and has been dealt with systematically in other sciences, e.g. telecommunications signal processing, it has been largely overlooked in the hydrology research literature until recently (Littlewood *et al.* 2010). Investigations of the effects of Δt on discrete-time rainfall–streamflow model parameters, and how to mitigate these effects, include work by Littlewood & Croke (2008), Wang *et al.* (2009), Littlewood *et al.* (2010, 2011), Clark & Kavetski (2010), Kavetski & Clark (2010), Young (2010) and Ostrowski *et al.* (2010).

Throughout this paper, precision means the spread of (random) error about a best estimate (expressed as $\pm X$) and accuracy means the (systematic) departure of a best estimate from the true value (expressed as $\pm Y$). Clearly, for an estimate to have low uncertainty it must have small X and Y close to zero. (Standard deviation is a measure of precision, and the term bias is often used instead of accuracy.) The focus of this paper is on the accuracy, rather than precision, of rainfall–streamflow model parameters as Δt changes.

Discrete-time model parameters can be extremely inaccurate when calibrated from coarsely sampled data. For example, Littlewood (2007) and Littlewood & Croke (2008) show that, for the 10.6 km^2 headwater catchment of the Wye at Cefn Brwyn in Wales, a quick-flow decay time constant calibrated with good precision (about ± 2%) over a 210-day period using daily data was +429% of the same model parameter calibrated over the same

doi: 10.2166/nh.2012.099

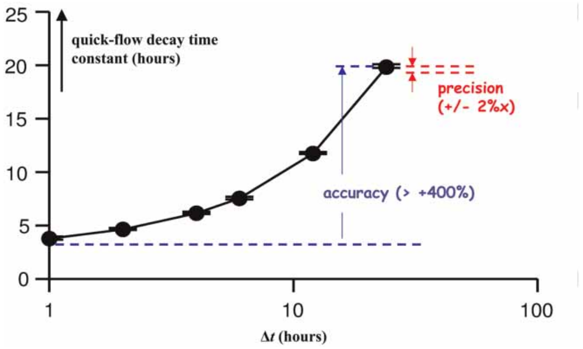

Figure 1 | Precision and accuracy.

period using hourly data. Figure 1 clarifies the terms precision and accuracy using this example. The rainfall–streamflow model had just five parameters. The inaccuracy of each of the other four parameters calibrated using daily data was less than for the quick-flow time constant but still large (Littlewood & Croke 2008): +192% for a catchment drying time constant; +132% for the depth of a catchment wetness store; +111% for a slow-flow decay constant; and –18 percentage points for a slow-flow index (SFI).

It is therefore suggested that the accuracy of some Cefn Brwyn model parameters calibrated using daily data, caused by Δt effects, is likely to be much larger than additional contributions to uncertainty due to (1) imprecision and inaccuracy in measurements of the rainfall and streamflow and (2) estimation of the basin rainfall from raingauge measurements (further work, outside the scope of this paper, is required to check this suggestion). However, as Figure 1 shows, by calibrating the quick-flow decay parameter using hourly data, its inaccuracy (and therefore its uncertainty) can be reduced substantially.

If the same Cefn Brwyn daily data (downloadable from http://tdwg.catchment.org/datasets.html) were used for calibrating any other conceptual discrete-time rainfall–streamflow model, at least some of its parameters would have large inaccuracy. Extending this argument to other catchments, at least some of the parameters of any discrete-time model calibrated using daily data will be very inaccurate for any basin that exhibits a highly dynamic, i.e. sub-daily, streamflow response to rainfall.

This paper expands upon the previous work introduced above. Further insights are developed that may help guide rainfall–streamflow modelling where the objective is more than simply data-fitting, e.g. quantitative characterisation and comparison of dynamic hydrological responses for a set of gauged catchments to assist with model parameter regionalisation and subsequent estimation of flows for ungauged (flow) catchments. Components of the inaccuracy in Cefn Brwyn unit hydrograph (UH) model parameters are identified corresponding to the separate losses of information in effective rainfall and streamflow as Δt increases ($1\ \text{hr} < \Delta t < 24\ \text{hr}$). Discussion covers the implications of Δt effects for (1) rainfall–streamflow model parameter regionalisation studies (including relevant database aspects) and (2) the estimation of model parameters to assist with catchment-scale process studies.

THE WYE AT CEFN BRWYN AND ITS DATA

The Wye at Cefn Brwyn is a 10.6 km^2 predominantly open-moorland headwater catchment in mid-Wales and is one of the wettest gauged basins in England and Wales; mean annual rainfall from 1951 to 2008 was 2,487 mm, of which about 85% generated streamflow (NERC 2008). Cefn Brwyn is one of the Plynlimon research basins operated by the Centre for Ecology and Hydrology (CEH), United Kingdom (e.g. Robinson & Dupeyrat 2005). Streamflow is measured at a weir (52°26′18.58’N, 3°43′25.77’W). The hydrometric data used in this paper comprise 15-minute flows and hourly catchment rainfalls for the 210-day model calibration period 6 December 1987–2 July 1988, from which 1-, 2-, 4-, 6-, 12- and 24-hr rainfall–streamflow datasets were prepared. The n-hr flows are averages of 15-minute flows over each n-hr period, expressed in mm per n-hr time-step. The corresponding n-hr catchment rainfall data are totals (mm) for each n-hr period. The hydrometric data were retrieved from the CEH Plynlimon data archive and employed at face value. Investigation of uncertainties in the basin rainfall and streamflow data lies outside the scope of this paper, but the Wye at Cefn Brwyn is a key UK research basin for which the data can be assumed to be of high quality.

THE MODEL AND MODELLING METHODOLOGY

The IHACRES (Identification of unit Hydrographs And Component flows from Rainfall, Evaporation and Streamflow data) model and modelling methodology applied here, their antecedents and how the model parameters are calibrated in the discrete-time domain have been described previously (e.g. Jakeman *et al.* 1990; Jakeman & Hornberger 1993; Littlewood & Jakeman 1994). The following outline provides necessary context for later sections of the paper.

The model comprises two modules in series: a non-linear loss module that has basin rainfall as input and effective rainfall as output, followed by a linear UH module that has effective rainfall as input and modelled streamflow as output. There are three dynamic response characteristics (DRCs) in the loss module (dimensions or units are given in brackets): the depth of a conceptual catchment wetness store c [L]; a catchment drying time constant τ_w [T] and a factor f ($^\circ C^{-1}$) that modulates τ_w according to air temperature. The UH module also has three DRCs: a quick-flow decay time constant $\tau^{(q)}$ [T]; a slow-flow decay time constant $\tau^{(s)}$ [T]; and a proportional volumetric contribution of modelled slow-flow to modelled streamflow, i.e. a slow-flow index SFI (dimensionless). All acronyms, symbols, etc. used in this paper are defined in Appendix 1 (Table A1, available online at http://www.iwaponline.com/nh/044/099.pdf).

For an exponential decay store (e.g. either of the quick- or slow-flow stores in the UH module, and ignoring any pure time delay), the modelled output from the store at time-step k (O_k) can be expressed:

$$O_k = -aO_{k-1} + bu_k \quad (1)$$

where a ($-1 < a < 0$) and b (>0) are transfer function parameters. The value of a in Equation (1) is very strongly influenced by the time-step in two ways: firstly through the linear dependence of a on Δt (as shown in Equation (2), which gives the conversion between a and the time constant τ from the continuous time form); and secondly through the loss of information due to the sampling frequency

$$\tau = \frac{-\Delta t}{\ln(-a)} \quad (2)$$

The time constants $\tau^{(q)}$ and $\tau^{(s)}$ are used to account for the linear dependence to the time-step Δt.

For the Cefn Brwyn model calibration period employed in this paper (6 December 1987–2 July 1988), temperature modulation of τ_w did not improve the model fit. Therefore, only the five DRCs (c, τ_w, $\tau^{(q)}$, $\tau^{(s)}$ and SFI) were calibrated.

At the time the analysis was undertaken, there were two IHACRES rainfall–streamflow modelling software packages freely available: PC-IHACRES (http://www.ceh.ac.uk/products/software/CEHSoftware-PC-IHACRES.htm) or PCI (Littlewood *et al.* 1997) and the more powerful IHACRES Classic Plus (http://icam.anu.edu.au/products/ihacres.html) or ICP (Croke *et al.* 2006). Other more recent IHACRES software is described by Andrews *et al.* (2011). For this paper, PCI (v1.03) and ICP (v2.1) were used. Both PCI and ICP allow calibration of a full rainfall–streamflow model, generating effective rainfall u_k internally. However, for later sections of the paper it should be noted that PCI also allows calibration of a UH module from effective rainfall computed externally, i.e. the prescribed loss module can be bypassed if the operator wishes to try using a different effective rainfall. Although ICP is generally the more powerful package, its functionality does not allow bypassing of the loss module.

Figure 2(a) shows the Cefn Brwyn ICP calibration model fit (daily data) 6 December 1987–2 July 1988, for which the coefficient of determination D is 0.89. Figure 2(b) shows the model applied in simulation mode 1 October 1980 to 1 March 1981 ($D = 0.78$).

THE EFFECT OF Δt ON CALIBRATED PARAMETER VALUES

Figure 3 shows the trajectories of calibrated Cefn Brwyn ICP-DRCs (hereafter DRC$^\#$s) for selected Δt for 1–24 hr. Indicative precisions are calculated as the 95% confidence bounds derived from a fine search in the vicinity of the optimised set of parameter values, selecting the DRC sets that gave a coefficient of determination D of more than 99.9% of its maximum value (Littlewood & Croke 2008). Each of the five calibrated DRC$^\#$s reaches or approaches a stable value as log Δt decreases to 1 hr,

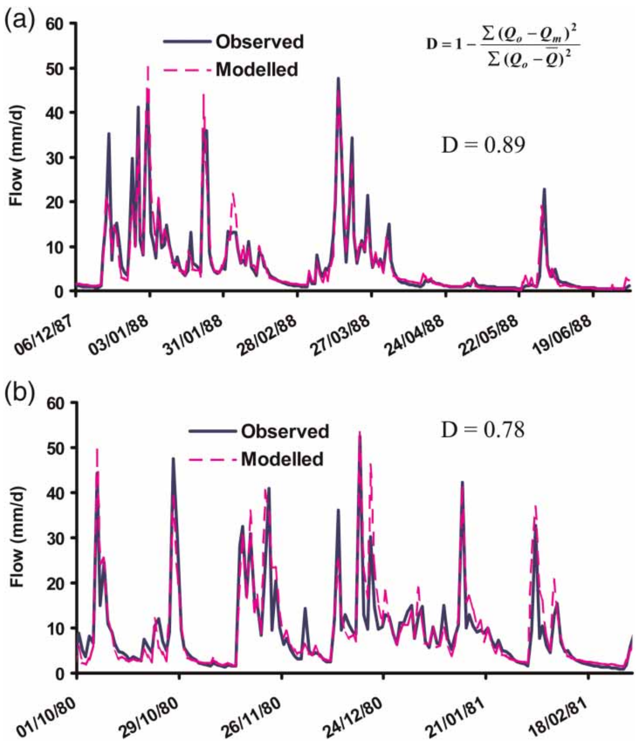

Figure 2 | Cefn Brwyn model fits: (a) calibration 6 December 1987–2 July 1988 and (b) simulation 1 October 1980–1 March 1981.

inviting the conclusion that the DRC$^{\#}$s estimated using hourly data are superior to DRC$^{\#}$s calibrated using data with $\Delta t > 1$ hr. On this basis, Littlewood (2007) and Littlewood & Croke (2008) proposed extrapolating the trajectories to $\Delta t = 0$ (0 hr) as an empirical-graphical method for estimating Cefn Brwyn DRCs that are essentially independent of Δt. For the UH module, this gives estimates of instantaneous unit hydrograph (IUH) parameters.

The UH DRC$^{\#}$s ($\tau^{\#(q)}$, $\tau^{\#(s)}$ and SFI$^{\#}$) for Cefn Brwyn have been compared with the same DRCs calibrated by a different method, as outlined in the next section.

COMPARISON OF UNIT HYDROGRAPHS IDENTIFIED BY IHACRES AND THE CT-DBM

The continuous-time data-based mechanistic (CT-DBM) modelling methodology (Young & Romanowicz 2004; Young & Garnier 2006; Young 2011) is a powerful technique that allows estimation of the continuous-time equivalents of discrete-time model parameters such as IHACRES $\tau^{(q)}$ and $\tau^{(s)}$ using discrete-time data. One of the key advantages of the continuous-time approach over the discrete-time approach is the greater stability of the model parameters. As discrete-time data is used as the input to the continuous-time models, the sampling interval has to be sufficiently short to permit an adequate reconstruction of the continuous-time signals.

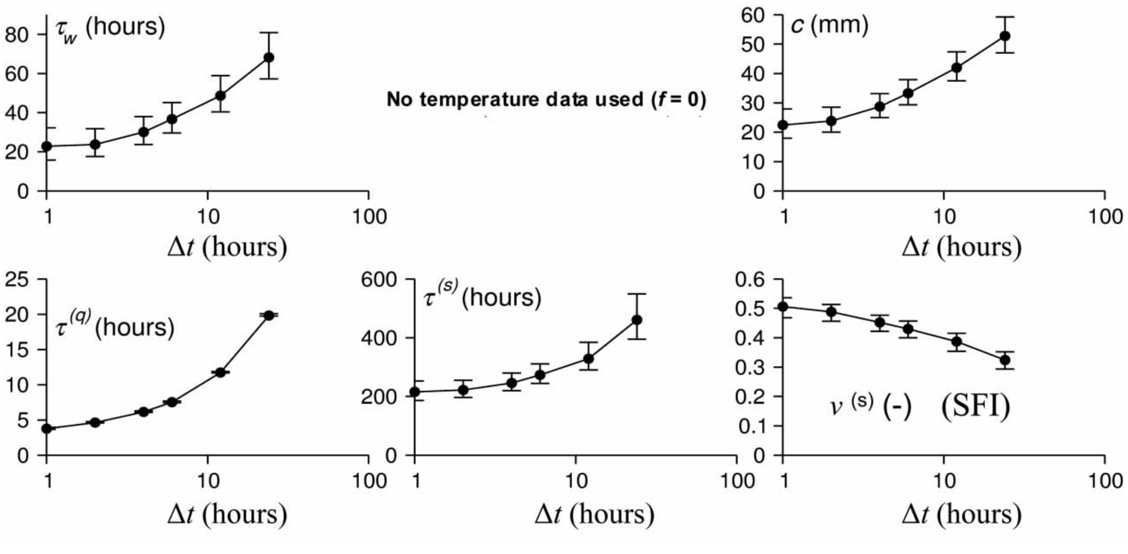

Figure 3 | Wye at Cefn Brwyn IHACRES dynamic response characteristics (DRCs) and indicative 95% uncertainties (Littlewood & Croke 2008).

Littlewood *et al.* (2010) applied the CT-DBM technique using the sequences of Cefn Brwyn effective rainfall generated during ICP calibration, i.e. $u^{\#}_{k}$, and their corresponding sequences of observed streamflow to calibrate UH DRC$^{\wedge}$s ($\tau^{\wedge(q)}$, $\tau^{\wedge(s)}$ and SFI$^{\wedge}$). Approximate convergences of the DRC$^{\#}$ and DRC$^{\wedge}$ trajectories at $\Delta t = 1$ hr (shown in Figure 4) confirm that the discrete-time IHACRES UH DRC$^{\#}$s for Cefn Brwyn calibrated using 1 hr data are, as proposed by Littlewood & Croke (2008), good approximations of IUH parameters.

Figure 4 shows that $\tau^{\wedge(q)}$ and $\tau^{\wedge(s)}$ for 1 hr $< \Delta t <$ 12 hr are much less sensitive to Δt than $\tau^{\#(q)}$ and $\tau^{\#(s)}$. It is interesting that SFI$^{\#}$ decreases with Δt, rather than increasing as for SFI$^{\wedge}$. This could be because $\tau^{\wedge(q)}$ is fairly constant with changing Δt while $\tau^{\#(q)}$ increases; the discrete modelling approach may associate more of the response to quick-flow, leading to a decrease in the slow-flow volume. For the CT-DBM model, the conversion of discrete data to continuous time is a possible cause of the slight increase in $\tau^{\wedge(s)}$ and SFI$^{\wedge(s)}$ (further work is required to investigate this). The CT-DBM method was unable to identify an effective rainfall–streamflow model from 24 hr data, and its application has yet to be extended to allow calibration of the full IHACRES rainfall–streamflow model structure applied here.

Ignoring all other sources of error, the (total) trajectories shown in Figure 3 represent the combined effects of losses of information in rainfall and streamflow time series as Δt increases. The next section provides insights about the separate effects of information losses in rainfall and streamflow data as Δt increases.

EFFECT OF COARSELY SAMPLED EFFECTIVE RAINFALL DATA

DRC trajectories representing the effects of increasing loss of information in effective rainfall, as Δt increases, were derived and compared with the total trajectories (Figure 3) as follows.

Given that the trajectories for $c^{\#}$ and $\tau^{\#}_{w}$ (Figure 3) reach or approach stable values as Δt decreases logarithmically to 1 hr, the sequence of effective rainfall generated by the loss module using hourly data ($u^{\#}_{k,1}$) is arguably superior to $u^{\#}_{k,\Delta t}$ with 2 hr $< \Delta t <$ 24 hr. Furthermore, the superiority of $u^{\#}_{k,1}$ might be expected to increase as Δt increases from 2 to 24 hr. The $u^{\#}_{k,1}$ times series was therefore considered to be quasi-optimal. Sequences of quasi-optimal effective rainfall for $\Delta t = 2$, 4, 6, 12 and 24 hr were derived by summing elements of $u^{\#}_{k,1}$ to form $u^{*}_{k,2}$, $u^{*}_{k,4}$, $u^{*}_{k,6}$, $u^{*}_{k,12}$ and $u^{*}_{k,24}$. For example, $u^{*}_{2,4} = u^{\#}_{5,1} + u^{\#}_{6,1} + u^{\#}_{7,1} + u^{\#}_{8,1}$. The sequence of data preparation and analysis is shown schematically in Figure 5. The $u^{*}_{k,\Delta t}$ time series, each with their corresponding Δt hr observed flow time series, were input to the PCI package to calibrate UH*s, bypassing the loss module. The resultant DRC*s were then compared to the DRC$^{\#}$s.

UHs calibrated by PCI (v1.03) and ICP (v2.1) were checked for consistency; Appendix 2 (available online at http://www.iwaponline.com/nh/044/099.pdf) shows that non-trivial differences between UHs calibrated by ICP (full rainfall–streamflow model calibration) and by PCI (effective rainfall–streamflow model calibration) are due only to differences between the effective rainfall series used.

Table 1 lists DRC$^{\#}$s, DRC$^{\wedge}$s and DRC*s. Although ICP ($^{\#}$) successfully identified a full rainfall–streamflow

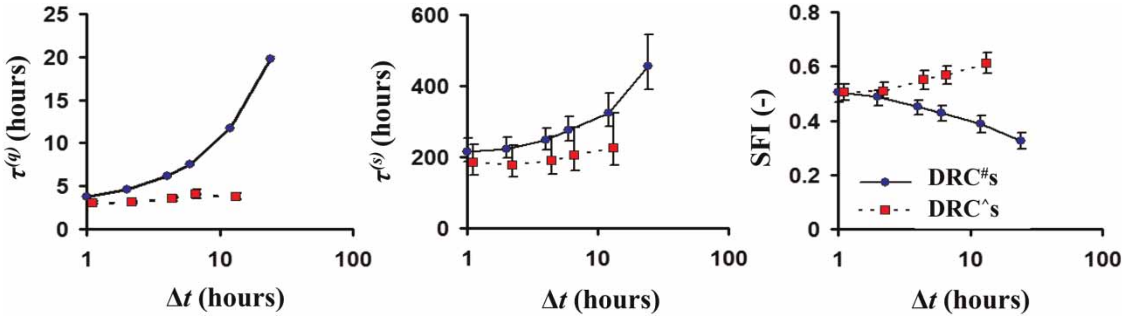

Figure 4 | Cefn Brwyn unit hydrograph DRCs calibrated using ICP ($^{\#}$) and CT-DBM ($^{\wedge}$).

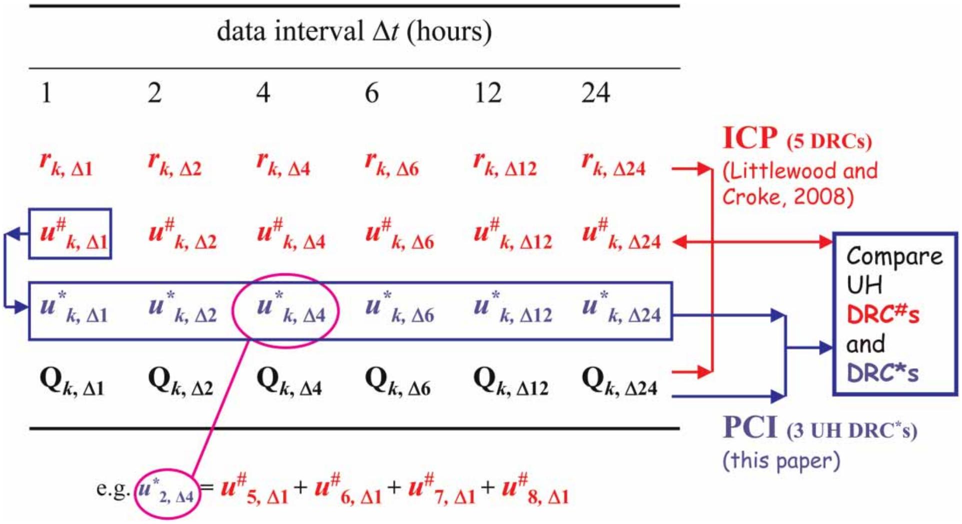

Figure 5 | Schematic of data preparation and analysis. *Notes*: $r_{k,\Delta t}$ is rainfall, $u^{\#}_{k,\Delta t}$ is effective rainfall generated during rainfall–streamflow model calibrations by ICP, $u^{*}_{k,\Delta t}$ ($\Delta t > 1$ hr) is quasi-optimal effective rainfall computed from quasi-optimal $u^{\#}_{k,\Delta 1}$ ($u^{*}_{k,\Delta 1} = u^{\#}_{k,\Delta 1}$) and $Q_{k,\Delta t}$ is observed streamflow.

Table 1 | Cefn Brwyn modelled DRCs (#: ICP, Littlewood (2007); Littlewood & Croke (2008); ^: CT-DBM, Littlewood *et al*. (2010); *: PCI, this paper; – means only the UH module was calibrated; NI: not identified)

	τ_w (hr)			*c* (mm)			$\tau^{(q)}$ (hr)			$\tau^{(s)}$ (hr)			SFI		
Δ*t* (hr)	#	^	*	#	^	*	#	^	*	#	^	*	#	^	*
1	23.0	–	–	22.5	–	–	3.76	3.08	NI	216	184	NI	0.505	0.503	NI
2	24.0	–	–	24.0	–	–	4.63	3.09	4.44	223	178	226	0.487	0.509	0.491
4	30.4	–	–	28.9	–	–	6.14	3.54	5.48	247	190	238	0.450	0.550	0.475
6	37.2	–	–	33.6	–	–	7.54	4.04	6.54	275	206	240	0.428	0.567	0.471
12	48.0	–	–	41.6	–	–	11.7	3.79	9.00	325	226	250	0.389	0.611	0.456
24	67.2	–	–	52.2	–	–	19.9	NI	13.2	455	NI	252	0.326	NI	0.443

model using hourly data, PCI (*) failed to identify a UH* from $u^{\#}_{k,1}$ exported from ICP. In Figure 6, each DRC* has a flatter trajectory than for the corresponding DRC#. Indeed, $\tau^{*(s)}$ is quite insensitive to Δt.

The sensitivity of $\tau^{*(q)}$ to Δt is much less than for $\tau^{\#(q)}$ but still quite marked. About 42% of the inaccuracy in $\tau^{\#(q)}$ calibrated using daily data appears to be accounted for by the loss of information in effective rainfall

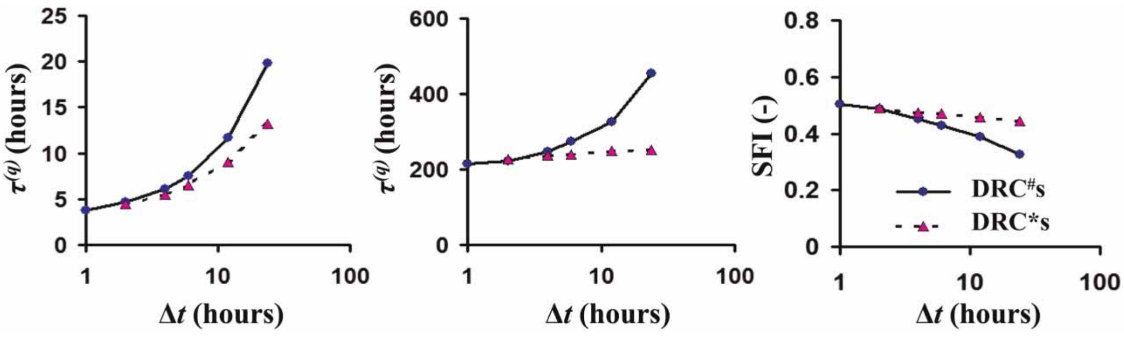

Figure 6 | Cefn Brwyn unit hydrograph DRCs calibrated using ICP (rainfall–streamflow, #) and PCI (quasi-optimal effective rainfall–streamflow, *).

((19.9–13.2)/(19.9–3.76) × 100%; see Table 1), leaving about 58% of the inaccuracy caused by loss of information in the streamflow data. For $\tau^{\#(s)}$ the corresponding percentages are 85% ((455–252)/(455–216) × 100%) and 15%. Thus, for Cefn Brwyn, the values of $\tau^{\#(q)}$ calibrated using daily data are affected much more by the loss of information in streamflow data ($\Delta t = 24$ hr) than values of $\tau^{\#(s)}$. This is intuitively reasonable given that the standardised ($\Delta t = 1$ hr) $\tau^{\#(q)}$ of 3.76 hr is much less than 1 day, whereas the standardised $\tau^{\#(s)}$ is much more than 1 day (216 hr = 9 days). Cefn Brwyn daily data are too coarse for quantifying its dominant quick-flow dynamic accurately, but they are close to being sufficiently frequent to accurately capture the dominant slow-flow dynamic.

UNIT HYDROGRAPH DRC TRAJECTORIES DUE SOLELY TO Δ*t*

The Cefn Brwyn UH DRC* trajectories (Figure 6) illustrate the effects of (1) the loss of information in observed streamflow as Δt increases from 1 to 24 hr and (2) Δt itself. The DRC$^{\#}$ trajectories illustrate the combined effects of (1) losses of information in both rainfall and streamflow, (2) different loss module parameters and (3) Δt. For comparison with DRC* and DRC$^{\#}$ trajectories, DRC** trajectories affected solely by Δt were derived as follows.

For $\Delta t = 1$ hr, $u^{**}{}_{k,\Delta t} = u^{\#}_{k,1}$ and $Q^{**}{}_{k,\Delta t} = Q^{\#}{}_{k,1}$, i.e. modelled flow was used instead of observed flow. For $\Delta t > 1$ hr, $u^{**}{}_{k,\Delta t} = u^{*}{}_{k,\Delta t}$ (see earlier for the simple aggregation process used). $Q^{**}{}_{k,\Delta t}$ ($\Delta t > 1$ hr) was also derived by aggregation, e.g. $Q^{**}_{3,6} = Q^{\#}_{13,1} + Q^{\#}_{14,1} + Q^{\#}_{15,1} + Q^{\#}_{16,1} + Q^{\#}_{17,1} + Q^{\#}_{18,1}$.

As Δt increases, the information content of $u^{**}{}_{k,\Delta t}$ (or $Q^{**}_{k,\Delta t}$) decreases due solely to Δt; there are no other causes of decreasing information in $u^{**}_{k,\Delta t}$ and $Q^{**}_{k,\Delta t}$. The $u^{**}_{k,\Delta t}$ and $Q^{**}_{k,\Delta t}$ time series were input to PCI to calibrate a UH module ($\tau^{**(q)}$, $\tau^{**(s)}$ and SFI**) for each Δt. As expected, when $\Delta t = 1$ hr (no information loss), D is very nearly 1 and the UH DRC**s are essentially the same as the UH DRC$^{\#}$s (Tables 1 and 2). As Δt increases to daily, D for the PCI** models decreases to 0.953; by this conventional measure of model fit there is only a small effect as Δt increases. However, as Δt increases from 1 to 24 hr, $\tau^{**(q)}$ increases by about 300%, $\tau^{**(s)}$ increases by about 10% and SFI** decreases by about 3 percentage points.

Table 2 | Cefn Brwyn modelled unit hydrograph DRC**s

Δ*t* (hr)	*D*	$\tau^{(q)}$ (hr)	$\tau^{(s)}$ (hr)	SFI (–)
1	0.999	3.76	214	0.505
2	0.995	4.50	223	0.493
4	0.981	5.76	234	0.479
6	0.969	6.90	236	0.477
12	0.958	9.72	240	0.474
24	0.953	15.1	237	0.473

A general implication of the results presented in this section and from the previous work outlined is that, provided the non-linear loss module is only weakly time-step dependent, the linear module (UH) of IHACRES (and most likely, other discrete-time effective rainfall–streamflow models) can give reasonably accurate measures of $\tau^{(s)}$ and SFI. However, $\tau^{(q)}$ is likely to be strongly affected by data time-step unless $\tau^{(q)} > \Delta t$. In comparison, the CT-DBM model can give significantly more accurate $\tau^{(q)}$ values, although the model may have difficulty converging for very coarsely sampled data ($\Delta t >> \tau^{(q)}$).

DISCUSSION

The analyses presented here reinforce messages from earlier work (Littlewood *et al.* 2010; Young 2010) concerning the need to use sufficiently high-frequency data when calibrating parameters of discrete-time rainfall–streamflow models. New insights have been presented about the relative contributions to the inaccuracy of Cefn Brwyn IHACRES UH model parameters due to losses of information in effective rainfall and streamflow time series as Δt increases from hourly to daily.

Flow-duration curves (FDCs) reveal that the Cefn Brwyn ICP$^{\#}$ models tend to perform less well at low flows than at high flows. Figure 7 shows FDCs for hourly and 2-hourly modelled flows and for 2-hourly observed flows over the calibration period, plotted on log-normal axes (log-normally distributed flows plot as a straight line). The FDC for

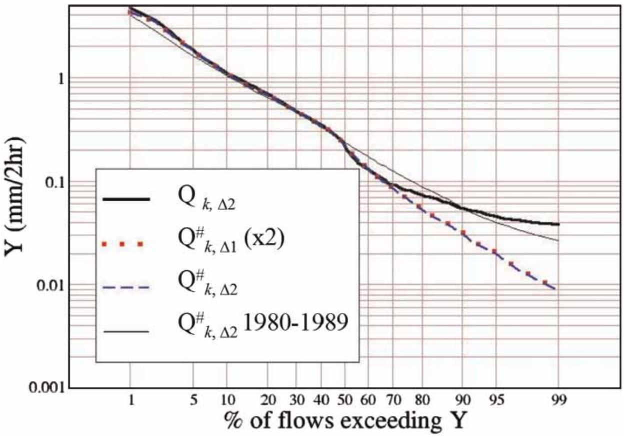

Figure 7 | Cefn Brwyn flow duration curves.

observed 2-hourly flows from 1980 to 1989 is shown for comparison; as expected, the distribution of $Q_{k,2}$ becomes closer to log-normal as record length increases. Over the model calibration period, $Q_{k,2}$ is approximately log-normal between flows exceeded for 1 and 50% of the time, when modelled flows $Q^{\#}_{k,1}$ and $Q^{\#}_{k,2}$ are, as expected, very close to $Q_{k,2}$. (The ordinates of the FDC for $Q^{\#}_{k,1}$ have been multiplied by 2 to facilitate comparison with the other FDCs.) The low-flow half of the FDC for $Q_{k,2}$ does not follow a log-normal distribution and hints that a third, very slowly decaying exponential store is needed which has not been identified from the time-series data.

This might explain some of the Δt sensitivity of SFI$^{\#}$ and $\tau^{\#(s)}$ due to the third component becoming more apparent at larger Δt. The FDCs for modelled $Q^{\#}_{k,1}$ and $Q^{\#}_{k,2}$ are approximately log-normal over the whole of the range shown and are therefore a poor match to the low-flow part of the FDC for $Q_{k,2}$. Consequently, the value of 216 hr for $\tau^{\#(s)}$ calibrated using hourly data (Table 1) is unlikely to be a very good reference value against which to assess the accuracy of $\tau^{\#(s)}$ values calibrated for $\Delta t > 1$ hr. To address this issue, the analysis presented in this paper must be repeated for many calibration periods of different durations to investigate whether it is possible to identify a value for $\tau^{\#(s)}$ that is more worthy than 216 hr of being considered a good reference value for Cefn Brwyn. This is beyond the scope of the current paper.

Effective rainfall cannot be measured. Consequently, it is very difficult to assess the effective rainfall generated by any loss module. However, the decrease in Cefn Brwyn Δt sensitivities of $\tau^{(q)}$,$\tau^{(s)}$ and SFI when u^{*}_{k} is used rather than $u^{\#}_{k}$ (Figure 6) supports the view that the values of $\tau^{\#}_{w}$ and $c^{\#}$ estimated using hourly data generate a superior sequence of effective rainfall than when $\tau^{\#}_{w}$ and $c^{\#}$ are calibrated using larger time-step data. It would have been very odd if the sensitivities had increased.

Possible directions for further research and suggestions for better practice will now be discussed in the context of two broad modelling objectives: rainfall–streamflow model parameter regionalisation (including relevant database aspects) and better understanding of catchment-scale processes.

Many studies have derived statistical relationships between discrete-time conceptual rainfall–streamflow model parameters and physical catchment descriptors for gauged basins, and applied these relationships to estimate model parameters for ungauged (flow) basins and thereby flows at ungauged sites from basin rainfall. Some regionalisation schemes have employed the IHACRES model structure applied in this paper (Sefton & Boorman 1997; Sefton & Howarth 1998; Post & Jakeman 1999). Other studies have regionalised the parameters of other rainfall–streamflow models (e.g. Merz *et al.* 2006; Young 2006). Typically, the precision associated with estimates of model parameters from physical catchment descriptors has not been good. Part of the reason for this could be that regionalisation studies usually employ daily data to calibrate a discrete-time rainfall–streamflow model for each of the gauged catchments considered (other contributions to the imprecision associated with model parameter regionalisation equations include noise in parameter estimates and the non-linear response of rainfall-streamflow models; Croke & Norton 2004).

The Cefn Brwyn analysis indicates that calibrated ($\Delta t =$ daily) rainfall–streamflow model parameters for gauged catchments with a highly dynamic (sub-daily) streamflow response to rainfall, especially parameters related to a dominant quick-flow component of streamflow, are likely to have been estimated with poor accuracy in some regionalisation studies. Further work is required to establish the proportion of imprecision in statistical parameter regionalisation relationships caused by the use of non-standardised (Δt-dependent) calibrated model parameters for the gauged basins involved. The IHACRES empirical/graphical and CT-DBM methods discussed in this paper offer potential ways forward for increasing the accuracy in DRCs for gauged catchments, thereby reducing the uncertainty in estimates of flow at ungauged sites.

Although daily data for gauged basins have often been used in the derivation of regionalisation schemes, in some cases sub-daily data exist from which the daily data were derived. Given that many gauged basins are required for regionalisation, daily flow data may have been used in some studies because they were relatively easy to access systematically, e.g. from national databases. Furthermore, sub-daily basin rainfall data may not have been readily available, and uncertainty in basin rainfall can increase as catchment size increases (especially in regions where predominantly convective rainfall affects parts of the catchment or where raingauge density is low). Model parameter regionalisation studies that have used daily data for all of the gauged basins considered have not always reported that the information in sub-daily data (if they exist) has effectively been discarded and that this may have adversely affected the precision associated with the model parameter regionalised relationships. Where sub-daily data exist, it would appear to be wise to use the extra information they contain, leading to more accurate calibrated model parameters. Where sub-daily data do not exist, or have not been used, estimated model parameters (and their stated uncertainties) should be qualified accordingly. Alternatively, a hybrid approach could be adopted where high-temporal-resolution streamflow data is used to estimate the parameters of the UH module directly from streamflow (Croke 2006). The technique described in Croke (2006) has been tested on ephemeral catchments, and further research is needed to investigate the application of this approach to very wet catchments.

The extent to which the structurally simple, spatially lumped, IHACRES model can assist with quantification of catchment-scale processes deserves further investigation. If model parameters are required to have low uncertainty (good precision and good accuracy) and be physical meaningful (not solely a means of fitting a model to data), the Cefn Brwyn modelling at different Δt presented in this paper has reinforced how important it can be to calibrate a model using sufficiently frequent data. This argument can be extended to any conceptual rainfall–streamflow model. Simply stating that model parameters represent physical processes does not mean their calibrated values (using daily data for example) are estimated with low uncertainty, even if they have good precision and the modelled flows are a good fit to observed flows.

CONCLUDING REMARKS

Each of the five IHACRES DRC$^{\#}$s for Cefn Brwyn were identified with sufficiently good precision to give a well-defined trajectory ($1 < \Delta t < 24$ hr). If the parameters of a model (IHACRES or other) are not identified with good precision, their Δt trajectories may not be so well identified. When a model is required to give accurate (and precise) estimates of DRCs (response times, etc.), it is proposed that Δt trajectories of model parameters are used as an additional diagnostic tool for helping to assess a model.

A discrete-time rainfall–streamflow model calibrated using daily data for a dynamically responsive catchment may have good conventional model-fit statistics (high coefficient of determination, low bias, etc.) but that does not guarantee anything more than fitting a model to the data. To move beyond data-fitting towards estimation of physically meaningful model parameters (e.g. to assist with regionalisation or process studies), developers and users of discrete-time models need to demonstrate that calibrated model parameters are essentially independent of Δt or, at least, how they have accounted for Δt dependency of the parameter values. It should be noted that the dependency on Δt will be dependent on catchment and climate characteristics.

In summary, provided that Δt is not too much greater than the value of the quick-flow decay parameter $\tau^{(q)}$ being sought, a continuous-time model will as expected give a more accurate estimate of $\tau^{(q)}$ than an equivalent discrete-time model. If $\Delta t > > \tau^{(q)}$, the continuous-time approach may fail to identify a model (as for CT-DBM when trying to estimate a $\tau^{(q)}$ for Cefn Brwyn of about 3 hr from daily data). A discrete-time model, e.g. IHACRES as applied in this paper, may identify a model for a wider range of $\Delta t \gg \tau^{(q)}$ than a continuous-time approach but with poorer accuracy on the model parameters. Care must therefore be taken when interpreting model parameters identified using discrete-time models, particularly for applications in ungauged basins which is often required in regions where only daily rainfall data are available. However, as the Δt trajectories for Cefn Brwyn IHACRES parameters indicate, a discrete-time approach can give a good approximation to the continuous-time approach when Δt is sufficiently small (about 1 hr for Cefn Brwyn).

ACKNOWLEDGEMENTS

The authors are grateful to reviewers for their comments and suggestions leading to improvements.

REFERENCES

Andrews, F. T., Croke, B. F. W. & Jakeman, A. J. 2011 An open software environment for hydrological model assessment and development. *Environmental Modelling and Software* **26** (10), 1171–1185.

Clark, M. P. & Kavetski, D. 2010 Ancient numerical daemons of conceptual hydrological modeling: 1. Fidelity and efficiency of time stepping schemes. *Water Resources Research* **46**, W10510.

Croke, B. F. W. 2006 A technique for deriving an average event unit hydrograph from streamflow-only for ephemeral quick-flow-dominant catchments. *Advances in Water Resources* **29**, 493–502.

Croke, B. F. W. & Norton, J. P. 2004 Regionalisation of rainfall-runoff models. In: *Complexity and Integrated Resources Management, Transactions of the 2nd Biennial Meeting of the International Environmental Modelling and Software Society*, Osnabruck, Germany, July 2004, Vol. 3, pp. 1201–1207.

Croke, B. F. W., Andrews, F., Jakeman, A. J., Cuddy, S. & Luddy, A. 2006 IHACRES Classic Plus: a redesign of the IHACRES rainfall-runoff model. *Environmental Modelling & Software* **21** (3), 426–427.

Jakeman, A. J. & Hornberger, G. M. 1993 How much complexity is warranted in a rainfall-runoff model? *Water Resources Research* **29**, 2637–2649.

Jakeman, A. J., Littlewood, I. G. & Whitehead, P. G. 1990 Computation of the instantaneous unit hydrograph and identifiable component flows with application to two small upland catchments. *Journal of Hydrology* **117**, 275–300.

Kavetski, D. & Clark, M. P. 2010 Ancient numerical daemons of conceptual hydrological modeling: 2. Impact of time stepping schemes on model analysis and prediction. *Water Resources Research* **46**, W10511.

Littlewood, I. G. 2007 Rainfall-streamflow models for ungauged basins: uncertainty due to modelling time-step. In: *Proceedings of the Eleventh Biennial Conference of the Euromediterranean Network of Experimental and Representative Basins, Uncertainties in the 'monitoring-conceptualisation-modelling' sequence of catchment research*, UNESCO Technical Documents in Hydrology Series No. 81, Paris, pp. 149–155.

Littlewood, I. G. & Croke, B. F. W. 2008 Data time-step dependency of conceptual rainfall-streamflow model parameters: an empirical study with implications for regionalisation. *Hydrological Sciences Journal* **53** (4), 685–695.

Littlewood, I. G. & Jakeman, A. J. 1994 A new method of rainfall–runoff modelling and its applications in catchment hydrology. In: *Environmental Modelling (Volume III)* (P. Zannetti, ed.). Computational Mechanics Publications, Southampton, UK, pp. 143–171.

Littlewood, I. G., Croke, B. F. W. & Young, P. C. 2011 Comment on 'Effects of temporal resolution on hydrological model parameters and its impact on prediction of river discharge'. *Hydrological Sciences Journal* **56** (3), 521–524.

Littlewood, I. G., Down, K., Parker, J. R. & Post, D. A. 1997 The PC Version of IHACRES for catchment-scale rainfall–streamflow modelling: User Guide. Institute of Hydrology Software report, pp. 89.

Littlewood, I. G., Young, P. C. & Croke, B. F. W. 2010 Preliminary comparison of two methods for identifying rainfall–streamflow model parameters insensitive to data time-step: the Wye at Cefn Brwyn, Plynlimon, Wales. In: *Proceedings of the Third International Symposium, British Hydrological Society*, 19–23 July 2010, Newcastle University, UK. Available from: http://www.ceg.ncl.ac.uk/bhs2010/ (accessed November 2012).

Merz, R., Blöschl, G. & Parajka, J. 2006 Regionalization methods in rainfall–runoff modelling using large catchment samples. In: *Large Sample Basin Experiments for Hydrological Model Parameterization* (V. Andreassian, A. Hall, N. Chahinian & J. Schaake, eds). IAHS Publications, Wallingford, UK, Vol. 307, pp. 11–125.

NERC 2008 *UK Hydrometric Register*. Natural Environment Research Council, Swindon.

Ostrowski, M., Bach, M., Gamerith, V. & De Simone, S. 2010 Analysis of the time-step dependency of parameters in conceptual hydrological models. Report, Technical University of Darmstadt. Available from: http://tuprints.ulb.tu-darmstadt.de/2099/ (accessed November 2012).

Post, D. A. & Jakeman, A. J. 1999 Predicting the daily streamflow of ungauged catchments in SE Australia by regionalising the parameters of a lumped conceptual rainfall–runoff model. *Ecological Modelling* **123**, 91–104.

Robinson, M. & Dupeyrat, A. 2005 Effects of timber harvesting on streamflow regimes in the Plynlimon catchments, mid-Wales. *Hydrological Processes* **19**, 1213–1226.

Sefton, C. E. M. & Boorman, D. B. 1997 A regional investigation of climate change impacts on UK streamflows. *Journal of Hydrology* **195**, 26–44.

Sefton, C. E. M. & Howarth, S. M. 1998 Relationships between dynamic response characteristics and physical descriptors of catchments in England and Wales. *Journal of Hydrology* **211**, 1–16.

Wang, Y., He, B. & Takase, K. 2009 Effects of temporal resolution on hydrological model parameters and its impact on prediction of river discharge. *Hydrological Sciences Journal* **54** (5), 886–898.

Young, A. R. 2006 Stream flow simulation within UK ungauged catchments using a daily rainfall-runoff model. *Journal of Hydrology* **320** (1), 155–172.

Young, P. C. 2010 The estimation of continuous-time rainfall-flow models for flood risk management. In: *Proceedings of the Third International Symposium*, British Hydrological Society, 19–23 July 2010, Newcastle University, UK. Available from: http://www.ceg.ncl.ac.uk/bhs2010/ (accessed November 2012).

Young, P. C. 2011 *Recursive Estimation and Time-Series Analysis*, 2nd edition. Springer, London.

Young, P. C. & Garnier, H. 2006 Identification and estimation of continuous-time, data-based mechanistic (DBM) models for environmental systems. *Environmental Modelling & Software* **21**, 1055–1072.

Young, P. C. & Romanowicz, R. 2004 PUB and data-based mechanistic modelling: the importance of parsimonious continuous-time models. In: *Complexity and Integrated Resources Management, Transactions of the 2nd Biennial Meeting of the International Environmental Modelling and Software Society*, Manno, Switzerland, 2004, Vol. 3, pp. 1141–1146.

First received 18 September 2011; accepted in revised form 28 August 2012. Available online 4 December 2012