accounting for aerodynamic blockage ahead of the point of separation would be useful.

Examination of Figs. 4 and 8 shows that the relative Mach number on the shroud suction surface is approximately constant over the last 25–35 percent of the passage. Comparison of these relative Mach number plots with the associated wake distribution plots (Figs. 2 and 7) shows that the start of the constant \( M_R \) region appears to correspond with a rapid increase in the wake width. The latter could be interpreted as the initiation and growth of the separated flow region. Hence, although no calculations were performed, it appears that the rule of constant relative Mach number on the jet-wake boundary is upheld in the separated regions.

The use of a quadratic wake distribution for design purposes is indeed a simplification and it would probably be a good idea to adjust this near the passage exit, if necessary to avoid the prediction of reacceleration of the shroud suction \( M_R \). An acceleration on the shroud mean streamline near the exit is expected from the jet-wake model [8], so that the jet exit \( M_R \) is equal to the \( M_R \) at the suction surface separation region.

It is unlikely that a two-dimensional wake structure occurs, since the hub streamlines are subject to different diffusion schedules and normal pressure gradient distributions than those along the shroud. This “non-two-dimensional” structure is clearly illustrated by the experimental data of Eckardt [9]. A two-dimensional wake was assumed in the data matching exercises both for simplicity and because no detailed knowledge of the wake structure can be gained from the available data. For design purposes a varying hub-to-shroud (i.e. non-two-dimensional) wake distribution can be utilized. Although a sharply delineated wake structure, as experimentally determined by Eckardt [9], cannot be directly employed in the calculation approach, it is thought that an approximation (e.g. a triangular shape) to Eckardt’s wake structure may be adequate for design purposes. Further calculations are needed to confirm this view.

Figs. 4 and 5 clearly illustrate the significant differences in relative Mach numbers (over the last 50 percent of the passage) that results when the effects of blockage are ignored, even if a distributed pressure loss is included. Calculations were also performed (not shown) without both a total pressure loss and blockage and lead to even lower relative Mach numbers over the last 50 percent of the passage. Also, the blade-to-blade loadings, which are important for assessing where flow separation is likely to occur, are higher without blockage than those calculated with blockage. Consequently, significant errors in the chosen impeller blade geometry, can occur if these blockage effects are ignored. Although inviscid flow methods are better than nothing for design purposes, they cannot supplant those methods based on more realistic flow models.

In principle, the approach of Bosman and El-Shaariawi [1] of iterating \( S_1 \) and \( S_2 \) solutions is the correct one for inviscid flow. However, as their Figs. 4 and 5 show, the difference between the \( S_1/S_2 \) solutions and those based solely on an \( S_2 \) calculation is very small, except near the impeller exit (and at the impeller entry on the hub streamline). These large discrepancies result directly from their assumption that the \( S_2 \) streamsurface is parallel with the blading, even near the impeller exit. The use of a simple slip model for the \( S_2 \) calculation would bring the two results into much closer agreement, since the very low number of blades leads to significant impeller slip. The general conclusion is that use of the \( S_1/S_2 \) iteration calculation is unwarranted for routine design calculations because of the large cost with little improvement in accuracy over an \( S_2 \) solution. A comparison with experimental data could have indicated whether the \( S_1/S_2 \) calculation approach more closely approximates real flow patterns.

Perhaps the most important point is that the Bosman and El-Shaariawi approach has not included viscous effects. It may be of interest to perform \( S_1/S_2 \) iteration calculations with the inclusion of blockage and to compare the results both with an \( S_2 \) solution and with experimental data.

**Quasi-Three-Dimensional Numerical Solution of Flow in Turbomachines**

R. G. BULLOCK. This paper is a good presentation of some interesting and valuable work. One result of potential importance is puzzling, however, and some remarks about the significance of dissipative forces should be elaborated. This discussion is directed to these comments.

The paper emphasizes the valid and important point that gross errors result from arbitrary assumptions about the shape of the surfaces within the confines of the blades. Figs. 3 illustrates the shape problem dramatically. Figs. 4 and 5 show the consequence of an arbitrary assumption. The distributions upstream and downstream of the blades, however, indicate that the shapes of the \( S_i \) surfaces in these regions were calculated for both the quasi-three-dimensional solution and two dimensional solution. This mixture of calculations and assumptions would be rather unusual in practice. If the arbitrary \( S_i \) surface for the two dimensional case were extended both upstream and downstream of the blades, the value of calculating this surface would be even more spectacular. The deficiencies of many techniques now in daily use would be clearly revealed.

The use of the dissipative force \( D \) in equation (3) of the paper has both a mathematical and a physical appeal; it explains that part of \( T S \) at any point that results from the local wall friction and Reynolds stresses. These forces are usually confined to regions very close to the surfaces of the flow boundaries, however. Moreover, they are functions of the local gradients in the flow. To properly incorporate the refinement of dissipative forces requires: (1) the use of very small grid spacings, especially adjacent to the flow surfaces; and (2) the knowledge of a rule for calculating \( D \) as a function of the flow gradients. If these features of dissipative forces are ignored, it may be more harmful to include \( D \) rather than exclude it.

D. JAPKE. The authors have reported a very careful study of the classical Wu \( S_i \) and quasi-three-dimensional calculation procedure. Several calculations have been performed for actual geometric configurations but without comparison to experimental data.

It often happens, when new numerical calculation procedures are employed, that distinct differences can be found by comparison to simpler calculation procedures. Yet, it may turn out that neither a simple or a more elegant calculation procedure brings our basic understanding closer to the real physical problem. Will the authors kindly consider in detail the recent experimental...
would render any two-dimensional based estimate inadequate of the loss mechanism as suggested by Bullock, would not improve from the calculation but it may be no improvement upon per­

sense it must be an improvement on the total omission of loss justifying the static pressure difference across the blade row to adjusting the static pressure difference across the blade row.

Otherwise it is not claimed that the physical model of the loss mechanism agreement more closely with experiment. In so doing it is anticipated has been improved. Such deficiencies as existed in the loss model which was formerly necessary input data. As far as the loss model is concerned, the method only claims to incorporate consistency both in the individual solutions and their combination, it is not claimed that the physical model of the loss mechanism has been improved. Such deficiencies as existed in the loss model of the original individual solutions will also appear in the combined solution.

The authors entirely agree with Bullock that the distribution of dissipative force across the entire flow field as a means of accounting for effects which are in fact confined mainly to boundary layers may not improve the detail of the results in some respects. This method does not model the loss mechanism, it merely provides a physically consistent empirical means of adjusting the static pressure difference across the blade row to agree more closely with experiment. In so doing it is anticipated that other flow features will also be in better agreement. In this sense it must be an improvement on the total omission of loss from the calculation but it may be no improvement upon performing a loss free calculation which includes boundary layer displacement effects on blade, hub and shroud surfaces. The authors feel that in many turbomachine flows, a refined model of the loss mechanism as suggested by Bullock, would not improve fine detail since the three-dimensional nature of the boundary layer and the main flow and their macroscopic mixing would render any two-dimensional based estimate inadequate for such detailed improvement. Improvements, to be meaningful at this level may require a full field, three-dimensional viscous solution to be effective.

The two-dimensional flows presented in the paper for comparison were performed for S2 streamsurfaces which were blade-like in shape and did not extend upstream and downstream of the blade row as inferred by Bullock. In regions outside the blade row the flow was treated as being axi-symmetric and therefore sustaining no tangential force.

Because of the geometric limitations of the S1 streamsurfaces to being surfaces of revolution and to a lesser extent the assumption that the S2 flow field can be adequately described by a single representative S2 streamsheet, the method is only likely to give improved results for machines in which such constraints are probable. This implied constraint of the S1 streamsurfaces is most likely to occur where the upstream vorticity is small and the blade loading light. The latter condition is common in axial flow pumps and low speed compressors, to a lesser extent in axial flow turbines and not at all likely in centrifugal compressors and radial turbines. One of the authors has applied a fully three-dimensional time marching program to the solution of flow through the Ricardo 'A' turbine mentioned in this paper. This solution reverses the gradient of meridional velocity found by the quasi-three-dimensional method here reported and shows good agreement with experimental slow response traverses. The flow leaving the blade suction side shows good qualitative agreement with detail of the traverse which is quite unobtainable from the quasi-three-dimensional solution. These observations are not surprising since the three-dimensional results indicate strong twisting of the S1 streamsurfaces and an S2 streamsurface which enters the blade channel at mid-pitch, twists to intersect the blade surfaces before reaching the trailing edge section. Clearly these conditions completely violate the assumptions underlying the quasi-three-dimensional calculation and its inadequacy in such a situation is obvious.

By the same token, the quasi-three-dimensional solution of flow through a centrifugal compressor will undoubtedly prove inadequate to the purpose. It was the authors' realization of the unsuitability of S1, S2 streamsheet techniques to such machines that motivated the work on three-dimensional time marching procedures. It may be interest to D. Japikse that the three-di­

mensional solution for reversed flow through the Ricardo 'A' turbine to simulate a centrifugal compressor demonstrates a strong jet-work pattern with 25 percent area at the suction shroud corner at blade exit being occupied by a low velocity wake with about 25 percent of the maximum remaining jet velocities. This pattern of flow, not unlike that reported by Eckardt, was obtained from a free slip solution with no imposed losses. Here again the three-dimensional solution indicates that the assumptions of the quasi-three-dimensional-method are completely violated. Because of this interesting result the three-dimensional program has been run with the Eckardt compressor geometry at Eckardt M1 flow condition, again without imposed loss and with a coarse grid dictated by computer store capacity, but in this case a much less significant jet-wake pattern emerged. There is no doubt that the quasi-three-dimensional program will not give satisfactory results for the type of machine but for those machine geometries to which it is applicable it is a numerically accurate method, whereas more work needs to be done on the three-dimensional approach to achieve comparable accuracy.

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*Acknowledgments* and *References* are omitted from this text. Please refer to the original paper for complete bibliographic information.