DISCUSSION

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The authors deal with the problem of computing the deflection of a surface caused by a pressure loading. In addition, an alternative relaxation procedure is introduced in the multilevel algorithm for solving Reynolds' equation. Both the new integration and relaxation techniques are of a high standard. Yet, it seems that these advanced techniques are applied in solving physical unrealistic problems, using needless dense and uniform grids.

With respect to the grid it may be observed that taking 10^{+5} or more gridpoints for a contact of 0.01 to 0.1 mm. gives a stepsize of a few Angström. Phenomenon on such scales is certainly not described by Reynolds' equation. So, why is the grid that dense?

Taking the parameter setting as in the paper, i.e., alfa = 1.7 10^{-8} Pa^{-1}, $p_0 = 1.98\ 10^{+8}$ Pa, $z = 0.68$, and maximum Hertzian pressures of 2, 4, or 8 GPa leads to viscosities up to the order of 1, 10^{+5}, and 10^{+15} GPa•s, respectively. These high values suggest the question: "What is the physical relevance of the presented results?"

More so since the viscous shear stresses in the fluid and at the fluid-solid contact boundaries also become extremely large. To illustrate this, note that

$$\eta \partial_y v \simeq \eta v_- / h \tag{11}$$

where y is the coordinate across the fluid film, v the velocity along the film and v_- the difference between the velocities of the surfaces. Evaluating (11) for $v_- = 1$ m/s, $\eta = 1$ GPa•s, and $h = 0.1$ micron gives a shear stress of approximately 10^{+7} GPa. It is unnecessary to say that this (tangential) loading is much larger than the pressure. Consequently, the deformation of the surfaces caused by the shear stresses cannot be neglected. A detailed analysis of the effect of the viscous stresses can be found in [22] or in a forth-comming paper [23]. In brief, the deformation caused by the shear stresses can be neglected compared to the deformation due to the pressure if $\eta v_- = \mathcal{O}(q^2) w$, where q denotes the quotient of the characteristic height and the characteristic length of the fluid domain.

References

22 Verstappen, R., "Elastohydrodynamic Lubrication: A Dynamic Variation Method," Ph.D. theses, University of Twente, Enschede, The Netherlands, 1989.

23 Verstappen, R., "A First Study of Shear Stress at an Elasto-Hydrodynamically Lubricated Contact," submitted to *Int. J. Engng. Sci.*

Authors' Closure

The authors wish to thank Dr. Verstappen for his careful reading of the paper and would like to make the following remarks:

- The authors agree with the discusser that a solution showing significant pressure variations at the scale of a few Ångström would, at least locally, be in contradiction with the assumption that the fluid behaves as a continuum on which Reynolds equation is based. However, the situation depicted by the discusser is somewhat exaggerated. In practical bearing applications the half width of the Hertzian contact varies from 0.1 to 0.5 mm. The size of the calculational domain is a number of times this half width and in most situations it is at least $O(1)$ mm. Assuming the most extreme situation of 10^5 nodal points on the full domain the resulting mesh size is still not "a few" Ångström.
This discussion on the physical relevance of the solution of the differential problem should, in our opinion, be viewed separately from the subject of accurate numerical solution of the problem. In our situation, the solution of the discretized equation is an $O(h)$ approximation of the solution of the continuous differential problem. Hence, in regions of large gradients, at least locally, a sufficiently small mesh size is required. This means that, using uniform grids, a relatively large number of nodes is needed. One might argue that a similar accuracy can be obtained using less nodes and a non-uniform grid. That argument however, does not hold in case of rough surfaces where only to describe the roughness profile already a considerable number of nodes will be required.
Nevertheless, solving the smooth surface stationary line contact problem with $O(10^5)$ nodal points gives results which are far more accurate than needed for practical applications. This large number of nodes should be seen as a demonstration that the presented techniques provide a solid basis for a fast solver of the point contact problem where such large numbers of nodes will be needed especially if surface roughness effects should be taken into account.
- In the paper it is explicitly stated that the solutions for extreme high loads such as 8.0 GPa are of little practical importance. In addition to the unrealistically high viscosities mentioned by the discusser one could for example mention that in case of steel surfaces beyond some 3.0 GPa plastic deformation will occur which is not taken into account in the model. Besides, at such high pressures the lubricant will most likely behave as a solid instead of a fluid.
To investigate the effects of surface roughness on pressure profile and film thickness in both stationary and transient situations will require a very stable algorithm. The extremely high loaded situations included in the paper should therefore be seen as a demonstration that the presented algorithm provides the possibility to carry out such studies. Another reason to include these situations was to demonstrate that solutions do exist. Hence, the problems with respect to the solution of highly loaded situations reported in the past were numerical problems.
- As far as shear stress calculations are concerned it is well known that, the use of the model should be limited to no slip conditions. When slip occurs, the model predicts unrealistically high shear stresses and coefficients of friction. According to the discusser, tangential deformation should be included in the model. If the purpose is to accurately predict shear stresses and coefficients of friction in practical situations the authors would recommend to take into account the effect of shear heating instead. The viscosity of the lubricant decreases significantly with increasing temperature and the resulting shear stresses will be small compared to the maximum pressure in the contact, e.g. [22].

Additional Reference

22 Ten Napel, W. E., Klein Meuleman, P., Lubrecht, A. A., Houpert, L., and Bosma, R., 1985, "Traction in Elastohydrodynamic Lubrication at Very High Contact Pressures," *Proc. 4th. European Tribology Conf. Eurotrib 1985*, Lyon, France.

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* Copies are available free; address the discusser.