J. KOTZUR. In the middle part of the flow ducts and for low rotational speeds the authors achieve a good coincidence with the measuring results by application of their calculation method. A comparison of the theoretical results with the results of actual measurements on rotating ducts shows that the greater the ratio of tip speed and flow speed in the duct, the less these results coincide, in particular near the duct outlet. This ratio is exactly the factor which determines the deflection angle of flow in the relative system after the flow has left the vane duct. The effect of this on the flow in the outer area of the vane duct can be seen, for instance, from the flow characteristics calculated by Stanitz (see enclosed Fig. from J. D. Stanitz and V. D. Prian "A Rapid Approximate Method for Determining Velocity Distribution on Impeller Blades of Centrifugal Compressors, 'NACA-TN 2421 (1951)).

When extending the calculation process, as the authors intend to do, they should consider this deviation of the direction of flow by reference to a known slip factor according to Busemann, in so far as this has not yet been done.

K. N. GHIA. The authors have successfully analyzed the complex problem of turbulent flow of an incompressible fluid in straight ducts of rectangular cross-sections, rotating about an axis normal to the longitudinal axis of the duct. However, it appears that the paper could be made even more valuable by including in it some essential details that seem to have been missed. With this additional information, the work could be more widely used by the readers. While most of the details are obtainable from previously published papers, a few important ones still remain unclear.

Of the two basic approaches for modelling turbulence near a solid wall, the wall-function method has been employed for the present problem. The agreement between the resulting solutions and the experimental data is so good for ducts of constant cross-section that Prof. Spalding owes it to the reader to explain the use of this approach more fully. For example, it would be useful to state the exact locations of the near-wall point P for the configurations studied. Also, it was suggested that, for flow developing rapidly in the streamwise direction, use must be made of the alternate method of modelling the low-Reynolds-number flow near the wall, employing differential equations. The extensive experience of Prof. Spalding in this area places him in an excellent position to comment on how rapid a flow development is rapid enough to warrant the use of this alternate method. Such comments would be very useful to other researchers in the field of turbulence.

At the time of presentation of this paper, Professor Spalding had briefly discussed some solutions obtained recently for turbulent flow in curved ducts, using a "partially parabolic" calculation procedure. It would be helpful to know what limitations, if any, Professor Spalding foresees on the problem parameters (Dean number for curved ducts and divergence and rotation for the ducts studied in the present paper) in order to obtain accurate results with the partially parabolic procedures.

Finally, it is suggested that the quantities \( T \nu \) and \( T \gamma \nu \) be defined in this paper. Similarly, definitions should also be included for the term "rotation number" and the symbol \( \dot{\gamma} \).

J. MOORE. The authors are attempting to calculate three dimensional flow using a finite difference marching integration scheme. Let us consider their attempts to calculate the developing flow in Moore's rotating, radial flow passage [6]. The most important influence on the flow development is the Coriolis acceleration (2\( \Omega \times \nabla \nu \)) and this is certainly included in the equations of motion which the authors attempt to solve. However, it is also necessary to ensure that the calculation scheme and its initial conditions allow Coriolis to act to modify the flow. The authors discuss the parabolic nature of the flow and the changes in the pressure field occurring through the flow. They attempt the calculation from an initial station where the fluid already has a Coriolis acceleration and therefore there is already a tangential pressure gradient, and yet they have no tangential pressure gradient as their initial condition (Fig. 12).

The equations attempt to allow low momentum boundary layer fluid to accelerate on the top and bottom walls in the tangential direction towards the suction side of the passage. Great care must be taken in the selection of grid points and the selection of "wall functions" if these secondary flows are to be computed with accuracy. The specific details of the authors’ calculation scheme could have prevented the calculated secondary flows being large enough to cause the observed thickening of the...
suction side boundary layer and thinning of the pressure side layer (Fig. 11).

Certainly there is room for more progress in the application of marching integration schemes to the calculation of turbomachinery flows.

**J. G. Moore**. Moore's flows [4, 6] are essentially "parabolic." That is \( \partial \delta p / \partial z \) and \( \partial \delta u / \partial z \) are approximately zero throughout the region of interest. A simple estimate of the pressure gradients across the flow may be obtained by making the very simple approximation \( u = v = 0, \tau = 0 \) and \( w \) is uniform in \( x \) and \( y \). Then by continuity \( w = \delta h / (x/L) \) where \( \delta h \) is the mass flow rate through the duct, and from equation (2), \( \partial \delta p / \partial z = -2\delta h \delta w \). Since the authors did not impose their artificial variation in the pressure gradient on the through-flow velocity calculation, but correctly considered the flow "parabolic," the velocity calculations should be unaffected.

Using an integral boundary layer method, Moore [10] successfully calculated the suction side "wake" by looking in detail at the secondary flows in the boundary layers on the four walls. The failure of the authors' method to calculate the suction side wake is probably a result of the numerical procedure. Possible problem areas include the wall function, the grid spacing and the high lateral flux modification.

**Roger Grundmann**? The comparison between the experimental and the numerical results gives a very good agreement. But looking at the governing equations it is to be seen that the centrifugal acceleration term appears in the momentum equation for the downstream direction only. Additionally this term is to be held constant for each cross-sectional plane, although it should be a function of two independent variables in the case of the cartesian coordinate system used here. As the centrifugal acceleration has a quadratical character it will influence the results near the side walls more than in the inner flow region. May be that this is an explanation to the differences in Figs. 6 and 7.

**Authors' Closure**

The authors greatly appreciate the useful comments made by the discussers.

**J. Moore**

The authors agree to Dr. Moore's remark about the existence of tangential pressure gradient \( (\partial \delta p / \partial z) \) at inlet, and it is present in their computations, as a consequence of having started from the inlet velocity distribution measured by the experimenters [4, 6, 7]. In fact, Fig. 12 represents the predicted tangential pressure distribution at different downstream stations; that for the inlet happens not to be plotted.

**J. G. Moore**

The authors do not agree with Dr. Moore's remark that Moore's [4, 6] flows are essentially "parabolic" throughout the region of interest. Especially in the rotating diffuser [6], the flow near the exit \( (e.g. \ z = 18.5 \text{ in., Fig. 11}) \) could be appreciably affected by the exit condition as the flow was discharged directly into the atmosphere. Under this condition the flow near the suction side experiences an adverse pressure gradient whereas on the pressure side the pressure gradient is much more favor-

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like (a); but it fails to predict the situation (b) or any intermediate situation.

With the partially-parabolic procedure, the present authors [17] have also been successful in predicting secondary flows which are in reasonable agreement with the expectation (Fig. 16) and with the measurements (Fig. 17) of Wagner and Velkoff [18].

K. N. Ghia

For the present set of computations the nearest to the wall point was located at a distance 5 percent of the duct width (e.g. \( x_B/B \approx 0.05 \)).

In the opinion of the authors, low-Reynolds-number turbulence modelling is not yet in a satisfactory state. It seems preferable in this region to concentrate on one-equation models; and the work of Hassid and Poreh [20] is suggestive.

The Dean number does not appear to be particularly relevant to the applicability of the partially-parabolic method; but the parabolic method should not be used when there are rapid changes in curvature, or rapid changes in Coriolis acceleration, with longitudinal distance.

J. Kotzur

The authors are in agreement with Dr. Kotzur’s comment that the deflection angle depends on the ratio of tip speed and average flow speed in the duct. However, the slip factor, which is a measure of the deflection angle, can be predicted accurately without reference to any empirical input either (a) by prescribing an appropriate exit boundary condition (Figs. 14 and 15), or (b) by extending the calculation domain into the diffuser scroll. The flow in the diffuser scroll can be appropriately handled by employing a periodicity condition in the direction of rotation in a way described by Singhal and Spalding [19].

R. Grundmann

Dr. Grundmann correctly points out the misrepresentation in the \( \theta \)-direction momentum equation which does not contain the component of centrifugal acceleration. However, the results are not affected because in computation the centrifugal acceleration was absorbed into the pressure field by redefining the pressure as:

\[
P = p - \frac{1}{2} \rho r^2 \Omega^2
\]

where \( r \) is the distance from the axis of rotation.

Therefore an incompressible flow calculation is unaffected by the centrifugal acceleration.

Additional References


The Calculation of Turbulent Boundary Layers on Spinning and Curved Surfaces

P. BRADSHAW

It is not clear from this very interesting paper whether the authors would recommend applying the same altera-

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