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DISCUSSION

K. Komvopoulos[1]

The authors are to be commented for presenting an interesting analysis of elastic contact of rough surfaces. The solution is based on the classical Hertz theory and the representation of a smooth cylindrical macrocontact with equally spaced cylindrical microcontacts of a smaller radius. Results are presented for the contact pressure and subsurface stress field in terms of the load, microcontact pitch distance, and coefficient of friction. There are a few points, however, that require further clarification.

(1) The results presented are for plane-strain conditions. Have the authors checked whether this condition, as well as the assumptions invoked in the Hertzian theory (e.g., indentation depth vs. asperity contact width and radius), have been fully satisfied at the asperity scale in all cases presented?

(2) The assumed microcontact radius of curvature of 1 mm is quite large for the present treatment to be applicable to the asperity dimensions. Also, the asperity distance cannot be considered constant at the micrometer-scale, unless all high-frequency components have been filtered out from the digidized signal of the surface profile. Was this considered in the present analysis? What is this cut-off frequency for grinding (Fig. 1)? How is it dependent on the abrasive grit size?

(3) Similar results have been presented in other studies (Dundurs et al., 1973; Komvopoulos and Choi, 1992), and it would be desirable if the authors could provide some comparisons, in particular, with respect to the effect of the macrocontact vs. asperity radius and reference to similar trends. For example, it has been shown that for a flat macrocontact the stress field is intensified at the edge microcontacts (Komvopoulos and Choi, 1992), while in the present study due to macrocontact curvature effects the higher subsurface stresses arise near the center of the macrocontact. It would be of interest, therefore, to interpret what would be the effect of sliding friction in the magnitude and location of the maximum effective stress as a function of the macrocontact and asperity radii.

(4) The effective stress for a Hertzian macrocontact in Fig. 10 is relatively constant and does not seem to reach a maximum value at the expected depth $y = 0.7a$.

(5) The interpretation of surface versus subsurface flow, including plastic deformation and fracture, is speculative since the present analysis is for purely elastic deformation. Strain hardening effects introduced due to sliding and machining (e.g., grinding) would change significantly the material behavior and, hence, the location and type of deformation. For instance, recent finite element results have shown significant effects of strain hardening on subsequent yielding characteristics and the accumulation of plasticity for frictionless interfaces (Kral et al., 1993). Furthermore, the present treatment is not applicable to media at their elastic shakedown limit because strain hardening of the surface would result in a layered half-space. Consequently, the present results should be interpreted in the context of initial yielding.

(6) The results for high friction coefficients cannot be rationalized in the presence of purely elastic material behavior. According to the experimental evidence, friction coefficients as high as 0.4, 0.6, and 0.8 (Figs. 6–8) cannot be produced from elastically deforming sliding surfaces. Such high friction values are normally accompanied with energy dissipation in the form of significant heating, inelastic material deformation, fracture and wear.

[1]Department of Mechanical Engineering, University of California, Berkeley, CA 94720.

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Authors' Closure

The authors thank Prof. Komvopoulos for his interest in our paper. We note that the last paragraph of the Introduction of Komvopoulos and Choi (1992) begins by stating that "it is apparent that information about the interactions arising at multiasperity contacts is relatively sparse." As detailed in the Introductions of the present paper and Komvopoulos and Choi (1992) one difficulty with drawing general conclusions in the rough surface contact problem is the interaction of global component size length scales, such as the cylinder radii, and local asperity size length scales, such as radius of curvature and asperity spacing. At Purdue we have developed an interest in this problem partially through our research into superabrasive grinding of hardened steel surfaces and the resulting effect of grinding parameters on rolling contact fatigue life. Ground surfaces are particularly appealing for the rough surface contact problem since their surface topography is anisotropic, allowing for a two-dimensional plane strain approximation of the local asperity sized contacts.

Prof. Komvopoulos asks that several enumerated points be clarified. The following enumerated clarifications correspond to his numbered queries.

1. We assume that each localized asperity contact results in a semi-elliptical Hertzian pressure distribution where the deflection at the center of each asperity is the sum of deflections caused by the effect of pressure applied to each asperity. This localized pressure distribution is symmetric about the centerline