Dilatancy and the Strength of Rocks containing Pore Water under Undrained Conditions

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Summary

Experiments have been carried out at atmospheric temperature and confining pressures up to 5 kb in which rocks, of different porosities, and fitted with flexible jackets, have been deformed up to 6 per cent axially while containing pore water under undrained conditions. Application of a confining pressure and axial compressive stress creates a pore pressure, which weakens and embrittles the rocks under these conditions. The initial pore pressure may take any value up to that of the confining pressure, depending on the amount of water available inside the jacket and sample. Dilatancy produced by crack propagation during shear deformation resulted in dilatancy hardening which prevented or delayed mechanical instability (as manifested by stress drops). The greatest amount of dilatancy occurred during the fracturing process, but dilatancy decreased as the effective confining pressure increased. Only a small amount of dilatancy occurred during subsequent movement on faults, and the dilatancy tended to reach a stable value at deformations of ~5 per cent (~1.5 mm of movement on a major fault surface). There is evidence of small quasicyclic variations of dilatancy at greater deformations, which produce small stress drops of ~50 bar. The implications for crustal faulting and earthquake premonitory effects are discussed.

1. Introduction

The significance of the effect of pore pressures on the strength and stability of consolidated rock has only been considered comparatively recently. Hubbert & Rubey (1959) proposed that pore pressures played an important part in the mechanics of overthrust faulting, and Griggs & Handin (1960) suggested that pore pressures could contribute significantly to the mechanics of earthquake faulting. Subsequently Frank (1965) (see also Frank 1966; Orowan 1966a, b) gave a detailed and perceptive discussion of this topic and dealt with the important role of dilatancy. A connection has since been established between underground fluid pressures and the occurrence of certain earthquakes in the Denver area (Colorado, USA) (Healey et al. 1968), and in Rangeley (Colorado, USA) (Raleigh, Healey & Bredehoeft 1972). More recently still evidence has been produced to show that the recurrence intervals of earthquakes may be controlled by the time taken for fluids to flow into the earthquake source region after faulting episodes which caused a localized reduction in pore pressure (Nur &

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A number of experimental and theoretical investigations have been made of the effect of pore pressures on the deformation and fracture of rocks. In most cases experiments were carried out with the sample connected to a reservoir of fluid maintained at a fixed pressure, so that at sufficiently low strain rates, or in rocks with a sufficiently high permeability to the fluid the pore pressure remained at a fixed value during an experiment. This we may call a 'drained' condition. (See Robinson 1959; Heard 1960; Handin et al. 1963; Boozer, Hiller & Serdengecti 1963; Murrell 1966; Brace & Martin 1968; Byerlee & Brace 1972). Murrell (1963, 1966) and Handin et al. (1963) showed that an effective stress law was obeyed in these experiments, where effective principal stress equals the difference between the applied principal stress and the pore pressure (taking compressive principal stresses as positive). It was shown by Murrell (1964) and Murrell & Digby (1970) that the effective stress law was a corollary of Griffith's theory of brittle fracture (see also Jaeger & Cook 1969).

The obverse of the 'drained' condition is the 'undrained' condition, in which the pore fluid remains trapped in the rock. The degree to which pore fluids can drain from a natural rock clearly depends on the permeability of the surrounding rocks to the fluid and this has been extensively discussed by Heard & Rubey (1966), Bredehoeft & Hanshaw (1968), Hanshaw & Bredehoeft (1968) and Smith (1971). The 'undrained' condition may be regarded as the extreme case of very low or zero permeability in the surrounding rock.

Undrained conditions have been used by Raleigh & Paterson (1965), Heard & Rubey (1966) and Ismail (1974) in their experimental studies of the strength of rocks undergoing chemical decomposition by loss of water at elevated temperatures and pressures. In addition, some isolated experiments on the effect of water or solutions under undrained conditions have been reported by Griggs et al. 1951, 1953; Lane 1969a, b; Robertson 1972; Goodman & Ohnishi 1973. Aldrich (1969) has reported a series of experiments on the deformation and fracture of the highly porous Berea sandstone (porosity ~ 19 per cent—Bruhn 1972) under undrained conditions at low confining pressures (up to ~0.5 kb), and this has been followed up by Bruhn (1972) with a series of very detailed experiments on the same material, in which total strains and pore pressures were also measured during the experiments. However, the strains were restricted to ~1 per cent, and the behaviour after fracture was not studied. Bruhn made a comparison of the behaviour of Berea sandstone with that of soils under similar test conditions, and showed that it was similar.

The present paper describes a series of undrained experiments, with distilled water as the pore fluid, carried out at atmospheric temperature and elevated pressures (up to 5 kb) on samples of sandstone, serpentinite, and dunite, which have mean porosities of 11.41, 4.23 and 0.54 per cent, respectively. Axial shortening was continued until it reached 6 per cent, so that the post-fracture behaviour could be studied.

The sandstone was from the same source as that used by Murrell (1966) in his experiments in which drained conditions were used. The choice of the other two rocks was determined by the fact that, concurrently with the present series, experiments were being carried out on the effect of pore pressures arising from decomposition of serpentinites at elevated temperatures.

2. Experiments

2.1. Technique

The rock deformation apparatus used was the one referred to in a previous paper (Edmond & Murrell 1972), which is at present operating at pressures up to 8 kb.
High temperature 'undrained' experiments carried out with the same apparatus will be described in later papers.

The accuracy of pressure control is \(~1\) per cent and pressure is measured to an accuracy of \(\pm 35\) b. Differential stresses are measured with an accuracy of \(\pm 20\) b, and strains are measured with an accuracy of \(~0.01\) per cent. The strain rate used in the experiments was \(~10^{-5}\) s\(^{-1}\).

The rock specimens used were cylinders of diameter 1 cm and length 3 cm cut from single blocks of rock and with a fixed orientation. They were provided with rubber jackets which were a close fit on the specimen and on the steel rams by means of which the axial deformation force was applied. This technique has been extensively used and leakages are readily recognized. There was no evidence of leakage in the experiments reported here.

2.2 Preparation of samples for testing

The specimens were oven dried at 105°C and stored in a desiccator for about 24 hr, after which they were weighed. They were then placed in a thick-walled flask which was evacuated for about 6 hr by means of a rotary vacuum pump, after which distilled water was allowed to enter the flask and saturate the rock. The saturated specimens were then weighed again.

Following this the specimens were jacketed and assembled onto the upper steel ram, and accurately measured quantities of distilled water were injected between the jacket and the saturated rock specimen by means of a micrometer pipette. Quantities up to 500 \(\mu\)l can be injected, and the volume is measured to the nearest \(0.1\) \(\mu\)l. In some experiments the rock sample was not saturated with water before assembly and quantities of water less than those required for saturation were injected.

2.3 Rocks tested

- **Sandstone**: this came from the same source (Darley Dale, Derbyshire) as the rock used by Murrell (1966) in earlier experiments. It consists of \(~75\) per cent quartz (with little or no fracturing), \(~15\) per cent feldspar (plagioclase and microcline), and \(~10\) per cent minor constituents. The rock is poorly-graded (grain diameters 80–800 \(\mu\)m) and the cementing material is siliceous.

- **Serpentinite**: this was collected from Oulx (Dora Riparia, Italy). It consists of \(~90\) per cent serpentinite (lizardite and chrysotile) and \(~8\) per cent magnetite. There are two sets of talc veins inclined at about 45° to each other, but the serpentine crystals are randomly oriented and show no obvious fracturing.

- **Dunite**: this came from the collection used by Murrell & Chakravarty (1973). It originated in Norway, and is composed mainly of olivine with a small degree of preferred orientation, together with some enstatite. There are a few very narrow and irregular veinlets of sheared talc between some of the olivine and pyroxene grains.

<table>
<thead>
<tr>
<th>Rock</th>
<th>Mean porosity (per cent)</th>
<th>No. of specimens</th>
<th>Porosity range (per cent)</th>
</tr>
</thead>
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<td>10</td>
<td>0.36–0.74</td>
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<tr>
<td>Serpentinite</td>
<td>4.23</td>
<td>10</td>
<td>3.98–4.66</td>
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<tr>
<td>Sandstone</td>
<td>11.41</td>
<td>12</td>
<td>10.95–12.15</td>
</tr>
</tbody>
</table>
FIG. 1. Undrained tests on sandstone at a confining pressure of 1.10 kb, and with the following volumes of pore water: (a) zero, (b) 170 μl, (c) 264 μl (saturated), (d) 316 μl, (e) 325 μl, (f) 363 μl. Note: in all stress–strain curves in this paper strain in the rock specimen is to be measured from the axis inclined to the vertical in the diagram. The strain measured from the vertical axis includes the strain in the deformation apparatus.

FIG. 2. Undrained tests on serpentinite at a confining pressure of 2.41 kb, and with the following volumes of pore water: (a) zero, (b) 95.0 μl, (saturated), (c) 104.2 μl (saturated), (d) 147.3 μl, (e) 210.1 μl.
Fig. 3. Undrained tests on serpentine at a confining pressure of 4.83 kb, and with the following volumes of pore water: (a) zero, (b) 100.2 μl (saturated), (c) 118.9 μl, (d) 148.8 μl, (e) 199.8 μl, (f) 295.8 μl.

Fig. 4. Undrained tests on dunite at a confining pressure of 4.83 kb, and with the following volumes of pore water: (a) zero, (b) 11.3 μl (saturated), (c) 42.4 μl, (d) 66.8 μl, (e) 117.3 μl, (f) 212.3 μl.
2.4 Porosity of rocks

The volume of pores was measured by weighing the rock samples when dry and after saturation with distilled water, as described above. The volume of the rock sample itself was obtained by measuring its diameter and length with a micrometer. The values obtained for the porosity are given in Table 1. The measured variability in porosity is ~4 per cent.

3. Results

Examples of the observed stress–strain behaviour are shown in Figs 1–4. The strain is determined from the axial shortening of the specimen, and after fracture this is largely caused by sliding on the macroscopic fracture surface (or surfaces) rather than by macroscopically homogeneous deformation. In Tables 2–4 are shown the strength parameters observed in each experiment (fracture stress or stress at 2 per cent axial deformation, stress after 4 per cent axial deformation, and stress drop). In the majority of cases the specimens failed by the formation of a brittle shear fault, though in some cases, at low effective confining pressures splitting parallel to the direction of greatest compression also occurred. In several sandstone and dunite specimens (see Tables 2 and 4) fractures perpendicular to the axis of shortening (greatest compression) also occurred. These probably formed during unloading of the specimen (cf. Murrell 1966). In several dunite specimens tested at high effective confining pressures failure intermediate between shear faulting and cataclastic flow was observed, and the sandstone specimen tested at an effective confining pressure of 4.14 kb (specimen DS.14) exhibited cataclastic flow. An important feature of the observations was the occurrence of dilatancy hardening (Frank 1965, 1966), which occasionally obscured the fact that faulting had occurred (see Figs 2(b), 3(c), 4(d)) and sometimes led to faulting which was distributed throughout a wide zone rather than being concentrated on one or two major shear faults. These matters will be discussed in detail below.

3.1 Effect of pore water on strength

After the initial elastic deformation two types of behaviour were observed. With increasing strain, either the strength of the rock passed through one or more maxima (and minima with concomitant stress drop), which is typical of brittle faulting, or else the strength increased continuously, though with occasional inflections in the stress–strain curve (see Figs 1(c), 2(b), 3(b), (c), 4(c), (d)). For the purpose of illustrating the effects of increasing volumes of pore water on strength we take values of the strength at the first fracture (i.e. the first maximum in the stress–strain curve) in the former case or the strength at 2 per cent strain in the latter (see column 4 in Tables 2–4). These values of strength are plotted as functions of pore water content (expressed as a percentage of the specimen volume) in Figs 5–7. It is seen that there is a sharp drop in the strength as soon as the rock is saturated with water (that is, when the volume per cent of water equals the rock porosity).

As the effective confining pressure increases the strength increases, and above a certain pressure there is a transition from brittle behaviour to ductile behaviour associated with cataclastic flow. The effect of pore water on the ductility of Oulx serpentinite is shown as a function of confining pressure in Fig. 8.

The main cause of the changes in strength produced by pore water in these experiments is the induced pore water pressure, although this has not been directly measured. This pore pressure is created by the hydrostatic confining pressure initially applied to the rock samples, together with the differential stress subsequently applied. This is discussed in detail in the next section.
Table 2

Experimental results. Sandstone*

<table>
<thead>
<tr>
<th>Expt.</th>
<th>$P_s$ (kb)</th>
<th>$V_o$ ($\mu l$)</th>
<th>$V_i$ ($\mu l$)</th>
<th>$R$ (kb)</th>
<th>$R_{14}$ (kb)</th>
<th>$\Delta R$ (kb)</th>
<th>ECP (kb)</th>
<th>PPF (kb)</th>
<th>ECP4 (kb)</th>
<th>PP4 (kb)</th>
<th>Def4(%)</th>
<th>Diff(%)</th>
<th>Def.</th>
</tr>
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<td>0</td>
<td>-</td>
<td>0.36</td>
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<td>0</td>
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<td>0</td>
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</tr>
<tr>
<td>DS. 28</td>
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<td>274.4</td>
<td>264.4</td>
<td>0.62</td>
<td>0.45</td>
<td>0.25</td>
<td>0.07</td>
<td>0.07</td>
<td>0.10</td>
<td>0.04</td>
<td>0.012</td>
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<td>0.35</td>
<td>0.35</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Sh.</td>
</tr>
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<td>150.0</td>
<td>-</td>
<td>1.49</td>
<td>1.22</td>
<td>0.33</td>
<td>0.31</td>
<td>0.04</td>
<td>0.33</td>
<td>0.02</td>
<td>0.006</td>
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<td>Sh.</td>
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<tr>
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<td>265.4</td>
<td>0.96</td>
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<td>0.17</td>
<td>0.18</td>
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<td>258.0</td>
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<td>0.05</td>
<td>0.01</td>
<td>0.34</td>
<td>0.14</td>
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<td>0.10</td>
<td>0.01</td>
<td>0.34</td>
<td>0.08</td>
<td>0.27</td>
<td>0.041</td>
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<tr>
<td>DS. 7</td>
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<td>0</td>
<td>-</td>
<td>2.89</td>
<td>2.00</td>
<td>0.95</td>
<td>0.69</td>
<td>0</td>
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<td>-</td>
<td>-</td>
<td>Sh.</td>
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<tr>
<td>DS. 27</td>
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<td>318.7</td>
<td>273.7</td>
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<td>0.02</td>
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<td>-</td>
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<td>3.00</td>
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<td>1.10</td>
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<td>1.10</td>
<td>0</td>
<td>-</td>
<td>-</td>
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<tr>
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<td>-</td>
<td>2.97</td>
<td>2.89</td>
<td>0.08</td>
<td>0.76</td>
<td>0.34</td>
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<td>0.04</td>
<td>0.087</td>
<td>-</td>
<td>Sh.Pf.</td>
</tr>
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<td>264.3</td>
<td>1.22+</td>
<td>1.67</td>
<td>0.15</td>
<td>0.24</td>
<td>0.68</td>
<td>0.53</td>
<td>0.57</td>
<td>0.124</td>
<td>0.101</td>
<td>Sh.Fr.Spl.</td>
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<td>316.4</td>
<td>286.4</td>
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<td>0.98</td>
<td>0.05</td>
<td>0.12</td>
<td>0.98</td>
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<td>1.275</td>
<td>Sh.</td>
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<td>324.7</td>
<td>274.7</td>
<td>0.59</td>
<td>0.62</td>
<td>0.13</td>
<td>0.06</td>
<td>1.04</td>
<td>0.15</td>
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<td>0.028</td>
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</tr>
<tr>
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<td>362.7</td>
<td>0.21</td>
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<td>0.01</td>
<td>0</td>
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<td>-</td>
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<td>6.88</td>
<td>-</td>
<td>4.14</td>
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<td>-</td>
<td>-</td>
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<tr>
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<td>1.56</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
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<td>-</td>
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<td>-</td>
<td>-</td>
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<td>Fr.</td>
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* For explanation of column-headings see footnote to Table 4.
Table 3

Experimental results. Serpentinite*

<table>
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<tr>
<th>Expt.</th>
<th>$P_0$ (kb)</th>
<th>$V_w$ (nl)</th>
<th>$V_s$ (nl)</th>
<th>$R$ (kb)</th>
<th>$R_{14}$ (kb)</th>
<th>$\Delta R$ (kb)</th>
<th>ECPF (kb)</th>
<th>PPF (kb)</th>
<th>ECPF (kb)</th>
<th>PP4 (kb)</th>
<th>Def4(%)</th>
<th>Diff(%)</th>
<th>Def.</th>
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<td>2</td>
<td>0·62</td>
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<td>0</td>
<td>0</td>
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<td>---</td>
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<td>Fr.</td>
</tr>
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<td>0·69</td>
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<td>0·58</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>---</td>
<td>---</td>
<td>Sh.</td>
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<td>95·0</td>
<td>4·56+</td>
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<td>1·38</td>
<td>1·78</td>
<td>0·63</td>
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</tr>
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<td>104·2</td>
<td>3·94</td>
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<td>0·69</td>
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<tr>
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<td>97·3</td>
<td>1·60</td>
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<td>2·35</td>
<td>0·52</td>
<td>1·89</td>
<td>0·069</td>
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<td>0·05</td>
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<td>11</td>
<td>4·14</td>
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<td>93·9</td>
<td>3·56+</td>
<td>4·45</td>
<td>0</td>
<td>0·30</td>
<td>4·53</td>
<td>1·69</td>
<td>3·14</td>
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<td>3·91</td>
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<td>4·77</td>
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<td>0·100</td>
<td>7·40</td>
<td>Sh.Fr.</td>
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* For explanation of column headings see footnote to Table 4.
### Table 4

*Experimental results. Dunite*

<table>
<thead>
<tr>
<th>Expt.</th>
<th>$P_c$ (kb)</th>
<th>$V_w$ (μl)</th>
<th>$V_d$ (μl)</th>
<th>$R$ (kb)</th>
<th>$R_{44}$ (kb)</th>
<th>$\Delta R$ (kb)</th>
<th>ECPF (kb)</th>
<th>PPF (kb)</th>
<th>ECP4 (kb)</th>
<th>PP4 (kb)</th>
<th>Df4(%)</th>
<th>Dif(%)</th>
<th>Def.</th>
</tr>
</thead>
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<td>—</td>
<td>—</td>
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<td>1.59</td>
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<td>2.57</td>
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<td>108.6</td>
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<td>2.41</td>
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</tr>
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<td>—</td>
<td>—</td>
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<td>—</td>
<td>—</td>
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<td>12.2</td>
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<td>117.3</td>
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<td>212.3</td>
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<td>0.72</td>
<td>4.11</td>
<td>0.117</td>
<td>7.38</td>
<td>Sh.Fr.Pf.</td>
</tr>
</tbody>
</table>

* $P_c$: confining pressure;  
* $V_w$: volume of pore water to saturate rock sample before deformation;  
* $R_{44}$: differential stress at 4 per cent axial deformation (residual frictional strength);  
* PPF: pore pressure at fracture;  
* PP4: pore pressure at 4 per cent axial deformation;  
* Df: volumetric dilatation before fracture;  
* Sh: major shear fault;  
* Shz: shear zone (minor shears);  
* Pf: fractures perpendicular to specimen axis;  
* Cat: cataclastic flow  

$V_w$: volume of pore water;  
$\Delta R$: differential stress at fracture or at 2 per cent strain (+);  
ECPF: effective confining pressure at fracture;  
ECP4: effective confining pressure at 4 per cent axial deformation;  
Df4: volumetric dilatation between fracture and 4 per cent deformation;  
Def: description of deformation;  
Shc: conjugate shear faults;  
Spl: split parallel to specimen axis;  
Fr: specimen partially or wholly fragmented.
3.2 Induced pore pressure

In the present experiments the pore pressures were created by the application first of all of a hydrostatic confining pressure to the rock samples contained in their flexible jackets, and subsequently by axial shortening of the specimen produced by the application of a differential axial compressive stress. In the experiments reported by Aldrich (1969), on the other hand, the initial confining pressure and the initial pore water pressure were built up independently to pre-determined values, and the pore pressure was measured throughout the experiment. In order to achieve this a pumping system, pressure gauge and valve had to be connected to the jacketed specimen. Earlier experience had suggested to us that with the relatively small samples used by us, and especially with rocks of low porosity, changes of volume and pressure external to the rock specimen (for example, in tubing and pressure gauge) and possible leakages of fluid (even in small amounts) which might be associated with the use of the experimental arrangement described by Aldrich made it inadvisable to use such an arrangement in our own experiments. Aldrich and his colleagues appear to have been able to overcome these problems, in part by using large specimens of highly-porous rock.

In our experiments, although the pore pressure is not otherwise directly controlled, the use of a flexible jacket on the specimen ensures that the pore pressure does not exceed the confining pressure.* If the amount of water added to the jacketed specimen exceeds the amount required to make the effective confining pressure zero (when the induced pore pressure equals the confining pressure) then the excess water forms a reservoir between the jacket and the specimen. As long as this reservoir remains the pore pressure equals the confining pressure, and the experiment is a drained one. However, although in several experiments the effective confining pressure at fracture was estimated to be zero, further axial shortening always resulted, because of dilatancy, in a reduction in the estimated pore pressure with concomitant increase in the effective confining pressure. It appears, therefore, that the experiments remained essentially in the undrained category.

During the application of the hydrostatic confining pressure the rock specimens undergo a reduction in volume, which raises the pressure of the pore fluid from zero to some initial value which depends on the confining pressure, the value of $V_w/V_s$ (where $V_w$ is the volume of water in the jacketed sample, and $V_s$ is the volume of water required to saturate the rock specimen), and on the elastic properties of the rock and the geometry of the pores and cracks in the rocks.

If

$$
(V_s/V_w) < V_{pe}
$$

(1)

where $V_{pe}$ is the specific volume of water at the confining pressure $P_c$ (and the temperature of the experiment) then the initial pore pressure will certainly equal the confining pressure. This was the case in all our experiments other than those in which the specimens were simply saturated (or, in two cases, undersaturated) with water.

Application of a differential axial compressive stress at first causes a further reduction in volume of the rock. In experiments in which the inequality (1) above pertains, water will be squeezed out of the rock in this stage of the rock deformation and the pore pressure will tend to remain equal to the confining pressure. However, in experiments in which the initial pore pressure is less than the confining pressure there is a further increase in the pore pressure. At a certain stage crack propagation is

* However, this point suffers from uncertainty about the speed with which variations in pore pressure between one part of a specimen and another can be smoothed out. The work of Brace & Martin (1968) suggests that in the case of our low porosity dunite a strain rate of $10^{-7}$ s$^{-1}$ would be required to maintain a uniform pressure of the water inside (and outside) the rock specimen. Nevertheless, the experimental results for the dunite do not indicate any significant difference in behaviour between this and the other rocks. In the case of the other two rocks the porosity was high enough to allow uniform pore pressures to be maintained at the strain rate of $10^{-5}$ s$^{-1}$ which was used.
initiated in the rock (usually when the differential stress reaches about half the ultimate strength of the rock) and at low enough effective confining pressures further axial shortening result in an increase in volume (Handin et al. 1963; Murrell 1966; Brace, Paulding & Scholz 1966; Lane 1969a; Edmond & Paterson 1972). The latter phenomenon is known as dilatancy. In the present undrained tests dilatancy results in a reduction of the pore pressure, and an increase in the effective confining pressure, once any water trapped between the jacket and the rock specimen has been forced back into the specimen.

Dilatancy occurs as brittle fracture is approached, but as the effective confining pressure is increased the proportion of dilatation which occurs before fracture decreases relative to the prior volumetric compression (Brace et al. 1966). The experiments of Handin et al. (1963) and Edmond & Paterson (1972) show that when the confining pressure reaches or exceeds a value at which unstable brittle faulting no longer occurs (that is, when there is no peak in the stress–strain curve in drained tests) then there is no net volumetric dilatation during the deformation, although in very porous sandstones the porosity may pass through a minimum value at a strain of 10–20 per cent. In undrained tests under such conditions pore pressures would increase throughout the deformation until a minimum porosity was attained.
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FIG. 6. Strength (at fracture or 2 per cent strain) of serpentine in undrained tests, as a function of pore water volume ($V_w, \mu l$), at different confining pressures: (A) 4.83 kb, (B) 2.41 kb.

4. Discussion of results

Aldrich (1969) and Bruhn (1972) have showed, in experiments on Berea sandstone, that in terms of effective stresses the strength at fracture is the same in both drained and undrained tests. This indicates that the specimens were in the same structural state at fracture. We did not measure the pore pressures developed in our experiments, but we assume that both the fracture strength and the residual strength (that is the stress required to cause sliding on the fault developed at fracture) depend only on the effective stresses (for a given pore fluid—a slight effect due to the surface activity of the fluid has been noticed by several workers, Boozer et al. 1963). Using this assumption we have estimated the effective confining pressure and the pore pressure, at fracture, and after axial shortening by 4 per cent, by comparing the stress–strain curves obtained in experiments with specimens dried at 105°C (the latter experiments will be reported separately), or, in the case of the Darley Dale sandstone, with specimens tested under drained conditions. The estimated values of the effective confining pressures are given in Tables 2–4.

4.1 Elastic deformation

Although the strength of a given rock apparently depends mainly on the effective stresses (and only in a minor way on the properties of the pore fluid) the elastic
behaviour before fracture or cataclastic flow takes place depends not only on the effective confining pressure (which changes in a systematic way with deformation in an undrained test) but also on the compressibility of the fluid contained in the pores. It is well known that pores and cracks cause a characteristic non-linearity of the elastic behaviour of rocks. As a result the shape of the stress–strain curve observed before fracture takes place at a given value of the effective confining pressure, depends on whether the rock is tested dry (with air in the pores) or under drained or undrained conditions (with water in the pores). This can be clearly seen in the results of Aldrich (1969). We have also observed this in our own experiments, although it has not been studied in detail. In Table 6 we show the values of Young's Modulus (tangent values measured in the central, linear sections of the stress–strain curves) for dunite tested at a confining pressure of 4.83 kb under undrained conditions, and compare them
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Fig. 8. Ductility of serpentinite as a function of confining pressure and volume of pore water in undrained tests.

with values for the dry rock tested at effective confining pressures equal to those which existed at fracture in the undrained specimens. There is a considerable reduction of the modulus in the undrained tests. Similar differences are illustrated for sandstone and dunite in Figs 9 and 10.

4.2 Strength, in undrained tests, of Darley Dale sandstone

Murrell (1966) carried out drained tests on this rock over a range of confining pressures and pore pressures, and in particular obtained sets of measurements of the fracture strength at confining pressures of 0.35 kb and 1.10 kb, respectively and with

<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>$V_\omega/V_\nu$</th>
<th>ECPF</th>
<th>$E$</th>
<th>$E$(dry)*</th>
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<tr>
<td>U.21</td>
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<td>4.83</td>
<td>1.15</td>
<td>1.15</td>
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<td>0.69</td>
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<td>0.67</td>
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<td>0.43</td>
</tr>
<tr>
<td>U.29</td>
<td>16.6</td>
<td>0</td>
<td>0.05</td>
<td>0.15</td>
</tr>
</tbody>
</table>

$V_\omega/V_\nu$ ratio of volume of water to volume required to saturate specimen

ECPF effective confining pressure at fracture

$E$ Young's modulus (tangent)

* Young's modulus of dry rock at confining pressure equal to ECPF.
Dilatancy and the strength of rocks

Table 6

Estimated pore pressure and strength at fracture, as a function of confining pressure, when the volumetric dilatation at fracture is 15 per cent of the initial porosity.

<table>
<thead>
<tr>
<th></th>
<th>Initial porosity (per cent)</th>
<th>Porosity at fracture (per cent)</th>
<th>Confining pressure (kb)</th>
<th>Pore pressure at fracture (kb)</th>
<th>Differential stress at fracture (kb)</th>
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</tr>
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<td>0.34</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>11.41</td>
<td>13.1</td>
<td>1.10</td>
<td>1.05</td>
<td>0.67</td>
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</tbody>
</table>

various pore pressures. These measurements are plotted in Figs 11 and 12. In the present, undrained, experiments it was found that with a confining pressure of 1.10 kb the fracture strength fell to a value equal to the uniaxial compressive strength with a volume of pore water lying between 325 µl and 363 µl (experiments DS.21 & 22, Table 2). We assume that the pore pressure in these circumstances is 1.10 kb, and we take the corresponding volume of pore water as 335 µl. If we further assume that the pore pressure induced at a given confining pressure, when a specimen is loaded to the point of fracture, is proportional to the volume of pore water then we may compare the results of drained and undrained tests in the same diagram, plotting the fracture strength as ordinate and the pore pressure or the corresponding pore water volume as abscissa. This is done in Fig. 11, from which it can be seen that the assumption of proportionality between pore-pressure and pore-water-volume is a good one. Further support for this is obtained from undrained tests at a confining pressure of 0.35 kb,

![Fig. 9. Comparison of elastic modulus of sandstone, when dry with modulus when initially saturated with water and tested at different confining pressures under undrained conditions. (a) dry, 0.17 kb; (b) saturated, 0.35 kb; (c) saturated, 1.10 kb. (Note: effective confining pressures are all similar.)](https://example.com/fig9.png)
in which the fracture strength fell to a value equal to the uniaxial compressive strength, indicating that the induced pore pressure was 0.35 kb, when the volume of pore water was 288.0 $\mu l$ (experiment DS.25, Table 2). Drained and undrained tests with a confining pressure of 0.35 kb are compared in Fig. 12.

4.3 Dilatancy

In experiments on rocks which are dry or in the drained condition (pore pressure constant and uniform throughout the specimen) brittle faulting is followed by a drop in stress to a value which can be supported by friction on the fault surface, and typically frictional strength is independent of strain (though under some conditions stick-slip occurs). A good example of such a stress–strain curve is shown in Fig. 13. At higher confining pressures large cataclastic deformations occur accompanied by strain-hardening, and faulting accompanied by a stress-drop does not occur at small strains.
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Fig. 12. Comparison of fracture strength of sandstone at confining pressure of 0.35 kb under drained (constant pore pressure) and undrained (constant pore-water volume) conditions. ○, drained experiments (Murrell 1966); △, undrained experiments (this work).

The strength under these conditions is high. An example of a stress–strain curve associated with cataclastic flow is shown in Fig. 14(a).

In undrained tests, however, faulting may not be accompanied by a detectable stress-drop, and the stress may increase steadily with strain, as in Fig. 14(b). Even when a stress-drop occurs there is a general tendency for the stress to increase steadily with strain during later stages of the deformation (see Figs 1(e) and 3(d)). These phenomena are associated with dilatancy and a tendency for the pore pressure to decrease. However, superimposed on the generally increasing stress–strain curve there

Fig. 13. Typical stress-strain curves for brittle rock under drained (dry) conditions. (a) serpentinite, conf. press. 0.35 kb; (b) serpentinite, conf. press. 1.38 kb. No dilatancy hardening.
are sometimes small stress decreases which may be caused either by the initiation of new faults or by increases in the pore pressure induced by the deformation.

4.3.1. Theoretical discussion. Schematic diagrams illustrating the changes in effective stress during loading under undrained and drained conditions are shown in Fig. 15(a) and (b) for the case where there is a single fault. $\tau_m$ and $\sigma_m$ are respectively the maximum effective shear stress and the corresponding effective normal stress at
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Undrained test: showing dilatancy hardening, with stress drop

Undrained test: cataclastic flow and no dilatancy hardening

Effective normal stress, corresponding to $\tau_m$

Fig. 15. Diagram to illustrate changes in effective stress during drained and undrained experiments. For explanation see main text.

any stage of the deformation. In the present experiments, taking compressive stresses as positive, the minor and intermediate principal stresses are equal to the confining pressure, denoted $P_c$, and the major principal stress is the axial compressive stress, which is given by $P_c + R$, where $R$ is the differential stress. If there is no pore-pressure we have

\[
\begin{align*}
\tau_m &= R/2 \\
\sigma_m &= P_c + R/2
\end{align*}
\]

If there is a pore-pressure $p$ we have

\[
\begin{align*}
\tau_m &= R/2 \\
\sigma_m &= P_c + R/2 - p
\end{align*}
\]
We denote the effective confining pressure \((P_c - p)\) by \(P^*\), so that
\[
\sigma_m = P^* + R/2
\]  
(4)

FF' represents the brittle fracture locus, and OS represents the residual strength locus (associated with sliding on a fracture or fault). In a typical experiment with no pore fluid or with pore fluid under drained conditions \((p = \text{const.})\) an effective confining pressure \(P^*\) (d) is first applied (Fig. 15(b)), and the stress path is represented by \(OP^*\) (d). An axial differential stress is then applied and the stress path is a line \(P_d^* F_3\) with a gradient of unity (see equations (3) above). Fracture occurs at \(F_3\) and the stress drops back to the point \(S_3\) and remains at that level while sliding continues on the fault produced by fracture.

By contrast, in a typical experiment with pore fluid under undrained conditions if the external confining pressure is built up to a value \(P_e\) the effective confining pressure achieves some lower value \(P_u^*\), so that an initial pore pressure is built up, given by
\[
p_i = P_e - P_u^*.
\]  
(5)

There are then two cases to consider (i) low effective confining pressure, when dilatancy occurs, and (ii) high effective confining pressure, when cataclastic flow occurs and the porosity continues to decrease as shear deformation proceeds.

(i) Low effective confining pressure, with dilatancy. When an axial differential stress is applied a typical stress path is then \(P_u^* CF_1\) (Fig. 15(a)) or \(P_u^* CF_4\) (Fig. 15(b)). Along \(P_u^* C\) volumetric compression results in a further increase in pore pressure (given by \(P_u^* - P_c^*\)), and the gradient of the stress path is greater than unity. However, at \(C\) crack propagation is initiated, leading to the onset of dilatancy. Taking as an example the case shown in Fig. 15(a) there is a reduction of pore pressure, between the onset of dilatancy at \(C\) and the occurrence of fracture at \(F_1\), which is given by \((P_f^* - P_c^*)\). After fracture there is further dilatancy, giving rise to a further reduction of pore pressure.

If fracture occurs at some point such as \(F_0\) (Fig. 15(a)) then the occurrence of a stress-drop depends on whether the reduction in pore pressure is less than or greater than \(F_0 E_0\) (where \(F_0 E_0\) is parallel to the \(\sigma_m\)-axis). In the former case there is a reduction in the differential stress typified by the stress path \(F_1 S_1\) (Fig. 15(a)). Sliding on the fault is accompanied by further dilatancy and the stress then rises along the path \(S_1 S\). Dilatancy-hardening is indicated by the increase in the fracture stress from \(F_1\) to \(F_4\), and by the increase in the residual (frictional) stress along the stress-path \(S_1 S\).

If the dilatancy at fracture is large a typical stress path is \(F_4 E_4 S_4 S\) (see Fig. 15(b)), in which case there is no sliding on the fault until the stress has risen to \(S_4\), after which further dilatancy requires that an increase of stress along the path \(S_4 S\) be applied in order to permit continued sliding on the fault. No stress drop is observed in such a case, but there may be an inflection in the stress-strain curve (as in Fig. 1(c)).

(ii) High effective confining pressure, with cataclastic flow and a continuous reduction in porosity. When an axial differential stress is applied a typical stress path is then \(P_s^* F_2 O\) (see Fig. 15(a)), with a continuous increase in pore pressure in an undrained test so long as there is a continuous decrease in porosity. The differential stress will pass through a peak followed by a slow decline, or at most a small rapid stress drop.

The above analysis applies when a single major fault develops. However, it was found that when, because of dilatancy, there was no stress drop during the deformation shear faulting was usually accompanied by additional faulting, for example fragmentation, fractures parallel or perpendicular to the specimen axis, the formation of several shear faults (sometimes conjugate) or the formation of a shear zone containing...
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**Fig. 16.** Effective confining pressure as a function of strain for dunite in undrained tests at different confining pressures and pore water contents. ○, 4.83 kb, 42.4 μl (U.32); □, 4.83 kb, 66.8 μl (U.31); △, 4.83 kb, 117.3 μl (U.30); ●, 2.07 kb, 45.0 μl (U.36); ■, 2.07 kb, 82.7 μl (U.35).

**Fig. 17.** Effective confining pressure as a function of strain for serpentinite in undrained tests at a confining pressure of 4.83 kb and different pore water contents. ○, 118.9 μl (OS.73); △, 148.8 μl (OS.70); □, 199.8 μl (OS.69).
Fig. 18. Effective confining pressure as a function of strain for sandstone in undrained tests at different confining pressures and pore water contents. \( \Delta \), 1\,\text{kb}, 264\,\text{ml}, saturated (DS.18); \( \square \), 1\,\text{kb}, 316\,\text{ml} (DS.19); \( \bigcirc \), 1\,\text{kb}, 324\,\text{ml} (DS.21); \( \triangle \), 0\,\text{kb}, 265\,\text{ml}, saturated (DS.23); \( \blacksquare \), 0\,\text{kb}, 288\,\text{ml} (DS.25); \( \times \), 4\,\text{kb}, 336\,\text{ml} (DS.31); \( \ast \), 4\,\text{kb}, 402\,\text{ml} (DS.32).

Fig. 19. Dry and undrained tests on sandstone exhibiting the same mean residual (frictional) strength. (a) undrained test, 4\,\text{kb} conf. press., 402\,\text{ml} water content (DS.32); (b) dry test, 0\,\text{kb} conf. press (DS.1) Note quasicyclic stress variation at large strains in the undrained test, preceded by continuous dilatancy hardening.

numerous minor shears. Details of the observations made for the complete set of experiments are given in Tables 2–4. This can be understood in terms of localized dilatancy hardening as follows: When the first fault or faults develop the pore pressure in these faults decreases, and this may be sufficient to inhibit further propagation of these faults. This permits a further increase in the differential stress to take place, and a new fault or faults can develop in parts of the rock where the pore pressure is still high. This process may be repeated, but as water flows from the unfaulted zones into the fault zones it becomes increasingly difficult to initiate new faults, and instead renewed propagation takes place on already existing faults.

4.3.2. Estimates of pore pressure and dilatancy. By comparing the stress–strain curves from the undrained tests with those for dry rocks it has been possible to estimate the effective confining pressure, and hence the induced pore pressure at different stages of deformation. Knowing the pore pressure at fracture and at an axial shortening of
Fig. 20. Dry and undrained tests on sandstone exhibiting the same mean residual (frictional) strength (a) undrained test, 4.14 kb conf. press., 265.1 µl water content (saturated) (DS. 30); (b) dry test, 0.97 kb conf. press. (DS. 8). Note small stress drops at large strains in the undrained test, preceded by continuous dilatancy hardening.

4 per cent, together with the volume of pore water, the specific volume of water at different pressures (from Kennedy & Holser 1966), and the volume of the rock specimen it is also possible to estimate the volumetric dilatation which occurs between fracture and an axial shortening of 4 per cent. In addition, by assuming that the pore pressure equals the confining pressure at the start of volumetric dilatation (see Section 3.2 above) it is also possible to estimate a minimum value for the dilatation which occurs before fracture, and hence obtain a value for the total dilatation which takes place during an axial shortening of 4 per cent. The estimated pore pressures and dilatations are given in Tables 2-4. For a number of experiments a more detailed analysis of the effective confining pressure was carried out, and the results are presented in Figs 16-18.

Fig. 21. Dry and undrained tests on serpentinite exhibiting the same mean residual (frictional) strength. (a) undrained test, 4.83 kb conf. press., 199.8 µl water content (OS. 69); (b) dry test, 1.10 kb conf. press. (OS. 3).
These show that during the first few per cent of deformation the effective confining pressure increases rapidly because of dilatancy, but that it tends to attain an approximately constant value after an axial shortening of ~6 per cent (corresponding to a displacement along the fault surface of ~1.5 mm). This suggests that a more-or-less stable value of porosity is achieved after this degree of axial shortening, corresponding perhaps to the critical voids ratio in soils (Leonards 1962). Further evidence for this is provided by the stress-deformation curves in some experiments, which are shown in Figs 19–21. These show that following an initial stage of dilatancy hardening the stress required for further axial deformation reaches a plateau. This is particularly clearly seen in experiments DS.32 and OS.69.

5. Dilatancy and crustal faulting

5.1 Premonitory effects

This has been discussed in some detail by Nur et al. (1973), in connection with the changes in crustal seismic velocity which have been observed before some earthquakes. They show that a 15 per cent fractional increase in porosity, with a porosity value of ~1 per cent, is sufficient to explain the observation.

It can be seen in Tables 2–4 that the volumetric dilatations are very similar to each other for all three rocks tested, in spite of the large differences in initial porosity. However, the dilatation and the pore water volume depend upon the effective confining pressure. Both increase as the effective confining pressure decreases.

On the other hand the dilatation after fracture is comparatively small and shows no systematic dependence on effective confining pressure.

It is difficult to reconcile the results with the calculations of Nur et al. (1973). In Table 6 we show the estimated pore pressure and fracture strength at several confining pressures (calculated by linear interpolation) corresponding to a fractional increase of porosity (due to dilatation) of 15 per cent. It is seen that, except for the sandstone at low confining pressures the strength is high compared with estimated crustal stresses, and the discrepancy is particularly marked for the low-porosity dunite.

On the other hand, when the strength is low, corresponding to an effective confining pressure which is near zero, the porosity at fracture is high (~10 per cent), and the percentage increase in porosity before fracture for dunite is much greater than 15. The percentage increases in porosity following fracture is, however, only ~1 per cent (see Table 7).

**Table 7**

*Estimated porosity at fracture and after 4 per cent axial deformation when the effective confining pressure is very low (~0) and the strength is low.*

<table>
<thead>
<tr>
<th>Rock and expt.</th>
<th>Conf. press. (kb)</th>
<th>Porosity at fracture (1) (per cent)</th>
<th>Strength (kb)</th>
<th>Porosity at 4 per cent def. (2) (per cent)</th>
<th>Strength (kb)</th>
<th>Per cent increase of (2) over (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dunite U.28</td>
<td>2.07</td>
<td>8.46</td>
<td>0.25</td>
<td>8.51</td>
<td>1.05</td>
<td>0.63</td>
</tr>
<tr>
<td>U.29</td>
<td>4.83</td>
<td>7.92</td>
<td>0.25</td>
<td>8.04</td>
<td>2.50</td>
<td>1.48</td>
</tr>
<tr>
<td>Serpentinite OS.74</td>
<td>2.41</td>
<td>8.16</td>
<td>1.12</td>
<td>8.20</td>
<td>1.10</td>
<td>0.44</td>
</tr>
<tr>
<td>OS.72</td>
<td>4.83</td>
<td>9.83</td>
<td>1.34</td>
<td>9.93</td>
<td>1.95</td>
<td>0.99</td>
</tr>
<tr>
<td>Sandstone DS.25</td>
<td>0.35</td>
<td>12.66</td>
<td>0.43</td>
<td>12.73</td>
<td>0.57</td>
<td>0.58</td>
</tr>
<tr>
<td>DS.22</td>
<td>1.10</td>
<td>15.46</td>
<td>0.21</td>
<td>15.51</td>
<td>0.35</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Dilatancy and the strength of rocks

5.2 The effect of dilatancy on faulting

Dilatancy under undrained conditions results in the strengthening or hardening of the rock as already discussed, and as a consequence it tends to reduce or eliminate instabilities in the deformation (as manifested in stress-drops). In granular solids deformed by shear, however, a state of maximum porosity is eventually achieved, and further shear deformation results in a reduction of porosity, which under undrained conditions, also results in a strength which decreases with further strain. Thus a mechanical instability caused by negative dilatancy results. This is discussed in detail by Frank (1965). As against this Casagrande has proposed that once the critical void ratio is achieved in a soil further deformation might occur at a fixed volume (Leonards 1962), in which case the deformation would remain stable. The process considered by Frank (1965) is one in which there is a cyclic increase and decrease of volume as the granular mass is sheared, a simple example of which is the shearing of a mass of regularly packed equal spheres. In such a regular, homogeneous system instability would be manifested by a cyclic variation in shear stress and/or displacement, but the strain would be homogeneously distributed on a macroscopic scale (that is, if groups of grains are considered). Any natural system is, however, not regular or homogeneous. The conditions for instability will therefore occur first in only one or a few zones in the mass. The latter will become zones of weakness, and their weakness will increase as deformation proceeds, so that the deformation will tend to become concentrated in these zones. A corollary of this is that it will become increasingly difficult for deformation to take place outside the weak zones. Thus in a natural system unstable deformation will be accompanied by the formation of macroscopic shear zones. Conversely stability will be associated with greater homogeneity of the deformation, as discussed in Section 4.3.1 above.

In broad terms we can say that in a natural dilatant system deformed under undrained conditions there would be a quasicyclic variation of stress and/or displacement, in which there would be increases of stress accompanied by macroscopically homogeneous deformation, followed by the development of instability with decrease of stress and the formation and propagation of macroscopic shear faults. A period of fault movement will be brought to a halt as fluid flows away from the fault into regions of lower pore pressure, further dilatancy will take place, and the whole cyclic process will then be repeated.

In four experiments on Darley Dale sandstone there is clear evidence for the development of instability after initial faulting and a period of dilatancy. In experiments DS.18 and 19, conducted with a confining pressure of 1.10 kb there were stress drops of 0.15 kb, after a deformation of 5.5 per cent, and 0.22 kb, after a deformation of 3 per cent, respectively (Fig. 1(c) and (d)). In experiment DS.32 (see Fig. 19(a)), carried out at a confining pressure of 4.14 kb, a constant mean porosity is achieved after a strain of 5.5 per cent, but superimposed on this is a quasiperiodic variation about the mean which gives rise to a sequence of stress drops with an amplitude of ~0.05 kb. A similar effect, but more irregular, was observed in experiment DS.30 (Fig. 20(a)), which was also carried out at a confining pressure of 4.14 kb.

Nur & Schulz (1973) have proposed a special model for dilatancy, which associates it with the irregular topography of fault surfaces. This could hardly give rise to the premonitory seismic velocity changes discussed by Nur et al. (1973), which require dilatancy throughout a substantial volume of rock. Our experiments provide no clear evidence for or against the model, but on theoretical grounds we are inclined to prefer some variant of Frank's (1965) model.

6. Conclusions

It has been shown that in rocks whose average initial porosities range from
0.54 per cent to 11.41 per cent dilatancy during fracture at near zero effective confining pressures, when the strength is low, causes the porosities after fracture to reach very similar values of ~8–16 per cent. For rocks whose initial porosity is low this implies very large increases in porosity. The percentage increase in porosity after fracture is however only ~1 per cent. These results are very difficult to reconcile with the theoretical calculations of Nur et al. (1973) concerned with earthquake premonitory effects.

The general effect of dilatancy has been to produce dilatancy-hardening (or strengthening) of the rock, which tends to reduce mechanical instability and oppose the formation of major faults. However, after deformations of ~5 per cent the porosity tends to become nearly constant, but superimposed on this may be small quasicyclic variations of porosity (as discussed by Frank 1965) which produce small stress drops of ~50 b.

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References


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