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The $p^0$ and $\rho^0$-meson photoproduction from hydrogen and deuterium is studied. In the relation between cross sections on hydrogen and deuterium target it is found that the deuteron form factor plays an important role. Effects of isoscalar part of photon and the Glauber correction are also discussed.

§ 1. Introduction

Recently, the $\rho^0$ photoproduction from hydrogen and deuterium has extensively been observed in multi-GeV energy region. In the present paper, we pay our attention to the results obtained by the Cornell 10 GeV electron synchrotron. Among others, the most interesting point is that the ratio $R(t) = (d\sigma_d/dt)/(d\sigma_p/dt)$ at $t=0$ is around 3.26 from 4 GeV to 9 GeV. If we take the impulse approximation this ratio is slightly less than 4, namely 4 minus the Glauber correction. This value is, however, considerably larger than the experimental one. For increasing momentum transfer $t$ the ratio $R(t)$ rapidly falls down to 1.9 or so. This fact is easily understood in terms of the impulse approximation with the Glauber correction.

In theoretical point of view, the $\rho^0$ photoproduction from a proton is well understood in terms of the vector dominance model. On the basis of a broken SU(3) quark model the cross section is related with pion-nucleon scattering cross sections, and theoretical prediction is quantitatively in agreement with experiment.

In this situation it would be worth while to investigate the $\rho^0$ photoproduction on deuterium in detail. First of all, it is pointed out that the ratio $R(t)$ is sensitive to the functional form of the deuteron form factor in the impulse approximation as well as in the Glauber correction. Second, the effects of isoscalar part of photon must be taken into account. It is obvious that these effects are small, but they can definitely reduce the ratio $R$. We calculate the $\pi^+$ exchange contribution in detail, and the $A_1$ exchange effects are also discussed. Finally, the cooperative phenomena are considered, which are characteristic of the two-nucleon problem and are possible correction to the impulse approxima-

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The recent part of the cooperative phenomena is found to be included in the Glauber correction. Cross section for \( \rho^0 \)-meson photoproduction is also observed from hydrogen and deuterium target, and the ratio \( R(t) \) is obtained. The same formalism developed for \( \rho^0 \) photoproduction is applicable to \( \varphi^0 \) production.

In § 2 the impulse approximation is discussed. Section 3 is devoted to estimating the Glauber correction, and we compare the results for two kinds of deuteron form factors. Correction due to the isoscalar part of photon is calculated in § 4. The cooperative phenomena are discussed in § 5. Application of the formalism developed so far to \( \varphi^0 \) photoproduction is made in the last section.

\section{2. Impulse approximation}

The \( \rho^0 \)-photoproduction amplitude \( M_d \) on deuteron is given by the sum of production amplitude on a single nucleon. Following Glauber, \(^3\) \( M_d \) with the deuteron at rest is written as

\[ M_d = e^{i q s} M_1^{(0)} \tau_1^{(0)} + e^{-i q s} M_2^{(0)} \tau_2^{(0)} + e^{i q s} M_1^{(+)} + e^{-i q s} M_2^{(+)}. \]  

(2.1)

Here \( q \) is the momentum transfer, which at high energies and small angles reduces to a three-dimensional vector perpendicular to the initial photon momentum \( k \). \( s \) is the component of the deuteron relative coordinate \( r \) in the plane perpendicular to \( k \). The subscripts 1 and 2 refer to the nucleon 1 and 2. The superscript \((0),(3)\) and \((+)) \) are understood as the same notation with CGLN as far as isotopic spin is concerned. \(^9\) In the present paper the \( D \)-state of the deuteron is neglected.

Applying (2.1) to the deuteron, we obtain

\[ M_d|d\rangle = (e^{i q s} M_1^{(0)} - e^{-i q s} M_2^{(0)}) |T=1\rangle + (e^{i q s} M_1^{(+)} + e^{-i q s} M_2^{(+)}) |d\rangle. \]  

(2.2)

Here we used the following relations:

\[ (\tau_1^{(0)} + \tau_2^{(0)}) |d\rangle = 0, \]

\[ (\tau_1^{(0)} - \tau_2^{(0)}) |d\rangle = 2 |T=1\rangle. \]  

(2.3)

Then the \( \rho^0 \)-photoproduction cross section from deuteron is given by

\[ \frac{d \sigma_d}{d \Omega} = \frac{1}{6} \sum_{\tau} \int d^3 r \langle f | M_d | d \rangle \langle f |. \]  

(2.4)

where \( r \) is the deuteron relative coordinate. \( \varepsilon \) and \( \rho \) are the polarizations of photon and \( \rho^0 \), respectively. We use here the closure approximation, namely

\[ \sum_{\tau} |f\rangle \langle f| = 1. \]  

(2.5)

This approximation means that both the deuteron and \( p+n \) states are taken in
the final state. Then the cross section reads
\[ \frac{d\sigma}{d\Omega} = \frac{1}{6} \sum_{i,\sigma} \int d^3 r \langle d| M_d^* M_d| d \rangle. \] (2.6)

Since the deuteron is spin triplet, one finds
\[ \frac{d\sigma}{d\Omega} = \frac{1}{6} \sum_{i,\sigma} \int d^3 r Tr_1 Tr_2 \left[ \frac{3 + \sigma \cdot \sigma}{4} M_d^* M_d \right] |\Psi_d(r)|^2. \] (2.7)

Decomposing the matrix element \( M_i^{(j)} \),
\[ M_i^{(j)} = A^{(j)} + B^{(j)} \sigma_i, \] (2.8)
and introducing the deuteron form factor
\[ S(q) = \int d^3 r e^{iqr} |\Psi_d(r)|^2, \] (2.9)
the cross section is finally written as
\[ \frac{d\sigma}{d\Omega} = \sum_{i,\sigma} \left[ \left( |A^{(0)}|^2 + |B^{(0)}|^2 \right) (1 - S(q)) + \frac{2}{3} |B^{(0)}|^2 S(q) \right. \]
\[ + \left( |A^{(+)}|^2 + |B^{(+)}|^2 \right) (1 + S(q)) - \frac{2}{3} |B^{(+)}|^2 S(q) \]. (2.10)

Let us remind that the \( \rho^0 \)-photoproduction cross sections on a single proton and on a neutron are written as
\[ \frac{d\sigma_p}{d\Omega} = \frac{1}{2} \sum_{i,\sigma} \left[ |A^{(+)}| + A^{(0)}|^2 + |B^{(+)} + B^{(0)}|^2 \right] \] (2.11)
and
\[ \frac{d\sigma_n}{d\Omega} = \frac{1}{2} \sum_{i,\sigma} \left[ |A^{(+)}| - A^{(0)}|^2 + |B^{(+)} - B^{(0)}|^2 \right]. \] (2.12)

Then we have the following result:
\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma_p}{d\Omega} + \frac{d\sigma_n}{d\Omega} \right) (1 + S(q)) - S(q) \sum_{i,\sigma} \left[ 2 |A^{(0)}|^2 + \frac{4}{3} |B^{(0)}|^2 + \frac{2}{3} |B^{(+)}|^2 \right]. \] (2.13)

From (2.13) one can see some basic relations. First of all, the contribution of isoscalar part of photon is considered to be small, namely \( A^{(0)} = 0 \) and \( B^{(0)} = 0 \), then \( d\sigma_p/d\Omega = d\sigma_n/d\Omega \) holds. The form factor \( S(q) \) rapidly decreases with increasing \( q \). Consequently, we obtain for finite \( q \),
\[ R(t) = \frac{d\sigma}{dt} \left. \frac{d\sigma_p}{dt} \right| \frac{d\sigma}{dt} = 2. \] (2.14)

Note that \( d\sigma_p/d\Omega \) \( \left/ \right/(d\sigma_p/d\Omega) \) \( \left/ \right/(d\sigma_p/d\Omega) \) \( \left/ \right/(d\sigma_p/d\Omega) \) \( \left/ \right/(d\sigma_p/d\Omega) \) \( \left/ \right/(d\sigma_p/d\Omega) \) is valid, because \( q \) is almost
independent of the target mass \( m_t \), \( q = \sqrt{t/4m_t^2} \approx \sqrt{-t} \) when \( t \) is small. On the other hand, for vanishing \( t \), we may have \( S(q) \approx 1 \), and the spin flip term \( B^{(+)} \) is small. Hence we get

\[
R(t) = \frac{d\sigma}{dt} \bigg| \frac{d\sigma}{dt} \approx 4.8 \tag{2·15}
\]

It must be emphasized from (2·13) that the contribution from \( A^{(0)} \), \( B^{(0)} \) and \( B^{(+)} \) can reduce the ratio \( R \) from 4.

§ 3. Rescattering correction

In the previous section we have obtained the cross section of \( \phi^0 \) photoproduction from deuteron in the framework of the single scattering approximation. For the deuteron target, there is the rescattering correction which is supposed to suppress the single scattering cross section through negative interference between single and double scattering. According to Glauber,\(^8\) the rescattering amplitude \( R_d \), which should be added to \( M_d \), is given by

\[
R_d = i \frac{\hbar}{2\pi k} \int d^2q' e^{iq' \cdot \mathbf{k}} \left[ f_1(-q' + q/2) \left( M_1^{(+))(q' + q/2)} + M_1^{(0)(q' + q/2)} \right) + f_1(-q' + q/2) \left( M_1^{(+))(-q' + q/2)} + M_1^{(0)(-q' + q/2)} \right) \right] \theta(z). \tag{3·1}
\]

The amplitude \( f_i(q) \) is that for the \( \phi^0 N \) elastic scattering on the \( i \)-th nucleon with momentum transfer \( q \), in which the spin flip part is ignored. The step function \( \theta(\mp z) \) with the \( z = (\mathbf{r}_1 - \mathbf{r}_2) \cdot \mathbf{k} \) ensures that \( \phi^0 \) is first produced on nucleon 1 and subsequently rescattered on nucleon 2, and vice versa.

Operating (3·1) on the initial deuteron state, we obtain the following correction terms corresponding to the single scattering amplitudes \( M^{(+) \, 0} \) and \( M^{(0 \, 0)} \):

\[
R_d^{(+)} \rangle d\rangle = i \frac{\hbar}{4\pi k} \int d^2q' e^{iq' \cdot \mathbf{k}} \left[ f_1(q' + q/2) M_1^{(+))(q' + q/2)} \right] \langle d\rangle \tag{3·2}
\]

and

\[
R_d^{(0)} \rangle d\rangle = i \frac{\hbar}{4\pi k} \int d^2q' e^{iq' \cdot \mathbf{k}} \left[ f_1(-q' + q/2) M_1^{(0)(q' + q/2)} \right] \langle d\rangle. \tag{3·3}
\]

Since the magnitude of these correction terms is sufficiently small compared with that of single scattering as will be seen later, we take only the interference term in the matrix element squared \( \langle f \rangle (M_d + R_d) |d\rangle \|^2 \). As in the case of single scat-
tering, the closure approximation leads to the following cross section from the interference term:

\[
\frac{d\sigma_\text{(int)}}{dQ} = -\frac{1}{6\pi k^2} \sum_{\alpha,\beta} \int d^2 q' \text{Im}[f(q - q') \{3(S(q') + S(q - q'))A^{(+)}(q)A^{(+)}(q') + 3(S(q - q') - S(q'))A^{(\text{g})}(q)A^{(\text{g})}(q') + (3S(q - q') - S(q'))B^{(\text{g})}(q)B^{(\text{g})}(q')\}].
\] (3.4)

Let us estimate (3.4) at small momentum transfer. The \(\rho^0 N\) elastic scattering amplitude \(f(q)\) is considered to be pure imaginary and of diffraction type (the Gaussian form in \(q\)); the latter assumption is also applied to the \(\rho^0\)-photo-production amplitude. It is therefore natural to put

\[
f(q) = i \frac{\sigma_T}{4\pi} k \exp(-\alpha_p q^2/2),
\] (3.5)

\[
A^{(+)}(q) = a^{(+)} \exp(-\beta_p q^2/2).
\] (3.6)

For the deuteron form factor we take the Hulthen type and the Gaussian type,

\[
S(q) = 2\alpha \beta (\alpha + \beta) \frac{1}{\alpha - \beta} \frac{1}{q} \left\{ \frac{\tan^{-1} q}{2\alpha} + \frac{\tan^{-1} q}{2\beta} - 2\tan^{-1} \frac{q}{\alpha + \beta} \right\}
\] (3.7)

and

\[
S(q) = \exp(-\gamma q^2).
\] (3.8)

If we neglect \(A^{(\text{g})}\), \(B^{(\text{g})}\) and \(B^{(\text{g})}\), the following ratio is obtained for small \(q\):

\[
\frac{(d\sigma_\text{d}/d\Omega)^{\text{(int)}}}{d\sigma_\text{d}/d\Omega} = -\frac{\sigma_T}{8\pi^2} \frac{\exp(\beta_p q^2/2)}{1 + S(q)} \int d^2 q' \exp(-\alpha_p (q - q')^2/2 - \beta_p q'^2/2)
\times [S(q') + S(q - q')],
\] (3.9)

where \(d\sigma_\text{d}/d\Omega\) means (2.13). In the case of the Hulthen form factor, (3.9) is calculated numerically, while for (3.8) one can easily see

\[
\frac{(d\sigma_\text{d}/d\Omega)^{\text{(int)}}}{d\sigma_\text{d}/d\Omega} = -\frac{\sigma_T}{4\pi} \frac{1}{1 + \exp(-\gamma q^2)} \frac{1}{\alpha_p + \beta_p + 2\gamma} \left\{ \exp \left( \frac{\beta_p^2 + 2\gamma (\beta_p - \alpha_p)}{2(\alpha_p + \beta_p + 2\gamma)} q^2 \right) \right\}
+ \exp \left( \frac{\beta_p^2}{2(\alpha_p + \beta_p + 2\gamma)} q^2 \right).
\] (3.10)

Values of the parameters in (3.5) ~ (3.8) are obtained as follows: The slope \(\alpha_p\) is simply assumed to be given in the relation

\[
\alpha_{ap} = (\alpha_p + \alpha_\pi)/2,
\] (3.11)

where \(\alpha_\pi\) is the slope of \(\pi N\) elastic differential cross section and \(\alpha_{ap}\) is that for
Fig. 1. The ratio \( R(t) = \frac{\langle d\sigma_d/dt \rangle}{\langle d\sigma_p/dt \rangle} \) vs \( t \). (a) shows \( R(t) \) for the Hulthen type deuteron form factor, and (b) is plotted for the Gaussian form factor. The solid curves show the impulse approximation, and \( R(t) \) with the Glauber correction (3.9) are plotted by the broken curves.

The reaction \( \pi N \rightarrow \rho N \). Experimental values, for example those given by Aderholz et al.\(^7\) at 8 GeV/c, give \( \alpha_p = 9.5 \pm 1.0 \) (GeV/c)\(^{-2}\) which is almost energy independent. The value of \( \beta_p \) shows a slight variation with energy, but this fact is not so essential for estimating (3.9) and we take \( \beta_p = 8.5 \) (GeV/c)\(^{-2}\). The parameters in the deuteron form factor of the Hulthen type are \( \alpha = 0.0457 \) GeV and \( \beta = 0.260 \) GeV, respectively.\(^8\) For the slope of the Gaussian \( S(q) \), we find somewhat different values in different references.\(^*)\) According to Verde\(^9\) the deuteron wave function \( \Psi_d(r) \) is given in the form \( \Psi_d(r) \propto \exp(-b^2 r^2) \), where \( b^2 = 0.0961 \times (10^{-13} \text{ cm})^{-2} \). This wave function corresponds to the form factor \( \exp(-\gamma q^2) \) with \( \gamma = 1/4b^2 = 65.1 \) (GeV/c)\(^{-2}\). Finally for the \( \rho N \) total cross section \( \sigma_T \), there is no definite information. A rough estimate can be made from the absorption model calculation as 35 \( \leq \sigma_T \leq 45 \text{ mb}.\(^{10}\)

Numerical results of (3.9) are given in Fig. 1 as the ratio \( R(t) \). It should be noted that the deuteron to hydrogen ratio is sensitive to the deuteron form factor \( S(q) \). It is found that choice of form factor is more important than other part, such as correction due to the isoscalar part of photon. Finally, one can see that \( R(t) \) is almost \( s \) independent, in fact, \( s \) dependence appears only through the parameters \( \sigma_T, \alpha_p \) and \( \beta_p \).

§ 4. Estimation of isoscalar part

So far the isoscalar part of photon in \( \rho^0 \) photoproduction is neglected in the

\(^*)\) In the paper by Cornell Group the value of \( \gamma \) is taken as 56 (GeV/c)\(^{-2}\) from electron scattering.\(^1\)
forward direction. Although we do not have any reliable method of calculating the isoscalar part, rough estimates are made for the diagram shown in Fig. 2. The interaction Hamiltonian used are the following:

\[ \begin{align*}
H(p \rightarrow \rho^0) &= i \frac{f}{m_\pi} \bar{p} \gamma^\mu \rho_\mu \rho_\nu \bar{\rho} \gamma^\nu, \\
H(\omega^0 \rightarrow \rho^0) &= g_{\omega \rho} \varepsilon_{\alpha \beta \gamma \delta} \partial_\alpha \omega_\beta \partial_\gamma \rho_\delta, \\
H(\omega^0 \rightarrow \gamma) &= \frac{e m_\omega^2}{2 \gamma^\mu} \partial_\mu A_\nu,
\end{align*} \]

where \( \pi, \rho, \omega \) and \( \rho \) stand for the wave functions of the respective particles, and \( m_\pi, m_\rho, m_\omega \) and \( m_p \) are masses of these particles. \( f, g_{\omega \rho}, \) and \( \gamma_\alpha \) are the coupling constants.

The matrix element of the process \( \gamma + p \rightarrow \rho^0 + p \) is proportional to

\[ -\frac{e f g_{\omega \rho}}{\gamma_\alpha} \frac{m_p}{m_\pi} \bar{p} (p_f) \varepsilon_{\alpha \beta \gamma \delta} \partial_\alpha \omega_\beta \partial_\gamma \rho_\delta \varepsilon(k)(l) \rho_\delta(l), \]

where \( p_i, p_f, k \) and \( l \) are the momenta of the initial and final nucleon, photon and \( \rho \) meson, respectively. \( \varepsilon(k) \) and \( \rho_\delta(l) \) are the polarization vectors of photon and \( \rho^0 \).

The differential cross section is given by

\[ \frac{d\sigma}{dt} = \frac{1}{16\pi} \left( e f \right)^2 \left( \frac{g_{\omega \rho}}{2 \gamma_\alpha} + \frac{g_{\rho \rho}}{2 \gamma_\alpha} \right)^2 \left( \frac{m_p}{m_\omega^2} \right)^2 \left( \frac{s - m_p^2}{|t - m_p^2|^2} \right)^2, \]

where the \( \omega-\rho \) mixing is taken into account. It is obvious from (4.4) that the matrix element \( M \) is always proportional to \( \sigma \), so that we have

\[ A^{(\rho)} = 0. \]

Numerical values of \( d\sigma/dt \) at small momentum transfer due to (4.5) is shown in Fig. 3. The differential cross section has a peak at \( t = -0.02 \) and a dip at \( t = -0.5 \). Here the coupling constants are chosen as \( f^2/4\pi = 0.08 \) and \( (g_{\omega \rho}/2 \gamma_\alpha + g_{\rho \rho}/2 \gamma_\alpha)^2 \approx 2.6 \) (GeV)^{-2}, which is obtained from the deviation of \( d\sigma_\rho/dt \) from the diffraction part \( |\alpha^{(\rho)}|^2 \exp(-\beta_\rho q^2). \) The correction due to (4.4) on \( d\sigma_\rho/d\Omega \)
is given by $-S(q) \times \sum_{\mu} \frac{4}{3} |B^{(0)}|^2$, as is seen from (2.13). This is of the order of a few $\mu$b, and is considered to be negligible.

It is interesting to see the relation between polarizations of photon and $\rho^0$. It is well known that these polarizations are parallel in $A^{(+)}$, while the polarizations are perpendicular in (4.4). We obtain the following ratio at $t = -0.02$,

$$\frac{d\sigma_{\perp}}{d\sigma_{\parallel}} = 0.095,$$  for $E_r = 3.5$ GeV

(4.7)

from (3.6) and (4.5) for hydrogen target. Here $d\sigma_{\perp}$ and $d\sigma_{\parallel}$ mean the differential cross sections with polarized gamma ray, in which the polarizations of photon and $\rho^0$ are perpendicular and parallel with each other. Equation (4.7) may be compatible with the experimental value $d\sigma_{\perp}/d\sigma_{\parallel} = 0.11 \pm 0.04$ obtained at Cornell.\(^\text{11}\) Note that the theoretical value (4.7) decreases with increasing energy, and has a maximum at $t = -0.02$ independent of photon energy.

Another candidate of the $I=1$ exchange is $\rho^0$ meson, but this is obviously excluded from the G-parity. Next possibility of the $I=1$ exchange is therefore expected with $A1$ and $A2$ mesons. Since $A1$ is pseudovector the coupling with nucleon may be of axialvector type. Then the matrix element always involves $\sigma$, and $A^{(0)}$ does not appear. In addition, the polarization of $\rho^0$ is always perpendicular to that of gamma ray. Anyhow, it is very difficult to find $A^{(0)}$ from one-particle-exchange amplitude.

§ 5. Cooperative phenomena

In this section we discuss a possible contribution to the $\rho^0$ photoproduction from deuteron due to another mechanism which has not so far been taken into account. There is a particular production mechanism for the deuteron target, the so-called cooperative process, which has not any correspondence in the photoproduction from a single nucleon. This process is illustrated in Fig. 4, where the intermediate state between both the nucleon vertices contains a summation over all possible mesons. It may be reasonable to approximate such a state as being composed of a sum of appropriate one-meson ($\pi, \rho, \cdots$) states. In this approximation, the contribution from the cooperative process can be expressed in the form quite similar to the Glauber rescattering correction given in § 3. The only difference is that here the amplitude $M$ in (3-1) should be read as that for the process $\gamma N \rightarrow BN$, while the amplitude $f$ for the process $BN \rightarrow \rho^0 N$, where
$B$ stands for appropriate mesons. In this respect, the rescattering correction can be regarded as one and the largest part of the cooperative process, but this effect has already been treated earlier, and we will not discuss further.

Among various other one-meson states, at first we consider the charged $p$-meson states, since $p$ mesons are most copiously produced in the photoproduction process at high energy. It should, however, be noted that in this case, the amplitude $f$ which couples with the $p^\pm$-photoproduction amplitude is that for the charge exchange process $p^\pm N \rightarrow p^0 N$. We have no experimental information on this process, but at high energy the cross section for such a process may be very small and may decrease rapidly with increasing energy. Accordingly the effect of intermediate charged $p$ mesons is at most one order of magnitude less than that of $p^0$ (the rescattering correction). Next, for the charged one-pion intermediate states, both the cross sections $\sigma(\gamma N \rightarrow \pi N)$ and $\sigma(\pi N \rightarrow p N)$ are smaller by about factor 10 than the cross sections $\sigma(\gamma N \rightarrow p N)$ and $\sigma(p N \rightarrow \pi N)$, respectively. Therefore this effect is also negligible. For other possible meson states, similar investigation leads to the conclusion that the contribution from these states is safely neglected in comparison with the $p^0$ rescattering effect. As a result, we can conclude that the effect of the cooperative process except for the $p^0$ rescattering is totally of the order of one per cent or less of the single scattering cross section.

§ 6. $\phi^0$-meson photoproduction

Recently, differential cross section for $\phi^0$-meson photoproduction from hydrogen and deuterium is reported. The argument developed in the present paper is valid mutatis mutandis. The only difference is that the role of isoscalar and isovector parts of photon is interchanged.

The cross section for $\phi^0$ production is much smaller than that of $p^0$, far less than $1.33/9$ times $p^0$ cross section. This fact is due to the smallness of $\phi^0 p$ total cross section. Hence the $I=1$ exchange contribution is relatively important, because this part is of the same order of magnitude with $(4\cdot 5)$. Actually, a pion exchange term gives the cross section

$$\frac{d\sigma}{dt} = \frac{1}{16\pi} (\alpha f)^2 \left( \frac{g_{\phi \pi \gamma}}{2 \Gamma_{\phi}} \right)^2 \frac{2m_p^2}{m_{\phi}^2} \frac{1-t}{(s-m_{\phi}^2)(t-m_{\phi}^2)}.$$  

(6.1)

Here the coupling constants are chosen as $g_{\phi \pi \gamma}^2/4\pi = 1.2$ (GeV)$^{-1}$ and $\Gamma_{\phi}/4\pi = 0.5$. This $g_{\phi \pi \gamma}$ comes from the decay width $\Gamma_{\phi \rightarrow \pi \pi} = 0.8$ MeV. Numerical values of
(6.1) are plotted in Fig. 5 together
with the empirical cross section from
a proton, \( 5.1 \exp(5.2t) \mu b \) (GeV/c)^{-2}
. According to SLAC data the differen­
tial cross section is much smaller,\(^{19}\)
but we take here the Cornell data,
because differential cross sections on
hydrogen and deuterium target are
both available. One can see that the
\( I=1 \) exchange contribution is no
longer a small correction, but is com­
parable with the experimental value.
This result is a little embarrassing,
and we have no idea as to how to
reduce (6.1).

Finally, the ratio \( R(t) \) is observ­
ed at \( t = -0.23 \) and \( t = -0.004 \) in \( E_r =
8.25 \text{ GeV} \). The ratio \( R(t) \) is given by (2.13):

\[
R(t) = 2(1 + S(q)) - \frac{8}{3} S(q) \frac{d\sigma}{dt}(6.1) / \left( \frac{d\sigma}{dt} \right)_{\exp},
\]

where \( d\sigma/dt(6.1) \) means the pion-exchange contribution given in (6.1). \( R(t) \)
thus obtained is compared with experiment taking the Glauber correction into
account,

\[
R(t = -0.004) = 3.14 (G), 3.12 (H) \quad (\text{calc})
\]

\[
= 2.71 \pm 0.6 \quad (\text{exp})
\]

\[
R(t = -0.23) = 1.93 (G), 1.87 (H) \quad (\text{calc})
\]

\[
= 1.57 \pm 0.15 \quad (\text{exp})
\]

where (G) and (H) denote the Gaussian and Hulthen form factors. Parameters
are chosen as \( \alpha_q = \alpha_p, \beta_p = 5.2 \) (GeV/c)^{-2} and \( \sigma_r = 15 \text{ mb} \) in (3.5) and (3.6).
It is found that the contribution of \( B^{(0)} \) in \( R(t) \) is still small, though the mag­
nitude of \( B^{(0)} \) is relatively large in \( \phi^0 \) production. The Glauber correction is
less important than that in \( \rho^0 \) photoproduction, because the \( \phi^0 \rho \)-scattering am­
plitude is small.

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Note added, in proof: The differential cross sections for \( \rho^0 \) production for both hydrogen and deuterium are measured at 9 GeV, and the ratio \( R \) is found to be \( R = \frac{d\sigma_\rho}{dt}/\frac{d\sigma_{\rho^0}}{dt} \approx 3.5 \pm 0.3 \) at \( t=0 \). See D. W. G. S. Leith, Invited talk presented at Third International Conference on High Energy Physics and Nuclear Structure, Columbia University, 1969 (SLAC-PUB-679).