Lepton Conservation, $\mu$-$e$ Symmetry, Neutral Currents and the Principle of Universality

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All the known interactions involving leptons, electromagnetic and weak, are consistent with:

(i) the existence of two different kinds of two-component neutrinos $\nu_e$ and $\nu_\mu$,
(ii) the separate conservation of the electron number $N_e$ and the muon number $N_\mu$, and
(iii) the $\mu$-$e$ symmetry.

In this note, we want to demonstrate that, in the framework of the two-distinct two-component neutrino theory, the charged weak lepton current $L_\mu$ consistent with
the conservation law of the total lepton number is always consistent also with the properties (ii) and (iii) above, provided that $L_i$ obeys the principle of universality.\textsuperscript{2}

For our present purpose, we state this principle as follows (see also below): the charged current $L_i$ together with its hermitian conjugate $L_i^\dagger$ should generate an SU(2) algebra of "charges".

Let us denote the two distinct two-component neutrinos by $\nu_1$ and $\nu_2$, and define $e^-$, $\mu^-$, $\nu_1$ and $\nu_2$ as leptons and $e^+$, $\mu^+$, $\bar{\nu}_1$, and $\bar{\nu}_2$ as anti-leptons. The most general form of the charged lepton current which satisfies the lepton conservation law can be written as

$$L_i = i\bar{\nu}_i \gamma_j a_i A \nu_j,$$  

(1)

where $L_i$ is given by Eq. (1) while $L_i^{(0)}$ is its neutral counterpart, the form of which is to be determined by the principle itself. The universality principle then insists that $Q^{(+)}$, $Q^{(-)}$ and $Q^{(3)}$ should obey the SU(2) algebra. It is then readily verified that $L_i^{(0)}$ is settled to be of the form

$$L_i^{(0)} = (i\bar{\nu}_i \gamma_j a_i A \nu_j - i\bar{\nu}_i \gamma_j A A^\dagger A \nu_j) / 2$$  

(2)

with $A$ subject to the condition

$$AA^\dagger A = A .$$  

(3)

Equation (3) is satisfied either if $A$ is any unitary matrix, or if $A$ is a singular matrix of the form, say,

$$A = \begin{pmatrix} \alpha & \beta \\ \kappa & \kappa \beta \end{pmatrix} ,$$

$$\left(1 + |\kappa|^2\right) (|\alpha|^2 + |\beta|^2) = 1 .$$  

(4)

We first consider the latter case. Inserting Eq. (4) into Eqs. (1) and (2), one finds

$$L_i = (1 + |\kappa|^2)^{-1/2} (i\bar{\nu}_i \gamma_j a_i A \nu_j + \kappa \bar{\nu}_i \gamma_j a_i \nu_j) ,$$  

(5)

$$L_i^{(0)} = i\bar{\nu}_i \gamma_j a_i \nu_j / 2 - (1 + |\kappa|^2)^{-1} (i\bar{\nu}_i \gamma_j a_i \nu_j + \kappa \bar{\nu}_i \gamma_j a_i A \nu_j^\dagger) / 2 ,$$  

(6)

with $\nu_j = (|\alpha|^2 + |\beta|^2)^{-1/2} (\alpha \nu_1 + \beta \nu_2)$ and $\kappa$ an arbitrary constant. We see that we have arrived at, essentially, a single two-component neutrino theory,\textsuperscript{4} which is of course excluded by experiments.\textsuperscript{1}

Next turn to the former solution: $A$ is unitary. If one defines two new fields $\nu_e$ and $\nu_\mu$ through

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = A \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} ,$$  

(7)

the whole Lagrangian including the free part\textsuperscript{5} can then be rewritten with $\nu_e$ and $\nu_\mu$, and Eqs. (1) and (2) now reduce, respectively, to

$$L_i = i\bar{\nu}_i \gamma_j a_i A \nu_j + i\bar{\nu}_i \gamma_j a_i \nu_\mu ,$$  

(8)

$$L_i^{(0)} = (i\bar{\nu}_i \gamma_j A \nu_e + i\bar{\nu}_i \gamma_j A \nu_\mu) / 2 .$$  

(9)

The charged lepton current $L_i$ in the above form is the familiar one and just fulfills all the properties listed in the beginning. We remark that the argument given above casts doubts on the recent attempt\textsuperscript{3} to extend the domain of the universality so as to incorporate possible nonconservation of $N_e$ and $N_\mu$ into the lepton currents.

We now recall that the principle in question is "universal" just because it is

\textsuperscript{4} Recall that the two-component nature together with the lepton conservation require both $\nu_1$ and $\nu_2$ to be of zero mass and that neutrinos undergo weak interactions only.
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applicable also to the hadron currents and make the “universal” $V-A$ current $\times$ current weak interaction acquire a meaning.\(^2\)

In fact, when combined with the octet current hypothesis, the principle determines the form of $J_i$ and $J_i^{(0)}$, the hadronic counterparts of $L_i$ and $L_i^{(0)}$, as

$$J_i = \cos \theta (j_i^{(1)} - ij_i^{(3)}) + \sin \theta (j_i^{(4)} - ij_i^{(5)}), \quad (10)$$

and the effective Lagrangian constructed from $L_i$ and $J_i$ with a common coupling constant $G$ ($G \approx 10^{-5} m_p^{-2}$)

$$\mathcal{L}_a = 2 \sqrt{2} G (L_i + J_i) \langle L_i + J_i \rangle \quad (12)$$

gives consistent explanation of all purely- and semi-leptonic weak processes.\(^1\)

Now that the charged currents $L_i$ and $J_i$ given by Eqs. (8) and (10) are “observable” through the weak interactions, it is reasonable to expect that their neutral counterparts $L_i^{(0)}$ and $J_i^{(0)}$ given by Eqs. (9) and (11) should also be “observable” through some interaction, provided that the universality principle does possess a deeper physical meaning. An example of such a hypothetical interaction, analogous to Eq. (12), is readily written down:

$$\mathcal{L}_F = 2 \sqrt{2} F (L_i^{(0)} + J_i^{(0)}) \langle L_i^{(0)} + J_i^{(0)} \rangle \quad (13)$$

This is a modified form of the neutral current $\times$ current interaction proposed by the present author before. The new interaction in the form (13) should appear as a superweak one ($F/G \leq 10^{-4}$).\(^4\)

Finally, we add a remark on the alternative form of the lepton conservation law, in which one postulates a single four-component neutrino $\nu$ and defines $e^-$, $\nu$ and $\mu^+$ as leptons. In this scheme, the most general form of the charged lepton current becomes

$$L_i = \alpha i e \gamma_\nu a \nu + \alpha' i \bar{e} \gamma_\nu a \nu + \beta i e \gamma_\nu a \nu + \beta' i \bar{e} \gamma_\nu a \nu \quad (14)$$

Although the universality principle restricts the values of the constants in Eq. (14) to be

$$\{ |\alpha|^2, |\beta'|^2 \}$$

we need some further criterion to single out the special choice: $|\alpha| = |\alpha'| = 1$ and $\beta = \beta' = 0$, this being such one compatible with experiment. Thus the universality principle provides a way to discriminate between the two possible forms of the lepton conservation law.

3) P. Chang, preprint.

\(^{4)} j_i^{(a)} = (V_j^{(a)} + A_j^{(a)})/2$, where $a$ is the $SU(3)$ index, and $\theta$ is the Cabibbo angle.