ERRORS IN RILEY ANALYSIS

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SUMMARY

The effect of random perturbations on the data used to calculate dead space/tidal volume ratio (VD/VT ratio) and percentage physiological shunt (Qs/Qt ratio) is examined. With a coefficient of variation of 1%, such random errors cause a true calculated VD/VT ratio of about 40% to lie within the range 38% to 43% (mean 40.5%, SD 0.8%), and a calculated Qs/Qt ratio to vary around a true value of about 20% from 10% to 28% (mean 20.4%, SD 1.5%). With a 2% coefficient of variation the corresponding ranges are 36% to 44% (mean 40.4%, SD 1.5%) for the VD/VT ratio, and 0% to 35% (mean 20.6%, SD 6.1%) for the calculated Qs/Qt ratio. Errors in the calculated Qs/Qt ratio may be minimized by deriving the arterial and mixed-venous oxygen contents, rather than measuring them directly. Errors having a 2% coefficient of variation then cause the calculated physiological shunt to lie between 16% and 24% (mean 20.1%, SD 1.4%).

In recent years qualification of patients’ defects of cardiopulmonary function in terms of the two parameters dead space/tidal volume ratio and percentage pulmonary venous admixture or physiological shunt has become a frequently-used investigational technique in clinical physiology laboratories. In honour of the originator of the concept (Riley and Cournand, 1951), this technique is sometimes referred to colloquially as “Riley analysis”. This paper considers the magnitude of the probable errors which may occur when this type of analysis is used clinically.

THEORY AND METHOD

Dead space/tidal volume ratio (VD/VT ratio) is calculated from the equation:

\[
VD/VT = \frac{(PA_{CO_2} - P_{E_{CO_2}})}{PA_{CO_2}} \quad (1)
\]

Percentage physiological shunt (Qs/Qt ratio) is calculated from the equation:

\[
Qs/Qt = \frac{(Cc'_{O_2} - Ca_{O_2})}{(Cc'_{O_2} - Cv_{O_2})} \quad (2)
\]

All abbreviations are standard and follow Pappenheimer and associates (1950).

The performance of these calculations requires knowledge of the various cardiorespiratory variables listed in table I. From these primary data, certain intermediate qualities are calculated (see Appendix) and used, together with the primary data, to solve equations (1) and (2). Such calculations are tedious and time-consuming, but have been greatly speeded recently by the use of digital computers (Kelman, 1966) or tables (Kelman and Nunn, 1968).

Errors may be of two kinds—random and systematic. The latter can usually be eliminated by careful analytical technique and application of appropriate correction factors. The former are, however, inevitable, and with physiological measurements such as those at present under consideration may amount to several per cent of the measured variable. With functions of several variables, small errors in the primary data may combine to cause quite large errors in the final calculated results; this is so even when the calculations themselves are performed exactly, as by a digital computer.

Random numbers normally distributed with unit variance about a zero mean were generated by a digital computer (ICL 4/50, University of Aberdeen). Appropriately scaled random perturbations were then applied to the variables listed in table I, so as to introduce into the data normally distributed errors with a coefficient of variation (standard deviation/mean) of 1 or 2%. Thus to each variable, x, in the table was added either 0.01 \( \times r \times x \) or 0.02 \( \times r \times x \), where \( r \) is a random variate of zero mean and unit variance. \( Pa_{eFF} \) and \( F_{I02} \) were not adjusted in this way, because the likely error in the physical quantity...
**RESULTS**

The effect of 5\% increases in the values of the primary variables of table I on the calculated values of $V_D/V_T$ and $Q_S/Q_T$ are shown in table II.

**Table II.** Effect on calculated values of $V_D/V_T$ and $Q_S/Q_T$ of 5\% increases in values of primary data.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$V_D/V_T$ (%)</th>
<th>$Q_S/Q_T$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{ao_2}$</td>
<td>40.6</td>
<td>7.3</td>
</tr>
<tr>
<td>$C_{vo_2}$</td>
<td>40.6</td>
<td>21.9</td>
</tr>
<tr>
<td>$F_{Eco}$</td>
<td>38.2</td>
<td>20.2</td>
</tr>
<tr>
<td>$F_{Eo}$</td>
<td>40.6</td>
<td>21.1</td>
</tr>
<tr>
<td>$F_{io_2}$</td>
<td>40.6</td>
<td>19.6</td>
</tr>
<tr>
<td>$Hb$</td>
<td>40.6</td>
<td>29.4</td>
</tr>
<tr>
<td>$P_{aco_2}$</td>
<td>43.4</td>
<td>19.8</td>
</tr>
<tr>
<td>$P_{bry}$</td>
<td>37.6</td>
<td>20.8</td>
</tr>
</tbody>
</table>

Random perturbations in the primary variables (with the exception of $P_{bry}$ and $Fi_{o_2}$) with a coefficient of variation of 1\% caused quite a marked scatter in the calculated value of $V_D/V_T$, from 38\% to 43\% (fig. 1). When the coefficient of variation was increased to 2\% the errors were correspondingly greater (fig. 2). The scatter was still more marked in the case of $Q_S/Q_T$, because the value of this ratio is dependent on more factors than is $V_D/V_T$. Random perturbations of the primary data of 1\% caused the $Q_S/Q_T$ ratio to spread from 10\% to 28\% (fig. 3); and, when the co-efficient of variation was increased to 2\% (fig. 4), 10\% of the calculated $Q_S/Q_T$ ratios lay outside the range included in the figure. (One combination of random errors gave a negative $Q_S/Q_T$ ratio of -0.3\%).

When $C_{ao_2}$ and $C_{vo_2}$ were assumed not to be measured directly, but to be calculated from arterial and mixed-venous oxygen tensions (mean values 56.9 and 27.4 mm Hg, respectively) subjected to the
same type of random error, the spread of the calculated values of \( Q_s/Qt \) was considerably reduced (fig. 5).

**DISCUSSION**

The assumption of a 2\% coefficient of variation for the random errors considered in this paper is probably reasonable. Certainly it accords with personal experience. For example, Kelman, Coleman and Nunn (1966) found that the coefficient of variation of the error in measuring \( P_{CO_2} \) by the Astrup interpolation technique was 2.5\% when the apparatus was used by an experienced technician. The results of other workers are similar.

The present calculations confirm that there is a tendency for small errors in the primary data on which the Riley analysis is based to combine to give quite large errors in the calculated \( Vd/Vt \) ratio, and particularly in the calculated physiological shunt, \( Qs/Qt \). While this is not surprising, it is somewhat disturbing; and it is a tribute to the analytical technique of the various groups who use this means of quantifying a patient’s defects of cardiopulmonary function that they are able to obtain physiologically valid results. Nevertheless it is clear that even a slight relaxation of analytical precision is likely to cause errors large enough to make the technique of doubtful value.

The wide spread of the calculated \( Qs/Qt \) ratios seen in figures 3 and 4 arises from the marked dependence of this parameter on \( CaO_2 \) (table II). This is so because the numerator of equation 2 represents a small difference between two relatively large quantities (\( Cc'O_2 \) and \( CaO_2 \)). Errors from this cause may be minimized by calculating \( CaO_2 \) from the arterial oxygen tension, rather than measuring it direct. The slope of the oxyhaemoglobin dissociation curve is such that (except in very severely hypoxaemic patients) variations in \( PaO_2 \) have relatively little effect on the calculated value of \( CaO_2 \). If \( CvO_2 \) is similarly determined from \( PFiO_2 \), the effect of inaccuracies in the measured haemoglobin concentration is markedly reduced, because they affect both the numerator and denominator of equation (2).

**APPENDIX**

The steps in calculating the \( Vd/Vt \) ratio and \( Qs/Qt \) ratio from the primary data of table I are as follows (symbols are standard):

\[
\begin{align*}
VD/VT & = 100 \times \frac{(PA_{CO_2} - PE_{CO_2})}{PA_{CO_2}} \\
Qs/Qt & = 100 \times \frac{(Cc'O_2 - CaO_2)}{(Cc'O_2 - CvO_2)}
\end{align*}
\]

**REFERENCES**


On a étudié les effets de perturbations dues au hasard en ce qui concerne les données utilisées en vue du calcul du rapport espace mort/air courant (Vd/Vt) et du pourcentage de shunt physiologique (Qs/Qt). Avec un coefficient de variation de 1%, de telles erreurs dues au hasard entraînent un rapport Vd/Vt calculé vrai d'environ 40%, se situant dans les limites de 38% à 43% (taux moyen: 40,5%, écart standard: 0,8%) et un rapport Qs/Qt calculé, variable aux alentours d'un chiffre de 20%, de 10% à 28% (taux moyen: 20,4%, écart standard: 1,5%). Avec un coefficient de variation de 2%, les limites correspondantes sont de 36% à 44% (taux moyen: 40,4%, écart standard: 1,5%) pour le rapport Vd/Vt et de 0,0% à 35% (taux moyen de 20,6%, écart standard: 6,1%) en ce qui concerne le rapport Qs/Qt obtenu par calcul. Les erreurs concernant le rapport Qs/Qt obtenu par calcul peuvent être réduites à un minimum en dérivant les taux d’oxygène artériel et d’oxygène artério-veineux, plutôt qu’en les mesurant directement. Des erreurs présentant un coefficient de variation de 2% amènent alors le shunt physiologique obtenu par calcul à se situer entre 16% et 24% (taux moyen: 20,1%, écart standard: 1,4%).

FEHLER IN DER RILEY ANALYSE

ZUSAMMENFASSUNG

Geprüft wird die Auswirkung zufälliger Störungen auf die Unterlagen, die dazu dienen, das Verhältnis von Totraum und zeitlichem Volumen (Vd/Vt Verhältnis) und die Prozentzahl des physiologischen (Qs/Qt Verhältnis) zu berechnen. Mit einem Änderungskoeffizienten von 1% verursachen solche zufälligen Fehler ein wirklich errechnetes Vd/Vt Verhältnis von ungefähr 40%, das in dem Bereich zwischen 38 und 43% liegt (mittl. Wert: 40,5%, S.D. 0,8%) und ein errechnetes Qs/Qt Verhältnis von ungefähr 20%, variierend zwischen 10 und 28% (mittl. Wert: 20,4%, S.D. 1,5%). Mit einem Änderungskoeffizienten von 2% liegen die entsprechenden Bereiche zwischen 36 und 44% (mittl. Wert: 40,4%, S.D. 1,5%) für das Vd/Vt Verhältnis und zwischen 0% und 35% (mittl. Wert: 20,6%, S.D. 6,1%) für das errechnete Qs/Qt Verhältnis. Fehler bei der Berechnung des Qs/Qt Verhältnisses könnten verringert werden, indem man den arteriellen und gemischt-venösen Inhalt in Sauerstoff ableitet, anstatt direkt zu messen. Dann verursachen Fehler, die einen Änderungskoeffizienten von 2% haben, einen errechneten physiologischen Nebenschluss zwischen 16 und 24% (mittl. Wert: 20,1%, S.D. 1,4%).

THE SOCIETY OF ANAESTHETIC LABORATORY TECHNICIANS

The Autumn Scientific Meeting of the Society will be held at the Department of Anaesthesia, Medical School, University of Newcastle upon Tyne, on September 15 and 16, 1972.

Submissions for papers should be to Mr N. Burn at that address.