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Practical Paper

Probabilistic analysis of underground pipelines for optimal renewal time

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ABSTRACT

In practical management of an entire water distribution system, it is very important to define optimal renewal time for pipelines. Reasons mainly include their deterioration, invisibility as an underground system and the adverse impact on society in the case of leakage. Thus, this paper proposes a mathematical model to satisfy this actual need. The model is developed based on Weibull hazard function and least life-cycle cost estimation approach. This model enables us to determine not only the deterioration rate of pipelines but also optimal renewal time. Attention is further focused on finding optimal pipeline materials. Empirical study was conducted on the dataset of a pipeline system in the city of Osaka, Japan.

Key words | LCC, optimal renewal interval, pipeline management, Weibull hazard function

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INTRODUCTION

Generally, in the field of asset management, the methodology in searching for the best possible maintenance and repair strategy depends on deterioration model, life-cycle cost estimation based on actual data from monitoring and inspection. However, monitoring and inspection of pipeline systems face great difficulties since the system is underground and therefore unseen. It is consequently a challenge to observe the performance and condition of pipelines, leading to difficulties in deterioration forecasting and life-cycle cost evaluation.

The Weibull hazard function is employed to address the elapsed time of each pipeline measured from when it was buried. The physical impact factors are in the form of risk factors with a certain probability or range. Each impact factor results in a particular risk level and is integrated into the hazard function. Expected life-cycle cost considers both direct replacement cost and indirect social cost. The model is used for forecasting the deterioration of pipelines and determining the optimal renewal time that offers the minimum expected life-cycle cost of each pipeline.

BACKGROUND

In the field of operation optimization, particularly with industrial machinery systems, studies on optimal replacement policies have been well documented (Heyman & Sobel 1990). Among those, a large number of researches focus on formulation of stochastic optimization models to investigate the deterioration process, maintenance and repair policies as well as cost evaluation (Heyman &

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Sobel 1990). However, many of the methods in deterioration prediction were in the form of a simple approach using only a binary condition state or homogeneous Poisson process, which might not reflect the actual behaviour of pipeline deterioration (Durango & Madanat 2002).

Further to the development of hazard models with optimal renewal time, a remarkable study by Aoki *et al.* (2005) using the Weibull hazard model and Markov process has been presented (Eckles 1968; Aoki *et al.* 2005). In the same line of development for hazard analysis, additionally, Tsuda *et al.* (2005) proposed a Markov chain deterioration model, which discusses the deterioration progress in a stochastic manner. The Markov chain model can be estimated by a multi-staged exponential hazard model developed by Tsuda *et al.* (2005). The estimated Markov chain model can be applied for the purpose of optimal renewal policies. Empirical testing has been conducted successfully on a pavement system.

The purpose of this paper is to propose a practical application on a pipeline system. Thus, a rigorous review of the above-mentioned references has led us to select the most practical mathematical model, which employs the Weibull hazard model with binary condition state for the actual dataset of the pipeline. Our methodology is presented below.

METHODS

Should an incident occur, especially in the megacities, tap water will spill over the surface of the road causing social damage. By substituting the old pipeline proactively, the risk of such an incident occurring could be mitigated. This is under the control and decision of the managers. As a matter of course, the substitution of pipeline demands an increase in the replacement cost. It is therefore important to harmonize the trade-off situation by introducing the optimal renewal interval with respect to the summation of total social cost and renewal cost as a whole.

Deterioration process

In hazard analysis, the deterioration of an element is subject to a stochastic process (Lancaster 1990). For a pipeline, as previously mentioned, there are two condition levels: E_1, E_2 in Figure 1. Level E_1 reflects a healthy

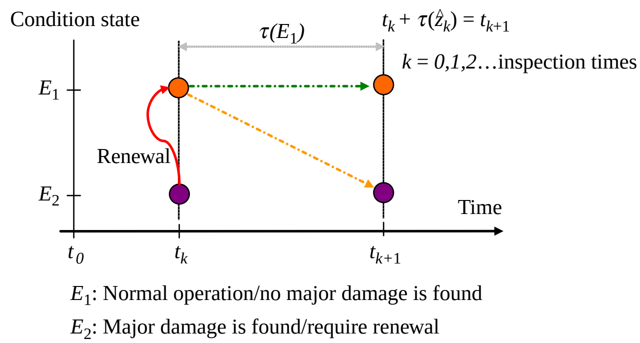

Figure 1 | The deterioration and decision framework.

condition, while level E_2 denotes that the pipeline is in a condition of leakage, damage or destruction. Whenever the condition level E_2 is detected, the damaged pipeline will be replaced by a new one immediately. The renewal is carried out at alternative time t_k ($k = 0,1,2,\ldots$). In this way, the next renewal time is denoted as $t = t_0 + \tau$, where τ indicates the elapsed time. The life span of the pipeline is expressed by a random variable ζ. The probability distribution and probability density function of the failure occurrence are $F(\zeta)$ and $f(\zeta)$, respectively. The domain of the random variable ζ is $[0,\infty]$. The living probability (hereafter known as survival probability) expressed by survival function $\tilde{F}(\tau)$ can be defined according to the value of failure probability $F(\tau)$ in the following equation:

$$\tilde{F}(\tau) = 1 - F(\tau) \tag{1}$$

The probability, at which the pipeline performs well until time τ and breaks down for the first time during an interval of $\tau + \Delta\tau$ can be regarded as the hazard rate and expressed in the following equation:

$$\lambda_i(\tau)\Delta\tau = \frac{f(\tau)\Delta\tau}{\tilde{F}(\tau)} \tag{2}$$

where $\lambda(\tau)$ is the hazard function of the pipeline. In reality, the breakdown probability depends largely on the elapsed time of the pipeline since its construction. Thus, the hazard function should take into account the working duration of the pipelines. In other words, the memory of the system should be inherited. The Weibull hazard function is satisfied in addressing this process.

$$\lambda(\tau) = \alpha m \tau^{m-1} \tag{3}$$

where α is the parameter expressing the arrival density of the pipeline, and m is the acceleration or shape parameter.

The probability density function $f(\tau)$ and survival function $\tilde{F}(\tau)$ in the form of the Weibull hazard function can be further expressed in Equations (4) and (5).

$$f(\tau) = \alpha m \tau^{m-1} \exp(-\alpha \tau^m) \tag{4}$$

$$\tilde{F}(\tau) = \exp(-\alpha \tau^m) \tag{5}$$

Risk factors and estimation approach for Weibull parameters

Risk factors

The corrosion process of the pipeline is affected by many internal and external factors. As mentioned before, the influential factors include material yield stress, length, radius, pipe wall thickness, traffic load, thermal expansion coefficient, internal fluid pressure and many others. These factors should be considered as either deterministic or random variables with specific mean and variance depending on the availability of gathered data and information. Evidently, these factors proportionally contribute to the deterioration level with different variation (Ahammed & Melchers 1997). It is, therefore, understandable to propose an integrated risk factor κ in the form of probability value. Estimation of risk factor can be retrieved from several physical models. Further expression of the hazard function considering the risk factor κ is as follows:

$$\lambda(\tau) = \kappa \alpha m \tau^{m-1} \tag{6}$$

The probability density function $f(\tau)$ in Equation (4) and survival function in Equation (5) $\tilde{F}(\tau)$ are further expressed as:

$$f(\tau) = \kappa \alpha m \tau^{m-1} \exp(-\kappa \alpha \tau^m) \tag{7}$$

$$\tilde{F}(\tau) = \exp(-\kappa \alpha \tau^m) \tag{8}$$

A further notice in the case of using the risk factor is that κ should be used for respective records available in the data set.

Covariates

In addition to the risk factor, another popular approach in addressing the impacts and correlations of characteristic variables (or covariates) is to consider location parameter α in additive form of covariates.

$$\alpha = \sum_{i=1}^{M} \beta_i x_i (i = 1, \ldots, M) \tag{9}$$

where m is total number of covariates and the value of the first covariate equals 1 as a constant value. Depending on the availability of the database, numbers of covariates are selected into the numerical calculation.

Estimation approach for Weibull parameter

It is assumed that the total number of recorded data is S, which is relatively equivalent to the entire length of the pipelines system. Each record refers particularly for s ($s = 1, \ldots, S$) unit of length (possibly in metres or kilometres). This type of separation is often found for the convenience of management of each city. Equations (4) and (5) are thus in the following formulas:

$$f(t_s) = \alpha m t_s^{m-1} \exp(-\alpha t_s^m) \tag{10}$$

$$\tilde{F}(t_s) = \exp(-\alpha t_s^m) \tag{11}$$

Deterioration of section s is supposed to be mutually independent from other parts of the pipelines system. For this reason, the simultaneous probability density of the deterioration is expressed in the following likelihood function:

$$\begin{aligned} L(\alpha, m : t_s) &= \prod_{s}^{S} \{\tilde{F}(t_s^m)\}^{(1-\delta_s)} \{f(t_s^m)\}^{\delta_s} \\ &= \prod_{s}^{S} \{\exp(-\alpha t_s^m)\}^{(1-\delta_s)} \left\{\alpha m t_s^{m-1} \exp(-\alpha t_s^m)\right\}^{\delta_s} \end{aligned} \tag{12}$$

in which δ_s is dummy variable receiving a value of 1 when leakage was encountered and 0 otherwise. For ease of mathematical manipulation, logarithm for both sides of Equation (12) is preferred. Thus, rewriting Equation (12), the log-likelihood function can be given by:

$$\begin{aligned} \ln L(\alpha, m : t_s) = \sum_{s}^{S} [(1 - \delta_s)(-\alpha t_s^m) \\ + \delta_s\{\ln \alpha + \ln m + (m - 1) \ln t_s - \alpha t_s^m\}] \end{aligned} \tag{13}$$

In order to obtain the two parameters α and m, the maximum likelihood estimation method is used.

The estimator of parameter value θ which maximizes the logarithmic likelihood function (13) is given as $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ $(\theta_1 = \alpha, \ \theta_2 = m)$ and must simultaneously satisfy the following condition:

$$\frac{\partial \ln L(\Xi, \hat{\theta})}{\partial \theta_i} = 0, (i = 1, 2) \tag{14}$$

Furthermore, the estimated value $\sum \theta$ of the asymptotic covariance matrix of the parameter can be expressed as follows:

$$\sum(\hat{\theta}) = \left[\frac{\partial \ln L(\Xi, \theta)}{\partial \theta \partial \theta'}\right]^{-1} \tag{15}$$

The optimal value of $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)$ are then estimated by applying a numerical iterative procedure such as the Newton method for simultaneous Equation (15) with two dimensions. This study employs the Newton-Raphson method. The statistical t-test is calculated by use of covariance matrix value $\sum \theta$.

Formulation of the optimal renewal interval model

The occurrence of an incident results in a certain amount of social cost, which is assumed to be a constant number, C. The expected social cost $EC(z)$ is estimated by use of the predetermined interval of renewal z. Thus, its value is followed in the probabilistic manner via the probability density function $f(\tau)$ defined in Equation (4). Over time, counting from the time of pipe burial or previous renewal, the expected social cost would be in the integral form as expressed in the following equation:

$$EC(z) = \int_0^z Cf(t) \exp(-\rho t) dt \tag{16}$$

The coefficient ρ is the instantaneous discounted rate of money over time, while another constant amount of money denoted as I is spent on renewal activities, which is subject to either the occurrence of an incident at time τ or the age of the pipeline reaching time z. It is therefore important to note that the renewal cost, when the age of the pipeline becomes z, must take the survival probability $\tilde{F}(\tau)$ into its calculation. Consequently, the present discounted cost of the next pre-determined renewal time $EL(z)$ can be expressed in the following form:

$$EL(z) = \int_0^z If(t) \exp(-\rho t) dt + \tilde{F}(z) I \exp(-\rho z) \tag{17}$$

The expected life-cycle cost (LCC) after the next renewal time is evaluated as the net present value of social costs, and renewal costs. As the social and renewal costs are fixed values, the expected LCC alters to be equal for every renewal time. In other words, the expected LCC at the next renewal time is equal to the expected LCC estimated at the present renewal. The expected LCC, denoted as $J(0{:}z)$, can be regulated through the regression estimation shown in Equation (18).

$$J(0:z) = \int_0^z f(t)\{c + I + J(0:z)\} \exp(-\rho t) dt + \tilde{F}(z)\{I + J(0:z)\} \exp(-\rho z) \tag{18}$$

The optimal value function $\Phi(0)$ can be expressed as the minimum expected LCC evaluated at the initial time:

$$\Phi(0) = \min_z \{J(0:z)\} \tag{19}$$

The estimation for the optimal interval z^* from Equation (19) can be handled.

Empirical study

Overview of empirical study

The water distribution network of Osaka city is approximately 5,000 km in length. The entire water distribution system comprises four distinct types, belonging to classes A, C, F and FL. The three types C, F and FL are the old cast iron types, which were buried 40 years or more ago. At present, those pipes (C, F and FL) are no longer manufactured. Type A is ductile cast iron with innovative material composition. For empirical analysis, type A is assumed to be the best type of pipe for replacement of the old types C, F and FL.

At the Osaka Municipal Waterworks Bureau, a pipeline information management system (a mapping system) has been utilized from 1999. The pipeline information such as the completion drawing in form to link to the pipeline

Table 1 | Estimation results for parameters of Weibull functions: types of pipeline

Pipeline type	Without covariate α	Without covariate m	With covariate β_1	With covariate β_2	With covariate m
C	1.11×10^{-5}	2.496	2.51×10^{-6}	1.49×10^{-4}	2.484
	(28.528)	(30.275)	(6.402)	(19.666)	(34.909)
	5,053.724		4,031.92		
F	2.55×10^{-5}	2.293	4.92×10^{-6}	3.25×10^{-4}	2.288
	(46.256)	(48.825)	(9.337)	(32.944)	(56.613)
	13,523.84		11,154.64		
FL	1.81×10^{-5}	2.4	6.73×10^{-6}	1.22×10^{-4}	2.391
	(14.537)	(15.432)	(4.375)	(7.365)	(17.79)
	1,331.98		1,114.08		
A	8.87×10^{-5}	1.907	8.27×10^{-6}	4.18×10^{-4}	2.144
	(29.416)	(31.38)	(10.117)	(26.642)	(35.865)
	6,654.54		5,588.27		

Note: Figures in parenthesis are *t*-test values; the figures in the third row for each type are the AIC values.

drawing of the city in pipe materials, coupling shape, a diameter, installation annual and an accident history are managed. Based on this information, the duration of survival before the pipeline is damaged is expressed using a Weibull hazard model. Firstly, about the social loss, follow consideration about the exchange priority of the pipeline of Taniguchi *et al.* (2004). Next, the exchange expense of the pipeline assumed it the results value of past exchange expense. In addition, by the following empirical study, pipeline equal to or less than pipe diameter $d = 300$ which information is recorded in pipeline information management system becomes an analysis object.

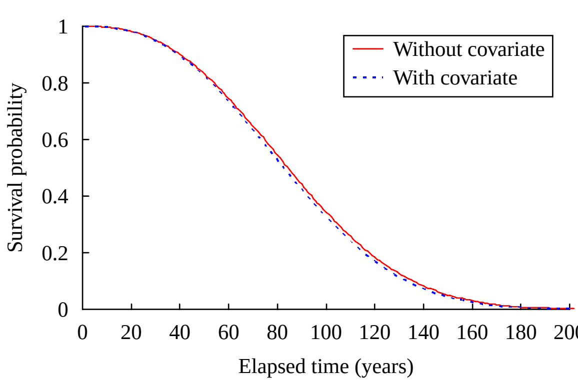

Figure 2 | Survival probability: pipeline type C.

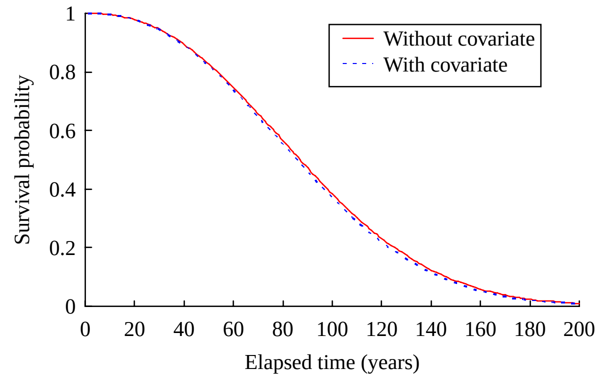

Figure 3 | Survival probability: pipeline type F.

Estimation results

The parameters α and m of the embedded hazard function are estimated by the maximum likelihood method with historical sectional records for each type of pipeline. Values of α and m are then verified with significant degree of t-test values. Table 1 presents the results of estimation for two comparative cases. In the first case, explanation variables were excluded from the estimation. In the second case, the effective length as a characteristic variable was considered in the estimation. Regarding the second case, as presented in Table 1, unknown parameter β_1 is a constant term with a value of 1 for characteristic variable x_1.

Unknown parameter β_2 refers to the effective length of the pipeline system. In this study, other characteristic variables, which reflect the influence of outer and inner rust, soil unit weight, top traffic volume and so on, were neglected because of their small impact or because data was unavailable. The value in parentheses in Table 1 refers to

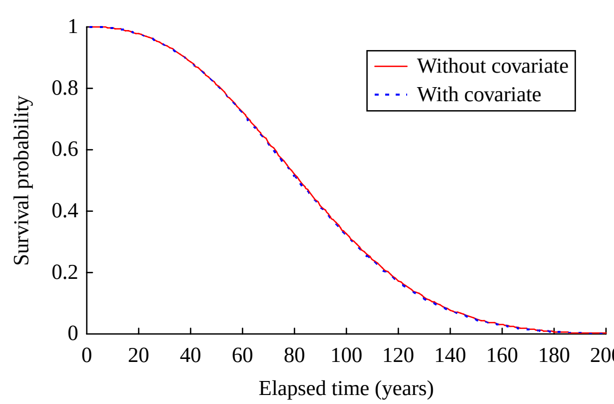

Figure 4 | Survival probability: pipeline type FL.

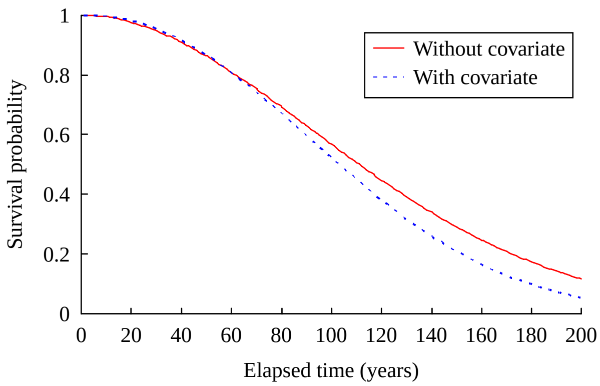

Figure 5 | Survival probability: pipeline type A.

the value of the statistical t-test. It is realized that, from the t-test value, the effective length of pipeline cannot be denied to have substantial effects on the deterioration process. This conclusion is further explained by comparison of AIC (Akaike Information Criteria; Akaike 1974) values. AIC values of the case considering effective length of pipeline are lower than the case without that covariate in the estimation.

$$AIC = -2^{*} \ln(\text{likelihood}) + 2^{*}K \tag{20}$$

However, as can be proved from Figures 2, 3 and 4, the differences in the decrease of survival probability over time are not significant for both cases of pipeline types C, F and FL. A considerable variation between two survival probability curves is realized only for pipeline type A shown in Figure 5.

Since the largest sampling population has been accumulated for pipeline type A (more than 15,000 data points), it can be concluded that the impact of effective pipeline length has a tendency to increase with the larger size of sampling population.

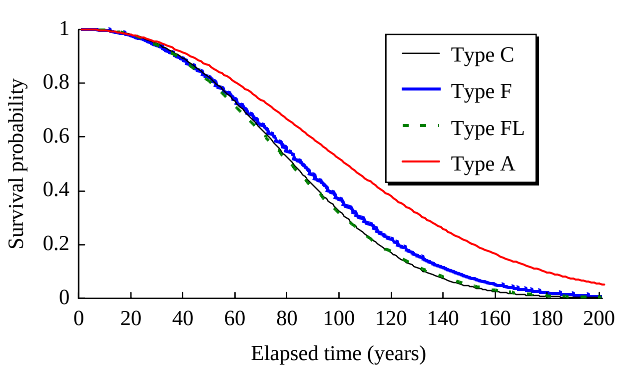

Figure 6 | Comparison of survival probability among the different types of pipeline.

A comparison of the survival probability curve of each pipeline type is drawn in Figure 6. As can be understood from this figure, pipeline types C and FL show a more rapid decrease than pipeline type F. However, all three of the older pipeline types have a 0.5 probability of being broken after 80 years in operation. On the other hand, pipeline type A has much longer life expectancy than the others.

The investigation into optimal renewal time and expected life-cycle cost is carried out in the second phase after obtaining the values for the parameters of the Weibull hazard function and the associated cost parameters. The optimal duration z^{*} is empirically analysed. It should be recognized that the optimal renewal duration is in the range of 50 to 60 years for the older types of pipeline and about 80 years for type A.

CONCLUSIONS

Water pipeline systems are underground, and it is difficult to detect the deterioration situation by conventional inspection. Following a leakage incident, a great amount of social expense occurs because of adverse impacts on road traffic. Therefore, it becomes necessary to proactively replace the pipelines in order to prevent such losses. Thus, in this study, we propose a methodology in which the deterioration process of the pipeline was expressed based on the data such as leakage incidents using the Weibull hazard model. Furthermore, we formulate a mechanism for life-cycle cost evaluation. Thus, the model enables the estimation not only for deterioration speed of pipelines but also for optimal renewal time with respect to least life-cycle cost analysis. The empirical study was conducted on the dataset for the water distribution system in Osaka City. From the standpoint of optimal renewal, this model can be extended to apply to other systems with similar characteristics.

REFERENCES

Ahammed, M. & Melchers, R. 1997 Probabilistic analysis of underground pipelines subject to combined stresses and corrosion. *J. Eng. Struct.* **19**(12), 988–994.

Akaike, H. 1974 A new look at the statistical model identification. *Autom. Control, IEEE* **19**(6), 716–723.

Aoki, K., Yamamoto, K. & Kobayashi, K. 2005 The estimation of the hazard model for deterioration predictions. *J. Civil Eng. JSCE* **791**(VI-67), 111–124. (in Japanese).

Durango, P. & Madanat, S. 2002 Optimal maintenance and repair policies for infrastructure facilities under uncertain deterioration rates: an adaptive control approach. *Transport. Res. A* **36**, 763–778.

Eckles, J. E. 1968 Optimal maintenance with incomplete information. *Oper. Res.* **16**, 1058–1067.

Heyman, D. P. & Sobel, M. J. (eds) 1990 *Stochastic Models, Handbooks in Operations Research and Management Science*, (vol. 2). North-Holland, Amsterdam.

Lancaster, T. 1990 *The Econometric Analysis of Transition Data*. Cambridge University Press, Cambridge.

Taniguchi, Y., Tanaka, H., Murakami, H. & Matsushita, T. 2004 A move method to be concerned with the decision of the improvement priority of the small diameter aging pipe watching from the viewpoint of prevention of water pipe accident. *48th Kansai JWWA Research Conference and Symposium*, Japan Water Works Association Kansai Branch, pp. 5–8.

Tsuda, H., Kaito, K., Aoki, K. & Kobayashi, K. 2005 Estimating Markov transition probabilities for bridge deterioration forecasting. *J. Civil Eng. JSCE* **801**(I-73), 69–82. (in Japanese).

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