Evaluation of the Traction Integral

An approach has been discussed for determining the shear stresses within an elastohydrodynamic film as a function of lubricant parameters and contact conditions. For many types of Newtonian lubricant models, these shear-stress expressions were in closed form where the only required numerical treatment is in converting these shear stresses into total tractions (i.e., equation (12)). A further improvement would, of course, be to eliminate this final numerical step for the simple model. Consequently, a method for approximating the traction integral for the model of equation (1) has been employed.

APPENDIX A

Appendix A

Fig. 5 Traction-function $g(\beta)$ for approximating forces


Appendix B

Approximation for Large Values of $\gamma_P$. For large values of $\gamma_P$, the parameter $\Lambda$ will be large and equation (16) may be approximated by

$$\tau = \mu_0 \exp \left( \frac{\gamma_P}{\Lambda h} \right) \frac{U_2 - U_1}{4} \ln (2A).$$

If it is assumed that this expression for shear stress predominates over the shear stresses in the contact zone, then equation (12) can be solved to yield

$$\psi = \frac{\mu_0 \delta}{8K} |U_2 - U_1|.$$

Higher Approximations for Large Values of $\Lambda$. A somewhat better asymptotic approximation may be obtained by Laplace method [20] to provide the result

$$F_T = \frac{3a}{h} \frac{2K\mu_0}{h} g(\beta)$$

and

$$g(\beta) = \int_0^\infty \exp \left( -u^2 \right) \sinh^{-1} \left( \beta \exp \left( -a^2 \right) \right) du.$$

The parameter $g(\beta)$ is presented as a function of $\beta$ in Fig. 5. It can be noted that both equations (10) and (20) overpredict the tractive forces whereas the lower of the two predictions will be the more valid. This form of approximation should be reasonably valid for values of $\gamma_P > 0$.

The maximum values of the shear stress occurs when $\beta = 2.2$ and is given by

$$F_{T_{\text{max}}} = 8.55 \sqrt{\frac{K\mu_0}{\gamma_P}} \exp \left( \frac{\gamma_P}{2} \right).$$

DISCUSSION

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The authors have extended the work of Crook [8] to include the use of more generalized functional relationships between shear stress, shear rate, pressure and temperature. Very thin EHD films (less than 10 $\mu$m.) may be treated adequately by considering the lubricant to be isothermal provided the solid boundaries are also isothermal. However, when the film thickness becomes much greater than 10 $\mu$m., appreciable temperature gradients may exist within the film and it is in this region that the authors’ procedure is most useful.

Recent work (reference [13] and the paper by the discussor) has shown that at high shear rates the assumption of a Newtonian model for the lubricant implies a viscosity-pressure relationship which at low pressures obeys the exponential law but at some pressure between 50,000 and 150,000 psi, the exponent is reduced to a much smaller value. This reduction is necessary to prevent the computed shear stresses from becoming impossibly large at high pressures. The pressure at which the exponent

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changes appears to be a function of shear rate. Therefore, for a squeeze film with no relative sliding the constant exponent may be valid but, for high shear rates, a modified exponent is necessary.

The authors' procedure may be combined with more sophisticated rheological models than those used heretofore and better correlation between experimental traction forces taken under a wide range of conditions may result. In this manner the authors' analysis may provide a means of synthesizing rheological models for various lubricants. This may well be the only method for determining rheological behavior since conventional viscometers are inadequate for measurements at the very high pressures and shear rates encountered in sliding Hertzian contacts.

A. Gu

The authors are to be congratulated for their contribution to the studies of the traction problem in elastohydrodynamic contacts. This problem has been the least understood area in elastohydrodynamic lubrication. Its main difficulty is the lack of knowledge on the lubricant behavior under the extremely complex contact zone conditions, such as high pressure, short duration time, and high shear rate. Rigorous methods, e.g., the method by Cheng [21] mentioned in the paper, based on the lubricant data taken under low pressure and static conditions have failed to predict traction correctly. The most important task in attacking the elastohydrodynamic traction problem is, therefore, the determination of the lubricant properties which correspond to the contact zone conditions. This involves very difficult and complicated experiments. Frequently an experiment can simulate only one, or several, but not all of those contact zone conditions [22, 23].

Before comprehensive experiments become available to determine lubricant properties under the onerous elastohydrodynamic contact conditions, it is the discussor's opinion that one of the following two approaches may be taken to study the elastohydrodynamic traction problem and to obtain data for the design of the lubricant property experiments. The first approach is to develop a simple but realistic traction analysis method by adopting a reasonble shear rate-stress or viscosity model of the lubricant with some flexibility in terms of certain free parameters. By studying the variation of these parameters which result from fitting the model to the available experimental data, some useful information about the lubricant behavior under the contact zone conditions may be extracted. Hopefully, but not very likely, a universal fit to the traction data over the entire range of sliding speeds and under different contact conditions might be obtained. The paper being discussed here is an excellent example of this type of approach. It is hoped, therefore, that exploration of this method to obtain further useful information will be continued.

The other type of approach to the elastohydrodynamic tractions is to carry out a systematic traction data analysis coupled to the necessary traction experiments. The traction experiments are comparatively simpler than those to determine the lubricant properties and the technique is known. The main purpose of this approach is to extract lubricant property information and to identify the relative importance of the effects of short time, high shear rate, etc., under different contact conditions. This discussor has tried to use this viscosity model to fit the Johnson and Cameron data and examined the discrepancy between the traction data obtained by their experiments and by Crook's simple thermal friction theory [25] to demonstrate the effects of contact duration time, shear rate, and thermal heating. The published data was found insufficient for the purpose of identifying the relative importance of these effects and conclusions could not be drawn. Therefore, further traction experiments are necessary and should be designed by bearing this purpose in mind in determining the range of operating conditions.

In the simplified traction analysis, the authors reduced the two dimensional momentum-energy coupled contact zone problem to a quasi one dimensional problem in y by considering the energy equation alone. The only dependence of the problem on x is through the contact pressure distribution, which is prescribed to be Hertzian (Assumption 2). Considerations of conservation of momentum and continuity are replaced by several assumptions. However, Assumption 2 and Assumption 4 (shear stress is constant across the film) are contradictory. The momentum equation, assuming inertia-free, is

\[ \frac{\partial p}{\partial x} = \frac{\partial T}{\partial y} \]

Therefore if the pressure distribution is Hertzian, the shear stress must vary with respect to y.

In Fig. 3 the comparison of the Crook's data with the simplified traction method based on two different viscosity models indicates that the exponentially temperature-dependent viscosity model, equation (1), is more realistic than the other viscosity model, equation (13), since the latter shows a wrong trend at high sliding speeds. The value of the constant \( \gamma \) to obtain the best data fitting for the viscosity model, equation (1), is 11.3 which is shown in Fig. 3. The pressure viscosity coefficient for most mineral oils is, however, only of the order of 10\(^{-4}\) in\(^2\)/lb based on the data taken at static and low pressure conditions. It appears that the constant \( \gamma \) loses its original physical meaning, even in the low pressure region in the constant zone, and acts only as a parameter to best fit the data. It would be more desirable and more informative to choose a viscosity model such that the parameters retained their original physical meanings and the model checked with experimental evidence in the limit of low pressure and long time.

In line with the first approach to the elastohydrodynamic traction problems mentioned above, Dr. H. S. Cheng and the discussor have selected a composite pressure-velocity model incorporating Crook's simple thermal friction method [25]. The composite viscosity model is characterized by the following pressure-velocity function:

\[ \ln \frac{\mu}{\mu_0} = \alpha p + \beta(T - T_0) = \xi \quad \text{for} \ p < p^* \]

\[ -\xi + \alpha'p + \beta(T - T_0) \quad \text{for} \ p > p^* \]

where \( \alpha \) and \( \beta \) are the pressure and temperature viscosity coefficients, respectively. The second pressure viscosity coefficient \( \alpha' \) and the break pressure \( p^* \) are the two free parameters. This model was initiated by the analysis of the apparent viscosity by Johnson and Cameron [24] based on their traction data. In the analysis they show such break point behavior in the pressure viscosity plot. The above composite viscosity model has the property that in the low pressure region the viscosity is proportional to \( \exp \) as is observed in experiments at low pressures and long time. \( \alpha' \) and \( p^* \) are considered to account for the deviation from the low pressure behavior at high pressures and short time. The discussor has tried to use this viscosity model to fit the Johnson and Cameron data by adjusting \( \alpha' \) and \( p^* \), but universality fails to exist. This is due to the complicated coupling of the various non-Newtonian effects. However, this model will be very useful in extracting information on lubricant properties under the elastohydrodynamic contact conditions. For example, the position of \( p^* \) is closely related to the short time effect. The shorter the contact duration time, the smaller the value of \( p^* \). This is discussed by Constantinacu, et al [23]. The reduction ration, defined as \( \alpha'/\alpha \), is an indication of the extent of the various kinds of non-Newtonian effects. The relative importance of the non-
Newtonian effects in elastohydrodynamic contacts can be identified, therefore, from a study of variations of the reduction ratio and the break pressure with respect to different contact conditions, provided there are sufficient traction data.

Additional References


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The authors have presented a technique which attempts to predict the traction in EHD contacts using a simplified analysis incorporating a general shear rate function $s(T, P, \tau)$. We contend that in order to make the analyses simple, the authors made assumptions leading to violations of at least one of the fundamental laws. Assumption (2) of the paper states that a Hertzian pressure profile is assumed but assumption (4) states that the shear stress does not vary across the lubricant film. Neglecting inertia terms, the $x$-component of the momentum equation for two-dimensional flow is

$$\frac{\partial P(x)}{\partial x} = \frac{\partial \tau_{xy}}{\partial y}$$

(1)

The left-hand side is zero only at the bearing centerline ($x = a$). In fact, the average absolute value of $\partial P/\partial x$ on each side of the centerline of a typical EHD contact is on the order of $10^5$ lbf/in.$^2$. Under such conditions, how can $\tau_{xy}$ be assumed independent of $y$? The assumption of a constant $\tau_{xy}$ allows equation (5) of the paper to be integrated to obtain equation (8) of the paper

$$K \frac{\partial T}{\partial y} = -\tau(u - U)$$

(2)

Without this assumption one could obtain only

$$K \frac{\partial T}{\partial y} = -\int_0^y \tau_{xy}(y) \frac{\partial u}{\partial y} dy = -\int_0^y \tau(y) du$$

(3)

In addition to requiring the $\tau = \text{constant}$ in order to obtain equation (2) above, the authors have assumed that

$$u = \frac{1}{2} (U_1 + U_2) \equiv U$$

(4)

at $y = 0$.

The idea of a constant center line ($y = 0$) velocity also contradicts the momentum equation. Under the authors' assumptions, a Newtonian lubricant would have a viscosity described by

$$\mu = \mu(T(y), P(x)) = \mu(\tau, y)$$

(5)

the momentum equation (1) then becomes

$$\frac{\partial P(x)}{\partial x} = \frac{\partial}{\partial y} \left[ \mu(\tau, y) \frac{\partial u}{\partial y} \right]$$

(6)

Since $\partial P/\partial x$ is not a function of $y$, equation (6) may be integrated to obtain

$$\frac{\partial P(x)}{\partial x} y + c_i(x) = \mu(\tau, y) \frac{\partial u}{\partial y}$$

(7)

or

$$\frac{\partial u}{\partial y} = \frac{1}{\mu(\tau, y)} \left( \frac{\partial P(x)}{\partial x} y + c_i(x) \right) = F(x, y)$$

(8)

or

$$u = \int F(x, y) dy + c_i(x)$$

(9)

Therefore, the velocity $u$ must be a function of both $x$ and $y$. The authors have stated, however, that

$$u(x, 0) = \frac{1}{2} (U_1 + U_2) = \text{const}$$

(10a)

$$u(x, h/2) = U_1 = \text{const}$$

(10b)

$$u(x, -h/2) = U_1 = \text{const}$$

(10c)

We doubt that physically possible velocity profiles can be drawn for various values of $x$ ($0 < x < 2a$) such that both equations (9) and (10) are satisfied.

We are also concerned about equation (17) of the paper which is said to represent a Maxwell model and was taken from reference (18) by Tanner. Tanner's development of that equation made use of a number of non-trivial assumptions (i.e., $U\mu/KG$ must be much less than one, but for typical numbers used in the paper this parameter is of order 10).

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