Duality Diagrams and Sum Rules

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We have written the sum rules for the amplitudes of scattering process which does not have legal diagram in the sense of quark structure of baryons and mesons. These sum rules are well satisfied when they are evaluated by taking contributions from all the known single-particle intermediate states.

§ 1. Introduction

Recently Harari and Rosner made use of the following physical assumptions to draw the legal diagram for the scattering process:

(a) All baryons can be described as a three-quark structure.

(b) All mesons can be described as a quark-antiquark structure.

(c) Except for the contribution of the Pomeranchukon, the total scattering amplitude in any channel can be described as a sum of single-particle states.

They found that the following processes cannot be represented by legal diagrams: $K^-B\rightarrow\pi^-B'$, $\pi^+B\rightarrowK^0B'$, $K^+B\rightarrowK^0B'$ and $K^-B\rightarrowM^0B$, where $B, B'$ are any nonexotic baryons and $M^0$ is any $Q=Y=0$ meson which does not contain a $\Lambda\bar{\Lambda}$ component. This concludes that at small $t$ (=square of four-momentum transfer between the two mesons or the two baryons) values the amplitudes of these scattering processes should be purely real at high energy or the average contribution from the $s$-channel intermediate states to the scattering amplitudes should be zero. Thus for

$$K^-p\rightarrow\pi^-\Sigma^+$$

(1)

we can write the following sum rules

$$\frac{1}{\xi} \int_0^\infty \text{Im} A_{t=0}(s, t=0)ds = 0,$$

(2)

$$\frac{1}{\xi} \int_0^\infty \text{Im} A_{t=0}(s, t=0)ds = 0,$$

(3)

$$\frac{1}{\xi} \int_0^\infty \text{Im} A_{t-1}(s, t=0)ds = 0,$$

(4)
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\[
\frac{1}{\bar{s}} \int_0^s \text{Im} \, B^{t=\bar{t}}(s, t=0) \, ds = 0,
\]

where \( \bar{s} \) is the square of the cutoff energy. The purpose of this paper is to test the validity of these sum rules.

§ 2. Calculations

The sum rules \((2) \sim (5)\) are evaluated by taking contributions from all the known\(^9\) intermediate states. Contributions from \(A(1115), Y_0^*(1405), \Sigma(1190)\) and \(Y^*_1(1385)\) evaluated from Born terms are given below, and contributions from the other intermediate states above threshold, evaluated from the Breit-Wigner terms, are given in Table I.

\[
A_{Iz=0} = \left( \frac{m_x + m_N}{2} - m_A \right) g_{A \pi K} G_{A^* \pi^-} / (s - m_A^2 + i\epsilon),
\]

\[
B_{Iz=0} = g_{A \pi K} G_{A^* \pi^-} / (s - m_A^2 + i\epsilon),
\]

\[
A_{Iz=0}^{\mu=0} = \frac{(m_Y - m_x)}{2} g_{A \pi K} G_{A^* \pi^-} / (s - m_Y^2 + i\epsilon),
\]

\[
B_{Iz=0}^{\mu=0} = g_{A \pi K} G_{A^* \pi^-} / (s - m_Y^2 + i\epsilon),
\]

\[
A_{Iz=1} = \left( \frac{m_x - m_z}{2} \right) g_{A \pi K} G_{A^* \pi^-} / (s - m_z^2 + i\epsilon),
\]

\[
B_{Iz=1} = g_{A \pi K} G_{A^* \pi^-} / (s - m_z^2 + i\epsilon),
\]

\[
A_{Iz=1}^{\mu=0} = ag_{A \pi K} G_{A^* \pi^-} / (s - m_Y^2 + i\epsilon),
\]

\[
B_{Iz=1}^{\mu=0} = bg_{A \pi K} G_{A^* \pi^-} / (s - m_Y^2 + i\epsilon),
\]

where

\[
a = \left( m_Y - m_x \right) + \frac{1}{2} m_x \left( -m_x^2 + m_N^2 \right) + \frac{1}{3} m_N m_x + \frac{E_1 m_x - E_2 m_N}{3} + \frac{2}{3} E_1 E_2
\]

\[
+ \frac{1}{2} m_N \left( -m_N^2 + m_x^2 + 2 E_1 E_2 \right) - \frac{1}{3} \left[ E_1 \left( m_x^2 - s + \frac{1}{2} m_N m_x \right) + m_N \left( \frac{1}{2} m_N E_1 + m_x^2 - s - \frac{1}{2} m_N m_x \right) \right],
\]

\[
b = -\frac{1}{3} \left[ \frac{3}{2} m_x^2 + m_N^2 - 2 E_1 E_2 + m_x E_1 + m_N E_2 + m_N m_x \right],
\]

with

\[
E_1 = \frac{s + m_N^2 - m_x^2}{2m_Y},
\]
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\[ E_n = \frac{s + m^2 - m^2}{2m_{Y^*}}. \]

On taking the contribution from all the known intermediate states, the sum rules (2) \(\sim\) (5) reduce, respectively, to

\[ \begin{align*}
-0.05 (g_{A^0K^-} A^0 K^- s^-/4\pi) + 2.47 (g_{Y^* (1405) pK^- Y^* (1405) s^-}/4\pi) &= 1.6430, \\
(g_{A^0K^-} A^0 K^- s^-/4\pi) + (g_{Y^* (1405) pK^- Y^* (1405) s^-}/4\pi) &= -2.4593, \\
-0.125 (g_{Y^* pK^-} Y^* pK^- s^-/4\pi) + 0.3633 (g_{Y^* (1385) pK^- Y^* (1385) s^-}/4\pi) &= -0.4595, \\
(g_{Y^* pK^-} Y^* pK^- s^-/4\pi) - 1.5274 (g_{Y^* (1385) pK^- Y^* (1385) s^-}/4\pi) &= 0.3947. 
\end{align*} \]

Solving Eqs. (6) and (7) and Eqs. (8) and (9) separately, we get

\[ \begin{align*}
g_{A^0K^-} A^0 K^- s^-/4\pi &= -3.07, \\
g_{Y^* (1405) pK^- Y^* (1405) s^-}/4\pi &= 0.60, \\
g_{Y^* (1385) pK^- Y^* (1385) s^-}/4\pi &= -3.24, \\
g_{Y^* (1385) pK^- Y^* (1385) s^-}/4\pi &= -2.38. 
\end{align*} \]

### § 3. Discussion

The SU(3) values of various coupling constants\(^{6-9}\) are: \(g_{NN}/4\pi = 14.7, g_{N^0K} = (1/\sqrt{3})(1 + 2\alpha)g_{NN}, g_{A^0K} = (2/\sqrt{3})(1 - \alpha)g_{NN}, g_{\Delta K} = (1 - 2\alpha)g_{NN}, g_{\Sigma K} = -2\alpha g\) and \(\alpha = 0.4\). These give \(g_{A^0K^-} A^0 K^- s^-/4\pi = -10.58\) and \(g_{Y^* pK^-} Y^* pK^- s^-/4\pi = -2.35\). Model dependent values of coupling constants are: \(g_{Y^* (1405) pK^- Y^* (1405) s^-}/4\pi = 0.5\) (Ref. 7) and \(g_{Y^* (1385) pK^- Y^* (1385) s^-}/4\pi = -5.61\) (SU(3) value), \(-3.66\) (Ref. 8)), \(-3.02\) (Ref. 9)). Thus we find that our values of various coupling constants are in agreement with the other model dependent respective values. The agreement of the values of these coupling constants with other model dependent calculations shows that the amplitude for the scattering process of illegal diagram is purely real. Schmidt\(^9\) has also used this result to write the correct Veneziano amplitudes for \(\pi N \rightarrow KA\) and \(\pi N \rightarrow K\Sigma\) scattering processes.

Biswas et al.\(^{11}\) have written the superconvergence sum rule for \(B_{t=0} \approx 0\) and have found that the sum rule is well satisfied when saturated with low-lying intermediate states. But they have used\(^{15,13}\) \(g_{\Sigma K} = 2\alpha g_{NN}\) and \(g_{\Delta K} = (1 - 2\alpha)g_{NN}\), which give \(g_{\Sigma K}, g_{\Delta K}\) a positive value. If instead the value is taken to be negative, then one will have to take the contribution from the high mass resonances because the decay width of higher \(Y^*\) resonances to \(KN\) is quite appreciable.

It is clear from Table I that the contributions of the resonances to the invariant amplitudes do not decrease with the increase of the mass of the resonance. This does not allow us to fix the value of cutoff energy. At present we can say that the sum rules must take contributions from pair of resonances.
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Table I. Resonance Parameters\(^5\) and their contributions to different amplitudes. \(\Gamma\) = Total decay width, \(x_t\) = Fraction to \(KN\) decay in % and \(x_f\) = Fraction to \(\Sigma\pi\) decay in %.

<table>
<thead>
<tr>
<th>Resonances</th>
<th>(I(J^P))</th>
<th>(\Gamma) in MeV</th>
<th>(x_t)</th>
<th>(x_f)</th>
<th>Contribution to (\frac{1}{4\pi^2} \int \text{Im} A(s, t=0) ds)</th>
<th>Contribution to (\frac{1}{4\pi^2} \int \text{Im} B(s, t=0) ds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_0^*(1518.8))</td>
<td>0((3^-/2))</td>
<td>16</td>
<td>45</td>
<td>45</td>
<td>-0.4418</td>
<td>0.9144</td>
</tr>
<tr>
<td>(Y_0^*(1670))</td>
<td>0((1^-/2))</td>
<td>25</td>
<td>14</td>
<td>45</td>
<td>0.0135</td>
<td>0.0049</td>
</tr>
<tr>
<td>(Y_0^*(1890))</td>
<td>0((3^-/2))</td>
<td>40</td>
<td>25</td>
<td>35</td>
<td>-0.4158</td>
<td>0.6148</td>
</tr>
<tr>
<td>(Y_0^*(1815))</td>
<td>0((5^-/2))</td>
<td>75</td>
<td>65</td>
<td>11</td>
<td>-0.9570</td>
<td>1.1068</td>
</tr>
<tr>
<td>(Y_0^*(1830))</td>
<td>0((5^-/2))</td>
<td>80</td>
<td>10</td>
<td>35</td>
<td>1.0104</td>
<td>-0.8304</td>
</tr>
<tr>
<td>(Y_0^*(2100))</td>
<td>0((7^-/2))</td>
<td>140</td>
<td>30</td>
<td>4</td>
<td>-0.9532</td>
<td>0.6488</td>
</tr>
<tr>
<td>(Y_1^*(1760))</td>
<td>1((3^-/2))</td>
<td>50</td>
<td>8</td>
<td>50</td>
<td>-0.4564</td>
<td>0.7020</td>
</tr>
<tr>
<td>(Y_0^*(1765))</td>
<td>1((5^-/2))</td>
<td>100</td>
<td>46</td>
<td>1</td>
<td>0.2909</td>
<td>-0.2907</td>
</tr>
<tr>
<td>(Y_1^*(1915))</td>
<td>1((5^-/2))</td>
<td>60</td>
<td>10</td>
<td>0.4</td>
<td>-0.0614</td>
<td>0.0615</td>
</tr>
<tr>
<td>(Y_1^*(2030))</td>
<td>1((7^-/2))</td>
<td>120</td>
<td>10</td>
<td>10</td>
<td>0.6864</td>
<td>-0.8675</td>
</tr>
</tbody>
</table>

with the same orbital angular momentum, parity and approximately the same mass but different total angular momentum as suggested by Sakita and Wali\(^40\) for their superconvergence sum rules.

Acknowledgments

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References

6) We have adopted the convention advocated by L. Levi Setti, in his report to the 1969 Lund Conference.