Implications of Duality for $KK$ Resonances and $A_2$ Splitting

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Recently Capps and Dixit\(^1\) have shown that even if the basic condition of local duality of King and Wali\(^2\) is weakened to one of semilocal duality, many resonances must still be accompanied by nearly degenerate subsidiary resonances. These methods have so far been applied to meson-baryon scattering by saturating the sum rules by the direct channel resonances corresponding to either the exotic $u$ or $t$ channel. The weakened method is simpler and can be more readily compared with experiment. In what follows we extend the idea to meson-meson scattering. The partial $KK$ decay width of $A_2$ meson is predicted to be $14\sim15$ MeV irrespective of whether $A_2$ is single or double structure so that the question on this resonant state in current literature may be clarified by the detailed study of the branching probability of $A_2$ in $KK$ decay.

The assumptions are that (i) the semi-local duality applies to different isospin amplitudes separately, (ii) the imaginary
Letters to the Editor 1993

part of the backward (forward) integrated amplitude with a definite isospin in a certain range of energy saturates the sum rule if the corresponding amplitude in the crossed \( u(t) \) channel belongs to the exotic state, (iii) the range of energy of integration is approximately \( \Delta s = 2(\alpha')^{-1} \approx 2 \text{(GeV)}^2 \), and (iv) the direct channel amplitudes may be approximated by the resonance states in the narrow width approximation.

The third assumption (iii) may not be considered to be a rigid one and be renounced later (see later discussion). A further implication along this line of thought is discussed in connection with higher spin states.

The amplitude for \( \bar{K}K \) scattering may be expanded in terms of partial waves,

\[
\sum_{J} (2J+1) f_J(r_{0,1}) P_J(\cos \theta),
\]

where \( s \) is the total energy square, \( J \) is the total angular momentum, the superscript \( I \) is the isospin and \( \theta \) is the scattering angle.

In general the partial wave amplitude \( f_J(s) \) may be approximated by the single level Breit-Wigner formula (or by multilevel formula if necessary)

\[
f_J(s) = \frac{1}{q} \frac{\Gamma_{(1c')}^{1/2} \Gamma_{(1c)}^{1/2}}{M_s^2 - s} \frac{1}{2M_i - i(\Gamma_i/2)},
\]

where \( q \) is the magnitude of the three-momentum in CM, \( \Gamma_{(1c')} \) is the partial width with level \( \lambda \) and channel index \( c(c') \) of the incident (outgoing) channel, \( M_i \) is the resonant energy and \( \Gamma_i \) is the total width of that state. The Mandelstam variables \( s, t \) and the four-scalar \( \nu = \frac{1}{2} (\rho + \rho') \times (q + q') \) are simply related by \( s = 2m_K^2 + 2\nu - 4t \).

The semilocal duality for backward \( \bar{K}K \) scattering may simply be expressed in the narrow width approximation by

\[
\langle \text{Im} f^I(s, \theta = \pi) \rangle = \sum_{J} (-)^J (2J+1) \frac{\Gamma_{(1)K}\Gamma_{(1)K}}{q_J} \text{Im} f^J(s, \theta = \pi) = \pi \sum_{J} (-)^J (2J+1) \frac{\Gamma_{(1)K}\Gamma_{(1)K}}{q_J} = 0,
\]

where \( N_0 \) and \( N \) are appropriate values to be fixed so as to satisfy the semilocal (or local) duality. We consider that their range may better be specified by examining the experimental data in connection with the sum rule (3).

The result of the calculation is indicated in Table I. The resonance below threshold, such as \( \omega(784) \), does not contribute to the imaginary part of the amplitude as far as the narrow width approximation is assumed, since the analytic continuation of the amplitude below threshold can be performed by replacing \( q \to q' \) in Eq. (2) which leads only the real part. It is reported that the \( \bar{K}K \) decay mode in \( \pi_N(1016) \) is seen. If we assume that it is at most a few per cent, we find the number listed for this state, e.g. for 3%, for definiteness.

Looking through the table in relation to Eq. (3), it seems natural to conclude that the saturation of the sum rule occurs for each of the adjacent pairs, viz. \( J = 2n, 2n+1 \) \((n=0, 1, 2, \cdots)\) by assuming that no low angular momentum states (daughter states) couple strongly with the \( \bar{K}K \) channel. This means that the range of energy is less than (a few hundred MeV)\(^2\), viz. the local duality holds approximately. The proposed conjecture suggests that the \( I=0, J^P=3^- \) states should be observed so as to cancel the contribution from \( f \) and \( f' \) mesons. If we assume that these 3- states appear as the Regge recurrences of the degenerate \( \omega-f \) and its parallel \( f' \) with masses 1600 and 1810 MeV, respectively, the sum of the \( \bar{K}K \) decay widths of these two resonant states is approximately 50 MeV. If we further generalize the idea to high \( n \) values, the difference of kinematical factors is not important, so that the \( \bar{K}K \) branching ratios
for the \( I=0 \) and \( I=1 \) adjacent pairs are separately equal for fixed \( n \) approximately. The part of the statements discussed above is consistent with the sum rule for the observed \( I=1 \) states, as will be seen below.

If we exclude \( \pi_N(1016) \) from the \( I=1 \) state sum rule we get

\[
\Gamma_{\bar{K}K}(A2) = 15 \text{ MeV}
\]

for a single \( A2(1300) \), (4)

but

\[
\Gamma_{\bar{K}K}(A2_L) + 0.96 \Gamma_{\bar{K}K}(A2_H) = 15 \text{ MeV}
\]

for double peaks, (5)

where the level index is specified by bracket notation for convenience. Actually \( \pi_N(1016) \) width for \( \bar{K}K \) decay is very small, and the inclusion of it modifies the numbers in Eqs. (4) and (5) to 14.7 and 14.4 MeV, respectively. Thus the bubble chamber study of the \( \bar{K}K \) decay branching probability of \( A2 \) will hopefully clarify the \( A2 \) structure in relation to the above prediction. At the present stage the sum of the observed total widths of \( A2_L \) and \( A2_H \) is 40 MeV\(^4\) so that the possible double peak structure cannot be excluded immediately.

There still remains the question how accurate the simple local sum rule is. However, the experimental errors in the meson decay widths are much smaller than those in the baryon ones; thus the sum rule will surely be tested better in the example considered. The question whether the 60 (70) % accuracy is insufficient for clarifying the \( A2 \) ambiguity or not should be consulted with experiment.

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3) A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30 (1958), 257.